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By Fitri maya Puspita

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Analytic Approach of Internet Pricing Scheme Model Based on function of Bandwidth Diminished with Increasing Bandwidth

Hartati Hartati[1,*], Fitri Maya Puspita[2] and Indrawati Indrawati[2]

[1] *Open University, Palembang South Sumatera Indonesia*
[2] *Sriwijaya University, Inderalaya Ogan Ilir South Sumatera Indonesia*

Abstract— Along with the changing times, the internet is becoming essential for everyday life. As the internet service providers, ISPs are required to provide a good service so as to give satisfaction to consumers at a low price. However, ISPs are expected to not only take into account customer satisfaction, but also take into account the advantages gained by considering all the factors that exist. Therefore, ISPs are given the option of pricing schemes, namely flat fee, usage-based, and two part tariff pricing schemes to be applied to the utility function as a function of bandwidth decreases with increasing bandwidth to maximize the benefits ISPs with regard to the level of user satisfaction. This study analyzed the two types of customers, namely homogeneous and heterogeneous consumers. Consumers are divided into heterogeneous consumer of willingness to pay (high end and low end) and with different consumption levels (high demand and low demand). In the case of consumers with homogeneous and heterogeneous consumer of willingness to pay (high end and low end), optimal pricing scheme is obtained if the ISP uses a flat fee and a two-part tariff schemes. As for heterogeneous consumers with different consumption levels (high demand and low demand), the scheme of two part tariff is the optimal scheme to generate maximum profits.

Keywords—Pricing scheme; Flat fee; Usage based; Two-part tariff; Optimal solution.

I. Introduction

Over time, the development of technology with the development of internet services is in balance. Internet Service Provider (ISP) is required to compete to provide maximum service but at minimum cost, because at minimum cost will increase consumer interest to use the service offered. However, as the company's internet service provider, ISPs also have to consider the profits and maintaining the quality of the service with minimum prices for consumers.

ISPs are trying to provide the best service with the best pricing scheme anyway. However, there is now a wide variety of user demand internet and various applications that make the Internet providers must take into account the quality of service (Quality of Service, QoS). Basically, QoS makes it possible to provide better service to a particular request. In demonstrating efficiency in service, it is a must for ISP to show interaction between price and QoS [1].

Not only ISP, there should be a separate consideration for consumers who use the internet service there must be compatibility between the prices given to customer satisfaction. As customer satisfaction can be viewed by using the utility function. Utility function describes the level of customer satisfaction in consuming a product [2]. Internet Service Provider (ISP) is currently handling a high demand to promote good quality information. But a new initiative to develop a new price that involves QoS is only a few [3-5]. Therefore, it is important to consider the ISP customer satisfaction in creating a scheme that will make the relationship between user satisfactions with the QoS provided by ISPs.

By taking into account all the factors of the providers and consumers then, three types of pricing schemes are most often used, namely a flat fee, usage-based, and the two-part tariff [6, 7] will be further analyzed. Analysis of these strategies is divided into two parts that considers homogeneous and

[*] Corresponding author can be contacted via the journal website.

heterogeneous customers. In the case of a homogeneous, all customers have the same utility on the level of consumption per day whereas in the case of heterogeneous, customers have two segments according to willingness to pay (willingness to pay) and the level of consumption (consumption level). Different analysis using different utility function has been introduced to show the variety of utility function applied to obtain new sight on choosing the satisfaction of the user both analytically or iteratively [8, 9] Furthermore, in the article, a modification of the other utility function except Cobb-Douglas utility function will be discussed. This research will seek better schemes by using a utility function as a function of bandwidth which decreases with increasing bandwidth since this utility function is the best utility function which deals with the various level of QoS [10-12] .

II. Research Methodology

The steps undertaken in this study are as follows:

1. Formulate a utility function as a function of bandwidth which decreases with increasing bandwidth (functions of bandwidth diminished with increasing bandwidth) based on three types of pricing schemes which are flat fee, usage-based, and the two-part tariff for two types of customers, namely homogeneous and heterogeneous consumers ,
2. Conduct an analysis of the utility function selected in order to obtain a model scheme based on the utility function.
3. Comparing the schematic model derived from the analysis.
4. Make the appropriate conclusions and comparative analysis obtained.

III. Result and Discussions

In next steps, we define the parameter and variables involved as follows.

P	:	Costs incurred if the following services rendered
P_x	:	Price given service provider during peak hours
P_y	:	Price given the current service providers off-peak hours
$\bar{X}_i$	:	The highest level of consumer *i* in using the service during peak hours
$\bar{Y}_i$	:	The highest level of consumer *i* in using the service when it is not rush hour
$X_i^* = X_i(P_X,P_Y,P)$	:	The level of customer service *i* consumption during peak hours
$Y_i^* = Y_i(P_X,P_Y,P)$	:	The consumption level of customer service *i* not at peak hours
$Z_i^* = Z_i(P_X,P_Y,P)$	:	The variable *i* consumer purposes with respect to its participation
$U_i(X_i,Y_i)$	:	The utility function of consumer *i* with X_i is the level of service usage during peak hours and Y_i is the level of service usage when not busy hours

In this case utility function as a function of bandwidth which decreases with increasing bandwidth of [11] will be applied:

$$U_{kj} = U_{0j} + W_j \ln \frac{X_{kj}}{L_{mj}} \quad (1)$$

Class J is divided into classes at busy times (x) and a class that is not at peak hours (y) thus obtained:

$$U_{kx} = U_{0x} + W_x \ln \frac{X_{kx}}{L_{mx}} \quad (2)$$

$$U_{ky} = U_{0y} + W_y \ln \frac{X_{ky}}{L_{my}} \quad (3)$$

with

$$U_0 = U_{0x} + U_{0y} \quad (4)$$

$$W_x = a \text{ and } W_y = b$$

$$X_{kx} = X \text{ and } X_{ky} = Y$$

$$L_{mx} = X_m \text{ and } L_{my} = Y_m$$

So it will become as follows.

$$U(x,y) = U_{0x} + W_x \ln \frac{X_{kx}}{L_{mx}} + U_{0y} + W_y \ln \frac{X_{ky}}{L_{my}} \quad (5)$$

$$U(x,y) = U_0 + a \ln \frac{X}{X_m} + b \ln \frac{Y}{Y_m}$$

With a few changes to simplify the calculation, then Eq. (5) is changed to:

$$U(x,y) = U_0 + a \ln \frac{X+1}{X_m+1} + b \ln \frac{Y+1}{Y_m+1} \quad (6)$$

This change was made to simplify the calculation when the minimum consumption level, X_m and Y_m and the level of consumption during peak hours and off-peak hours respectively, X dan Y can produce a minimum value of 0 rather than making value becomes negative.

A. Homogeneous Consumer

In the case of homogeneous consumers, it was assumed that all consumers have the same level of satisfaction and the maximum the same level of use in peak hours and in off-peak hours, i.e. $\bar{X}$ and $\bar{Y}$.

The optimization of consumer problem will be as follows:

$$\max_{X,Y,Z} U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - P_X X - P_Y Y - PZ \quad (7)$$

Subject to

$$X \le \bar{X}Z \quad (8)$$

$$Y \le \bar{Y}Z \quad (9)$$

$$U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - P_X X - P_Y Y - PZ \ge 0 \quad (10)$$

$$Z = 0 \; or \; 1$$

The optimization of supplier problem will be as follows:

$$\max_{P,P_X,P_Y} \sum_i (P_X X_i^* + P_Y Y_i^* + P Z_i^*) \quad (11)$$

with $(X^*, Y^*, Z^*) = \text{argmax } U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - P_X X - P_Y Y - PZ$ (12)

subject to

$$X \le \bar{X}Z \quad (13)$$

$$Y \le \bar{Y}Z$$

$$U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - P_X X - P_Y Y - PZ \ge 0 \quad (14)$$

$$Z = 0 \; or \; 1$$

The selection of the best scheme is determined by analysis of derivatives as follows:

Lemma 1: If ISPs use flat fee pricing scheme, the fee paid will be $U_0 + a\ln\frac{\bar{X}+1}{X_m+1} + b\ln\frac{\bar{Y}+1}{Y_m+1}$ and the maximum profit will be $\sum_i \left(U_0 + a\,ln\frac{\bar{X}+1}{X_m+1} + b\,ln\frac{\bar{Y}+1}{Y_m+1}\right)$.

Proof of Lemma 1

If the ISP chooses to use a flat fee pricing scheme then PX = 0, $P_Y = 0$, dan $P > 0$ will be set. With the enactment of these provisions, the level of consumer spending will be $X = \bar{X}$ dan $Y = \bar{Y}$ so Eq. (7) will be as follows.

$$\max_{X,Y,Z} U_0 + a\ln\frac{\bar{X}+1}{X_m+1} + b\ln\frac{\bar{Y}+1}{Y_m+1} - PZ \quad (15)$$

According to Eq. (14), it will become

$$U_0 + a\ln\frac{\bar{X}+1}{X_m+1} + b\ln\frac{\bar{Y}+1}{Y_m+1} - P \ge 0 \quad (16)$$

In order to keep the maximum level of customer satisfaction then the upper limit of Equation (16) is so $P = U_0 + a\ln\frac{\bar{X}+1}{X_m+1} + b\ln\frac{\bar{Y}+1}{Y_m+1}$. Then the fees charged on a flat fee pricing scheme is set at $U_0 + a\ln\frac{\bar{X}+1}{X_m+1} + b\ln\frac{\bar{Y}+1}{Y_m+1}$ and the maximum profit will be $\sum_i \left(U_0 + a\ln\frac{\bar{X}+1}{X_m+1} + b\ln\frac{\bar{Y}+1}{Y_m+1}\right)$.

Lemma 2: If ISPs use usage-based pricing scheme, the optimal price is $P_X = \frac{a}{\bar{X}+1}$ and $P_Y = \frac{b}{\bar{Y}+1}$ with the maximum profit of $\sum_i \left(a\left(1-\frac{1}{\bar{X}+1}\right) + b\left(1-\frac{1}{\bar{Y}+1}\right)\right)$.

Proof of Lemma 2

If ISP choose usage based pricing scheme then $P_X > 0, P_Y > 0$, dan $P = 0$. So Eq. (7) will be

$$\max_{X,Y,Z} U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - P_X X - P_Y Y - 0 \cdot Z$$

$$= \max_{X,Y,Z} U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - P_X X - P_Y Y \quad (17)$$

To maximize Eq. (17), ISP should lower the value of P_X and P_Y. Firstly, to optimize the consumer problem, we derive Eq. (17) to x and y.

$$\frac{a}{X^*+1} - P_X = 0 \Leftrightarrow \frac{a}{X^*+1} = P_X \text{ then } X^* = \frac{a}{P_X} - 1 \quad (18)$$

$$\frac{b}{Y^*+1} - P_Y = 0 \Leftrightarrow \frac{b}{Y^*+1} = P_Y \text{ then } Y^* = \frac{b}{P_Y} - 1 \quad (19)$$

The optimization of supplier problem will be:

$$\max_{P_X,P_Y} \sum_i (P_X X^* + P_Y Y^*) = \max_{P_X,P_Y} \sum_i (a - P_X + b - P_y) \quad (20)$$

Since X and Y are restricted so X^* and Y will be $\bar{X}$ and $\bar{Y}$. In other words, P_X and P_Y will be $P_X = \frac{a}{\bar{X}+1}$ and $P_Y = \frac{b}{\bar{Y}+1}$ with maximum profit of $\sum_i \left(a - \frac{a}{\bar{X}+1} + b - \frac{b}{\bar{Y}+1}\right) = \sum_i \left(a\left(1-\frac{1}{\bar{X}+1}\right) + b\left(1-\frac{1}{\bar{Y}+1}\right)\right)$.

Lemma 3: If ISP applies two part tariff pricing scheme, the optimal price will be $P_X = \frac{a}{\bar{X}+1}$, $P_Y = \frac{b}{\bar{Y}+1}$ and $P = U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - \frac{a\bar{X}}{\bar{X}+1} - \frac{b\bar{Y}}{\bar{Y}+1}$ with the maximum profit of $\sum_i \left(U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1}\right)$.

Proof of Lemma 3

If ISPs chooses two part tariff pricing scheme, then the requirements are $P_X > 0$, $P_Y > 0$, and $P > 0$. Eq. (7) will be

$$\max_{X,Y,Z} U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - P_X X - P_Y Y - P \quad (21)$$

We derive the Eq. (21) toward x and y to maximize P_X and P_Y so it will be Eq. (18)-Eq.(19).

The optimization of supplier problem will be as follows.

$$\max_{P,P_X,P_Y} \sum_i (P_X X^* + P_Y Y^* + PZ^*) = \max_{P,P_X,P_Y} \sum_i \left(P_X\left(\frac{a}{P_X} - 1\right) + P_Y\left(\frac{b}{P_Y} - 1\right) + P\right)$$

$$= \max_{P,P_X,P_Y} \sum_i (a - P_X + b - P_y + P) \quad (22)$$

since X and Y are restricted then, X^* and Y^* will be $\bar{X}$ and $\bar{Y}$. P_X and P_Y will be $P_X = \frac{a}{\bar{X}+1}$, $P_Y = \frac{b}{\bar{Y}+1}$ dan $P = U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1} - \frac{a\bar{X}}{\bar{X}+1} - \frac{b\bar{Y}}{\bar{Y}+1}$. So the prices charged according to usage of the consumer will be $a\left(1 - \frac{1}{\bar{X}+1}\right) + b\left(1 - \frac{1}{\bar{Y}+1}\right) = \frac{a\bar{X}}{\bar{X}+1} + \frac{b\bar{X}}{\bar{Y}+1}$ with the maximum profit of $\sum_i \left(U_0 + a\ln\frac{X+1}{X_m+1} + b\ln\frac{Y+1}{Y_m+1}\right)$.

Then the type of flat fee and a two-part tariff pricing schemes generate huge profits equally according to maximize profit-making for the ISP (supplier) while the usage-based pricing scheme is different from the two schemes.

B. Heterogeneous Consumer: High-End and Low-End

Lemma 4: If ISP applies flat fee pricing scheme then the price set up to consumers will be $U_{02} + a_2\ln\frac{\bar{X}+1}{X_m+1} + b_2\ln\frac{\bar{Y}+1}{Y_m+1}$ with the maximum profit of $(m+n)\left(U_{02} + a_2\ln\frac{\bar{X}+1}{X_m+1} + b_2\ln\frac{\bar{Y}+1}{Y_m+1}\right)$.

Proof of Lemma 4

Appling flat fee scheme, then we have $P_X = 0$, $P_Y = 0$, dan $P > 0$. If the consumers choose to join the services, then the maximum level of satisfaction can be determined by choosing the level of consumption of $X_1 = \bar{X}$, $Y_1 = \bar{Y}$ or $X_2 = \bar{X}$, $Y_2 = \bar{Y}$. This makes ISP can give the prices for each high-end consumer of no more than $U_{01} + a_1\ln\frac{X_1+1}{X_m+1} + b_1\ln\frac{Y_1+1}{Y_m+1}$ and low end consumer of no more than $U_{02} + a_2\ln\frac{X_2+1}{X_m+1} + b_2\ln\frac{Y_2+1}{Y_m+1}$. So, it is easier to assume that $a_1 < \frac{m+n}{m}a_2$ and $b_1 < \frac{m+n}{m}b_2$. Then, ISP will set the price of $U_{02} + a_2\ln\frac{\bar{X}+1}{X_m+1} + b_2\ln\frac{\bar{Y}+1}{Y_m+1}$ to high-end consumers and low-end consumers make profit to ISP for $(m+n)\left(U_{02} + a_2\ln\frac{\bar{X}+1}{X_m+1} + b_2\ln\frac{\bar{Y}+1}{Y_m+1}\right)$.

Lemma 5: If ISP applies the usage based pricing scheme, then the optimal price will be $P_X = \frac{a_2}{\bar{X}+1}$ and $P_Y = \frac{b_2}{\bar{Y}+1}$, the maximum profit is $(m+n)\left(\frac{a_2\bar{X}}{\bar{X}+1} + \frac{b_2\bar{Y}}{\bar{Y}+1}\right)$.

Proof Lemma 5

If ISP chooses to use usage based pricing scheme then we set $P_X > 0$, $P_Y > 0$, dan $P = 0$. Then, Eq. (23) will be

$$\left(U_{01} + a_1\ln\frac{X_1+1}{X_m+1} + b_1\ln\frac{Y_1+1}{Y_m+1} - P_x X_1 - P_y Y_1 - 0\cdot Z_1\right) + \left(U_{02} + a_2\ln\frac{X_2+1}{X_m+1} + b_2\ln\frac{Y_2+1}{Y_m+1} - P_x X_2 - P_y Y_2 - 0\cdot Z_1\right)$$

$$= \left(U_{01} + a_1\ln\frac{X_1+1}{X_m+1} + b_1\ln\frac{Y_1+1}{Y_m+1} - P_x X_1 - P_y Y_1\right) + \left(U_{02} + a_2\ln\frac{X_2+1}{X_m+1} + b_2\ln\frac{Y_2+1}{Y_m+1} - P_x X_2 - P_y Y_2\right) \quad (26)$$

Derive the Eq. (26) to optimize the consumer problem toward X_1, X_2, Y_1 and Y_2.

$$\frac{a_1}{X_1^*+1} - P_X = 0 \Leftrightarrow \frac{a_1}{X_1^*+1} = P_X \text{ then } X_1^* = \frac{a_1}{P_X} - 1 \quad (27)$$

$$\frac{b_1}{Y_1^*+1} - P_Y = 0 \Leftrightarrow \frac{b_1}{Y_1^*+1} = P_Y \text{ then } Y_1^* = \frac{b_1}{P_Y} - 1 \quad (28)$$

$$\frac{a_2}{X_2^*+1} - P_X = 0 \Leftrightarrow \frac{a_2}{X_2^*+1} = P_X \text{ then } X_2^* = \frac{a_2}{P_X} - 1 \quad (29)$$

$$\frac{b_2}{Y_2^*+1} - P_Y = 0 \Leftrightarrow \frac{b_2}{Y_2^*+1} = P_Y \text{ then } Y_2^* = \frac{b_2}{P_Y} - 1 \quad (30)$$

The supplier problem will be

$$\max_{P_X,P_Y} m(P_X X_1^* + P_Y Y_1^*) + n(P_X X_2^* + P_Y Y_2^*)$$

$$= \max_{P_X,P_Y} m(a_1 - P_X + b_1 - P_Y) + n(a_2 - P_X + b_2 - P_Y) \quad (31)$$

To maximize Eq.(31), ISP should lower the value of P_X and P_Y. Since X_1, X_2, Y_1, dan Y_2 are restricted, then X_1^*, X_2^*, Y_1^*, and Y_2^* cannot be more than $\bar{X}$ and $\bar{Y}$. For seeking the optimal price, first, we do the analysis on peak hours.

The supplier optimization problem will be as follows.

$$\max_{P_X} m(P_X X_1^*) + n(P_X X_2^*) = \max_{P_X} m(a_1 - P_X) + n(a_1 - P_X) \quad (32)$$

To maximize Eq. (32) the supplier should lower the value of P_X so the best value of P_X cannot be greater than $\frac{a_1}{\bar{X}+1}$. In other words, if the supplier gives the price below $\frac{a_2}{\bar{X}+1}$ than we do not obtain maximum profit since the values of X_1^* and X_2^* are not greater than $\bar{X}$ and the consumer demand will increase toward the decrement of the price. So, the best price P_X should be between $\frac{a_1}{\bar{X}+1}$ and $\frac{a_2}{\bar{X}+1}$. When the price is between that interval, high-end consumer demand will stay in $\bar{X}$ will for low-end consumer will increase proportionally to the decrement of the price. Then, P_X and P_Y will be $P_X = \frac{a_2}{\bar{X}+1}$ and $P_Y = \frac{b_2}{\bar{Y}+1}$ and the maximum profit is $(m+n)\left(\frac{a_2\bar{X}}{\bar{X}+1} + \frac{b_2\bar{Y}}{\bar{Y}+1}\right)$.

Lemma 6: If ISP uses two part tariff scheme, the optimal price will be $P_X = \frac{a_2}{\bar{X}+1}$, $P_Y = \frac{b_2}{\bar{Y}+1}$ and $P =$

$U_{02} + a_2 \, ln \frac{\bar{X}+1}{X_m+1} + b_2 \, ln \frac{\bar{Y}+1}{Y_m+1} - \frac{a_2 \bar{X}}{\bar{X}+1} - \frac{b_2 \bar{X}}{\bar{Y}+1}$ and the maximum profit is $(m+n)\left(U_{02} + a_2 \ln \frac{\bar{X}+1}{X_m+1} + b_2 \ln \frac{\bar{Y}+1}{Y_m+1}\right)$.

Proof of Lemma 6

If ISP chooses to adopt two part tariff pricing scheme, then it is set that $P_X > 0, P_Y > 0$, and $P > 0$. Then, Eq. (23) be:

$$\left(U_{01} + a_1 \ln \frac{X_1+1}{X_m+1} + b_1 \ln \frac{Y_1+1}{Y_m+1} - P_x X_1 - P_y Y_1 - P\right) + \left(U_{02} + a_2 \ln \frac{X_2+1}{X_m+1} + b_2 \ln \frac{Y_2+1}{Y_m+1} - P_x X_2 - P_y Y_2 - P\right) \quad (33)$$

First, to optimize the consumer problem, we need to derive Eq. (33) toward X_1, X_2, Y_1 and Y_2 so we have Eq. (27)-Eq. (30). After that, we will have P and P_Y whose value between $\frac{a_1}{\bar{X}+1}$ and $\frac{a_2}{\bar{X}+1}$ So, the best P_X is between $\frac{a_1}{\bar{X}+1}$ and $\frac{a_2}{\bar{X}+1}$. When the price is between that interval, high end consumer demand will remain the same in $\bar{X}$ whereas the low-end consumer demand will stay proportional to decrement of the value. Then, P_X and P_Y will be $P_X = \frac{a_2}{\bar{X}+1}$, $P_Y = \frac{b_2}{\bar{Y}+1}$ and $P = U_{02} + a_2 \ln \frac{\bar{X}+1}{X_m+1} + b_2 \ln \frac{\bar{Y}+1}{Y_m+1} - \frac{a_2 \bar{X}}{\bar{X}+1} - \frac{b_2 \bar{X}}{\bar{Y}+1}$. Assume that $a_1 < \frac{m+n}{m} a_2$ and $b_1 < \frac{m+n}{m} b_2$, then the maximum profit will be $(m+n)\left(U_{02} + a_2 \ln \frac{\bar{X}+1}{X_m+1} + b_2 \ln \frac{\bar{Y}+1}{Y_m+1}\right)$.

Then the type of financing schemes of flat fee and a two-part tariff generate huge profits equally in maximizing profit-making for the ISP (supplier) while the usage-based financing scheme obtain different value from the two schemes.

C. *Heterogeneous Consumer: High Demand and Low Demand*

Lemma 7: If ISP chooses flat fee, then the price paid will be $P = U_{02} + a \ln \frac{\overline{X_2}+1}{X_m+1} + b \ln \frac{\overline{Y_2}+1}{Y_m+1}$ and the maximum profit will be $(m+n)\left[U_{02} + a \, ln \frac{\overline{X_2}+1}{X_m+1} + b \, ln \frac{\overline{Y_2}+1}{Y_m+1}\right]$.

Proof of Lemma 7

For flat fee scheme, we set $P_X = 0$, $P_Y = 0$, dan $P > 0$. If consumers choose to join the service then the maximum satisfaction level will be obtained by choosing consumption level $X_1 = \overline{X_1}, Y_1 = \overline{Y_1}$ or $X_2 = \overline{X_2}$, $Y_2 = \overline{Y_2}$ with maximum satisfaction level $U_{01} + a \ln \frac{\overline{X_1}+1}{X_m+1} + b \ln \frac{\overline{Y_1}+1}{Y_m+1}$ or $U_{02} + a \ln \frac{\overline{X_2}+1}{X_m+1} + b \ln \frac{\overline{Y_2}+1}{Y_m+1}$. So, ISP cannot charge the price greater than $U_{01} + a \ln \frac{\overline{X_1}+1}{X_m+1} + b \ln \frac{\overline{Y_1}+1}{Y_m+1}$ to every high demand consumer and also to low demand consumer with $U_{02} + a \ln \frac{\overline{X_2}+1}{X_m+1} + b \ln \frac{\overline{Y_2}+1}{Y_m+1}$. Since applying flat fee scheme, then ISP cannot differentiate the price between high demand and low demand consumers. So ISP should choose to charge $U_{01} + a \ln \frac{\overline{X_1}+1}{X_m+1} + b \ln \frac{\overline{Y_1}+1}{Y_m+1}$ so the high demand consumer can apply the service with the price of $U_{02} + a \ln \frac{\overline{X_2}+1}{X_m+1} + b \ln \frac{\overline{Y_2}+1}{Y_m+1}$ then high demand and low demand consumers can join the service. If we assume that $m\left[U_{01} + a \ln \frac{\overline{X_1}+1}{X_m+1} + b \ln \frac{\overline{Y_1}+1}{Y_m+1}\right] < (m+n)\left[U_{02} + a \ln \frac{\overline{X_2}+1}{X_m+1} + b \ln \frac{\overline{Y_2}+1}{Y_m+1}\right]$, the best price that can be charged by ISP will be $U_{02} + a \ln \frac{\overline{X_2}+1}{X_m+1} + b \ln \frac{\overline{Y_2}+1}{Y_m+1}$ for high and low demand consumers. So, the maximum profit obtained by ISP will be $(m+n)\left(U_{02} + a \ln \frac{\overline{X_2}+1}{X_m+1} + b \ln \frac{\overline{Y_2}+1}{Y_m+1}\right)$.

Lemma 8: If ISP uses the usage based scheme then the price will be $P_X = \frac{a}{\overline{X_1}+1}$ and $P_Y = \frac{b}{\overline{Y_1}+1}$ also the maximum profit is $m\left(\frac{a\overline{X_1}}{\overline{X_1}+1} + \frac{b\overline{Y_1}}{\overline{Y_1}+1}\right) + n\left(\frac{a\overline{X_2}}{\overline{X_1}+1} + \frac{b\overline{Y_2}}{\overline{Y_1}+1}\right)$.

Proof of lemma 8

Choosing the usage based scheme means that $P_X > 0$, $P_Y > 0$, and $P = 0$. First order derivative for both consumers are as follows.

$$\frac{a}{X_1^*+1} - P_X = 0 \Leftrightarrow \frac{a}{X_1^*+1} = P_X \Leftrightarrow X_1^* = \frac{a}{P_X} - 1 \quad (34)$$

$$\frac{b}{Y_1^*+1} - P_Y = 0 \Leftrightarrow \frac{b}{Y_1^*+1} = P_Y \Leftrightarrow Y_1^* = \frac{b}{P_Y} - 1 \quad (35)$$

$$\frac{a}{X_2^*+1} - P_X = 0 \Leftrightarrow \frac{a}{X_2^*+1} = P_X \Leftrightarrow X_2^* = \frac{a}{P_X} - 1 \quad (36)$$

$$\frac{b}{Y_2^*+1} - P_Y = 0 \Leftrightarrow \frac{b}{Y_2^*+1} = P_Y \Leftrightarrow Y_2^* = \frac{b}{P_Y} - 1 \quad (37)$$

The optimization problem of supplier will be

$$\max_{P_X, P_Y} m(P_X X_1^* + P_Y Y_1^*) + n(P_X X_2^* + P_Y Y_2^*)$$

$$= \max m(a_1 - P_X + b_1 - P_Y) + n(a_2 - P_X + b_2 - P_Y) \quad (38)$$

since X_1, X_2, Y_1 and Y_2 are restricted then X_1^*, X_2^*, Y_1^* dan Y_2^* will be $\overline{X_1}$, $\overline{X_2}$, $\overline{Y_1}$ and $\overline{Y_2}$. From Eq. (38), it can be seen that to maximize the profit, ISP should lower the value of P_X and P_Y. Consider the peak hour time. Value of P_X cannot exceed $\frac{a}{\overline{X_2}+1}$. If P_X is set to lower than $\frac{a}{\overline{X_1}+1}$ then the profit cannot be optimal when X_1^* and X_2^* are not greater than $\overline{X_1}$ and $\overline{X_2}$ and also the consumer demand cannot increase due to value decrement. So, the best price for P_X should be between $\frac{a}{\overline{X_1}+1}$ and $\frac{a}{\overline{X_2}+1}$. When the price is between that interval, low demand consumer remains in $\overline{X_2}$ and high demand consumer will increase proportionally to price decrement. The, the best price for P_X will be $\frac{a}{\overline{X_1}+1}$. Also, for off-peak hour time, the price will be $P_Y = \frac{b}{\overline{Y_1}+1}$. So, P_X and P_Y will be $P_X = \frac{a}{\overline{X_1}+1}$ and $P_Y = \frac{b}{\overline{Y_1}+1}$ and the maximum profit is $m\left(\frac{a\overline{X_1}}{\overline{X_1}+1} + \frac{b\overline{Y_1}}{\overline{Y_1}+1}\right) + n\left(\frac{a\overline{X_2}}{\overline{X_1}+1} + \frac{b\overline{Y_2}}{\overline{Y_1}+1}\right)$.

Lemma 9: If ISP uses two part tariff scheme, the price charged will be $P_X = \frac{a}{\overline{X_1}+1}$, $P_Y = \frac{b}{\overline{Y_1}+1}$ and $P = U_{02} + a\ln\frac{X_2+1}{X_m+1} + b\ln\frac{Y_2+1}{Y_m+1} - \frac{a\overline{X_2}}{\overline{X_1}+1} - \frac{b\overline{Y_2}}{\overline{Y_1}+1}$ and the maximum profit is $m\left(a\frac{\overline{X_1}-\overline{X_2}}{\overline{X_1}+1} + b\frac{\overline{Y_1}-\overline{Y_2}}{\overline{Y_1}+1}\right) + (m+n)\left(U_{02} + a\ln\frac{X_2+1}{X_m+1} + b\ln\frac{Y_2+1}{Y_m+1}\right)$.

Proof of Lemma 9

If ISP choose to use two-part tariff scheme, then $P_X > 0, P_Y > 0$, dan $P > 0$.

We use first order derivative to obtain Eq. (34)-Eq. (37). Then the supplier optimization problem will be

$$\max_{P_X,P_Y} m(P_X X_1^* + P_Y Y_1^* + P) + n(P_X X_2^* + P_Y Y_2^* + P)$$

$$= \max m(a - P_X + b - P_Y + P) + n(a - P_X + b - P_Y + P) \quad (39)$$

From Eq. (39) it can be seen that to obtain the maximum profit than the value of P_X and P_Y should be as low as possible so the value of P remains large. So, X_1, X_2, Y_1 and Y_2 are restricted then X_1^*, X_2^*, Y_1^* and Y_2^* will be $\overline{X_1}$, $\overline{X_2}$, $\overline{Y_1}$ and $\overline{Y_2}$. P_X will be between $\frac{a}{\overline{X_1}+1}$ and $\frac{a}{\overline{X_2}+1}$ and so does P_Y that will be between $\frac{b}{\overline{Y_1}+1}$ and $\frac{b}{\overline{Y_2}+1}$. When the price is between that interval, the demand of low demand consumer will stay in $\overline{X_2}$ and the demand of high demand consumer will increase proportionally to value decrement. Then, the best price for P_X will be $\frac{a}{\overline{X_1}+1}$. Then, for the analysis for non $P_Y = \frac{b}{\overline{Y_1}+1}$. In other words. P_X and P_Y will be $P_X = \frac{a}{\overline{X_1}+1}$ dan $P_Y = \frac{b}{\overline{Y_1}+1}$ with $P = U_{02} + a\ln\frac{X_2+1}{X_m+1} + b\ln\frac{Y_2+1}{Y_m+1} - \frac{a\overline{X_2}}{\overline{X_1}+1} - \frac{b\overline{Y_2}}{\overline{Y_1}+1}$. The maximum profit will be $m\left(a\frac{\overline{X_1}-\overline{X_2}}{\overline{X_1}+1} + b\frac{\overline{Y_1}-\overline{Y_2}}{\overline{Y_1}+1}\right) + (m+n)\left(U_{02} + a\ln\frac{X_2+1}{X_m+1} + b\ln\frac{Y_2+1}{Y_m+1}\right)$.

Lastly, from the analysis of high demand and low demand consumers, it can be seen that for each scheme, the maximum profit obtained are different. Due to the differences, we can conclude that two-part tariff achieves maximum(optimal) value.

IV. Conclusions

Based on the analysis conducted applying utility function as a function of bandwidth which decreases with increasing bandwidth by using three types of existing pricing schemes on homogeneous consumers and heterogeneous consumers we obtained optimal financing scheme on each consumer. Homogeneous consumer pricing schemes if ISPs choose to use flat fee or two-part tariff then ISP can obtain optimal profit.

Whereas in the case of heterogeneous customers which have two types, i.e., according to willingness to pay (willingness to pay) and the level of consumption (consumption level). For heterogeneous consumers with a desire to pay (high end and low end) ISP will gain maximum benefit when using flat fee or a two part tariff schemes. For heterogeneous consumers with different consumption levels (high demand and low demand) optimal scheme exists when ISPs choose to use two part tariff scheme of scheme.

The submitting author is responsible for obtaining agreement of all coauthors and any consent required from sponsors before submitting a paper. It is the obligation of the authors to cite relevant prior works in the paper. Authors of rejected papers may revise and resubmit.

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