

# The Comparison of Bundle-Pricing Scheme Models Using Quasi-Linear Utility Function

*By Fitri maya Puspita*

## The Comparison of Bundle-Pricing Scheme Models Using Quasi-Linear Utility Function

Fitri Maya Puspita<sup>1\*</sup>, Evi Yuliza<sup>1</sup>, Muthia Ulfa<sup>1</sup>

**Abstract**— In this paper, we formulate bundle-pricing modified models involving pricing scheme based on quasi-linear utility function as one of internet service provider (ISP) main goal is to maximize their profit. The model formed by setting cost of creating bundle and total reservation price of customer  $i$ 's. LINGO 11.0 is used to solve the models to obtain the optimal solution. The solver result for each case either from original model and modified model are compared to obtain the optimal solution. The result showed that for each case based on 3-pricing schemes which are flat fee, usage based and two-part tariff, ISPs gain the same profit with the original model but ISPs save more time in terms of resources rather than the original model. ISP's may use this model as consideration for setting cost of creating a bundle and total reservation price of customer in maximizing profits and also to provide better service quality for customers with their preferences exactly.

**Keywords**— bundle pricing, utility function, pricing strategies, optimal solution.

### I. INTRODUCTION

Developments in the field of information science and technology impact also on one of the human needs in this modern era, namely the internet. ISP (Internet Service Provider) are now competing in offering products or services that can compete in the market and to provide benefits for the ISP itself.

ISPs often face with the problem of the uncertainty of the customer in choosing a product that bundles present as one way to deal with these problems [1, 2]. Bundling is the sale of two or more products in one package form to attract the customers' attention to buy the product.

Pricing for the internet has been the critical issue for the decades. Many approaches are introduced to highlight the possibility for provider to get higher profit with many types of variables and parameters involved. If we consider base price and quality premium, then the goal of getting maximum profit can be achieved. These parameters were introduced by [3] then to be applied by [4-7] in multi-class and multi-service network.

The price sensitivity for users was discussed to show user's sensitivity for the goods. This terms previously introduced by [8] then applied and improved by [4, 9]. This variable is known to be one important variable to measure the satisfaction of the users over the service or product. The utility function shows the user's preference on buying goods according to their budgets where user's price sensitivity is included in this function as one of the term.

Bundle-pricing models previously been investigated by [1, 2] using lagrangian approach can generate maximum revenue.

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The model that has been established by modifying model, the cost of making bundles and the total cost of the reservation for each bundle is able to produce the optimal solution. Other approach of bundle pricing models was the attempt to solve the model using branch and bound solver by LINGO 13.0 [10]. The results show, the application of branch and bound solver is very useful to achieve higher profit rather than applying lagrangian approach.

Model of bundling involving the utility function is seldom to apply. In fact, by utilizing this variable, providers can also promote the user's satisfaction variable as well as achieve higher profit. That is why, in this paper, we seek to formulate the bundle pricing schemes involving three pricing strategies with the utilizing of utility function. Then, the comparison of the original model of bundle- internet pricing scheme without the utility function and involving utility functions is also discussed. The improved model involving utility functions in consideration of customer satisfaction in every service and product sales using the corresponding pricing scheme with the expectation that service providers are able to compete in the market as well as considering customer satisfaction.

### II. RESEARCH METHODS

LINGO 11.0 is applied to obtain the optimal solution of linear and non-linear equations. The model is established by the parameters and variables that are used to solve the optimal problem. To analyze the case on pricing bundle-we need data. The data used in this study was obtained from one of the local server in Palembang. The optimal solution obtained can help in showing the existing problems involving pricing strategies, bundle price, the level of satisfaction and utility functions.

The model used is introduced in [1, 2], by setting the price of making a bundle ( $B$ ), the marginal cost ( $m$ ), cost of the bundle ( $P$ ) and total of reservation cost ( $R$ ). We proceed the analysis by applying the same model but using different utility function. Then, we obtained two models namely the original model and modified models. On modified models, subdivided into three cases which is financing schemes flat fee, usage-based and two-part tariff.

### III. MODELS

#### A. Original Model Bundling

This model was adopted from [2] as follows

$$\text{Max } R = \sum_{i=1, \dots, I} \sum_{j=1, \dots, J} (P_j - B_j) X_{ij} - \sum_{j=1, \dots, J} M Y_j \quad (1)$$

subject to :

$$S_i \geq (R_i - P_j) Y_j, \quad i = 1, \dots, I; j = 1, \dots, J \quad (2)$$

$$S_i = \sum_{j=1, \dots, J} (R_i - P_j) X_{ij}, \quad i = 1, \dots, I \quad (3)$$

$$(R_i - P_j) X_{ij} \geq 0, \quad i = 1, \dots, I; j = 1, \dots, J \quad (4)$$

$$X_{ij} \leq Y_j, \quad i = 1, \dots, I; j = 1, \dots, J \quad (5)$$

$$\sum_{j=1, \dots, J} X_{ij} \leq 1, \quad i = 1, \dots, I \quad (6)$$

$$S_i \geq 0, i = 1, \dots, I \quad (7)$$

$$P_j \geq 0, j = 1, \dots, J \quad (8)$$

$$X_{it} = 0 \text{ or } 1, i = 1, \dots, I; j = 1, \dots, J \quad (9)$$

$$Y_j = 0 \text{ or } 1, j = 1, \dots, J \quad (10)$$

With the parameters used in the original model, namely :

$R$  : Function for income

$B_j$  : Bundle cost of creating a bundle of  $j$  goods. This may include the sum of marginal production cost, distribution cost, transaction cost, any binding cost, etc. We assume this is the same for any kind of bundles of  $j$  goods.

$i$  : Total  $I$  potential customers in our target market.

$j$  : The vendor has total  $J$  different kinds of information goods in hand.

$M$  : Marginal cost if we add one more bundle choice on the menu.

$V_{ii}$  : Customer's  $i$ 's reservation price for his  $k$ th favorite information goods.

$R_{ii}$  : Total reservation price of customer  $i$ 's top  $j$ 's favorite goods. i.e.,  $R_{ii} = \sum_{k=1, \dots, j} V_{ii}$

$P_j$  : The price assign to the bundle of  $j$  goods.

$S_i$  : Consumer surplus for customer  $i$ .

$X_{ii}$  : The decision variabel which is one if consumer  $i$  chooses to buy the bundle of  $j$  goods, and zero for the otherwise.

$Y_j$  : The decision variabel which is one if the vendor chooses to offer the bundle of  $j$  goods on the menu, and zero for the otherwise.

#### B. Improved Models

This models was developed by [11]. This model actually is the original pricing strategy without involvement of bundle strategies except it already used the utility function. To seek the better solution, we proceed to adapt this model with the addition of new parameters. The parameters used in the improved model, namely

$P$  : Costs to be incurred consumers to participate in this service.

$P_x$  : Unit price set by the service provider not in peak hour

$P_y$  : Unit price set by the service provider in peak hour

$X_{it}$  : Consumer consumption level of service at peak hours

$Y_t$  : The level of consumer spending on services in off-peak hours

$Z_t$  : Decision variables are worth 1 if consumers choose to join the program and be 0 if do not want to join.

$\bar{X}$  : The highest consumption level of consumer  $i$  in using the program during peak hour

$\bar{Y}$  : The highest consumption level of consumer  $i$  in using the program during off-peak hours

next part, the improved models are divided into three pricing schemes, which are flat fee, usage based and two-part tariff schemes, the major differences among these three types of strategies are basically on the unit price setting up by ISP in peak or non-peak hours and the price set up to subscribe the services.

#### Modified Bundle-Pricing Using Quasi Linear Utility Function Case Flat Fee Pricing Scheme

An objective function for bundle-pricing models using Quasi-Linear Utility Function with flat fee pricing scheme given below :

$$\begin{aligned} \text{Max } R = & \sum_{i=1, \dots, I} \sum_{j=1, \dots, J} (P_j - B_j) X_{it} - \sum_{j=1, \dots, J} M Y_j \\ & - (a + f(Y)) + (P_x X_t) + (P_y Y_t) + (P Z_t) \end{aligned} \quad (11)$$

subject to :

$$X_{it} \leq \bar{X}_i Z_t \quad (12)$$

$$Y_t \leq \bar{Y}_t Z_t \quad (13)$$

$$-U_{(X_t, Y_t)} + P_x X_t + P_y Y_t + P Z_t \geq 0 \quad (14)$$

$$Z_t = 0 \text{ or } 1, i = 1, \dots, I \quad (15)$$

$$P_x = 0 \quad (16)$$

$$P_y = 0 \quad (17)$$

$$P > 0 \quad (18)$$

#### Modified Bundle-Pricing Using Quasi Linear Utility Function Case Usage Based Pricing Scheme

An objective function for bundle-pricing models using Quasi-Linear Utility Function with usage based pricing scheme given below

Max Eq. (11)

Subject to Eq. (12)-Eq. (15) and

by modifying the pricing scheme then add constraints :

$$P_x > 0 \quad (19)$$

$$P_y > 0 \quad (20)$$

$$P = 0 \quad (21)$$

#### Modified Bundle-Pricing Using Quasi Linear Utility Function Case Two-Part Tariff Pricing Scheme

An objective function for bundle-pricing models using Quasi-Linear Utility Function with two-part tariff pricing scheme given below :

Max Eq. (11)

Subject to Eq. (12)-Eq. (15) and

by modifying the pricing scheme then add constraints :

$$P_x > 0 \quad (22)$$

$$P_y > 0 \quad (23)$$

$$P = 0 \quad (24)$$

## IV. RESULT AND DISCUSSIONS

To solve the case LINGO 11.0 program are applied. Models created are Integer Nonlinear Programming and Integer Linear Programming depending on the models assigned by using the branch and bound iterative solver built up in LINGO. Based on Objective Function (1) subject to Eq. (2)-(10), the optimal solution for original models proposed by [2] using our solver are shown in the Table 1.

Table 1. Optimal Solution for Original Model

Variables	Value
Model Class	ILP
State	Global Optimal
Objective	56999.8
Infeasibility	0
Iterations	188
GMU	27K
ER	0s

Based on Table 1 the value will achieve the most optimal results of 56999.8. These results will be obtained by iterating as many as 188 iterations with the infeasibility of 0. Generated Memory Used (GMU), the total allocation of

memory used is equal to 27K and Elapsed Runtime (ER) shows the total time used to generate and solve the model that is 0 seconds. The model is integer linear programming (ILP) since the objective function and the constraints are in linear form. Since we deal with ILP, the state condition usually turns out to be global optimal. Next, in Table 2 actually explains the value of decision variables in original model.

Table 2. Variable Values for Original Model

Variables	Value
$P_1$	0.1
$P_2$	31499.9
$B_1$	2000
$B_2$	2000
$X_1$	0
$X_2$	0
$X_3$	1
$X_4$	1
$Y_1$	0
$Y_2$	1
$S_1$	0.1
$S_2$	9800.1

Table 2 shows variables value from the models using Lingo 11.0 program.  $P_1$  and  $P_2$  represent the price assigned to the bundle of  $j$  goods which is equal to 0.1 and 31499.9 successive.  $B_1$  and  $B_2$  represent bundle cost of creating a bundle of  $j$  goods set with value 2000 for each variables.  $X_1$ ,  $X_2$ ,  $X_3$  and  $X_4$  is a decision variable which is 1 if customer choose to buy bundle and 0 for the otherwise.  $Y_1$  and  $Y_2$  is a decision variabel which is 1 if th vendor chooses to offer bundle and 0 for the otherwise. And the last one  $S_1$  and  $S_2$  represent value of surplus that customer earn choose each bundle, which equal to 0.1 and 9800.1 successively.

Based on Eq (11) to Eq (24) <sup>4</sup> divided each case pursuant to their pricing scheme namely flat fee, usage based, and two-part tariff, and the optimal solution for each case are solve using Lingo 11.0 will given in the Table 3.

Table 3. Optimal Solution for Improved Models

Variables	Modified Models Using Quasi-Linear Utility Function		
	Flat Fee	Usage Based	Two-Part Tariff
Model Class	INLP	INLP	INLP
State	Feasible	Local Optimal	Local Optimal
Objective	56999.8	56999.8	56999.8
Infeasibility	$3.63798 \times 10^{-12}$	$1.81899 \times 10^{-12}$	$1.81899 \times 10^{-12}$
Iterations	187	130	151
GMU	27K	27K	27K
ER	6s	0s	0s

Based on Table 3, value will achieve the most optimal results in usage based pricing scheme which is equal to 56999.8. The result will be obtained with lower iteration than the others which is 130 iterations with the infeasibility of  $1.81899 \times 10^{-12}$ . Generated Memory Used (GMU), the total allocation of memory used is equal to 27K and Elapsed Runtime (ER) shows the total time used to generate and solve the model that is 0 seconds.

Table 4. Variable Values for Improved Models

Variables	Modified Models Using Quasi-Linear Utility Function		
	Flat Fee	Usage Based	Two-Part Tariff
$P_1$	41299.90	31680.43	31680.43
$P_2$	0.1	31499.9	31499.9
$B_1$	2000	2000	2000
$B_2$	2000	2000	2000
$X_1$	1	0	0
$X_2$	1	0	0
$X_3$	0	1	1
$X_4$	0	1	1
$Y_1$	1	0	0
$Y_2$	0	1	1
$S_1$	80200.1	0.1	0.1
$S_2$	0.1	9800.1	9800.1

Based on Table 4, it can be seen that the values of variables for case flat fee very different from the other case. And for usage based and tow-part tariff cases we have a same value of each variable. It means that ISPs can choose either to apply usage based or two part tariff with the same variable values.

To seek better revenue for ISP, we need to compare our improved models with original model proposed by [2]. In Table 5 we summarize for each case that are gruping based on their pricing scheme. Then, the comparison with the original model to show which model give the most optimal solution are discussed.

Table 5. Comparison Solution for Model Original and Modified Model

Variables	Original Model	Modified Models		
		2 Flat Fee	Usage Based	Two-Part Tariff
Model Class	ILP	ILP	ILP	ILP
State	Global Optimal	Local Optimal	Local Optimal	Local Optimal
Objective	56999.8	56999.8	56999.8	56999.8
Infeasibility	0	$3.63798 \times 10^{-12}$	$1.81 \times 10^{-12}$	$1.81 \times 10^{-12}$
Iterations	188	187	130	151
GMU	27K	27K	27K	27K
ER	0s	6s	0s	0s

Based on Table 5, the optimal solutions were obtained for each case on both models, which shows the same profit compared to original model. As in the improved model solution, the most optimal solution in the case using usage based pricing scheme by set the price in peak and not peak hours more than 0 which gained income in the amount of 56999.8 rupiah with number of iterations of 130 compared to other models. It means, provider can save time and resources if applying this scheme rather than others. Time taken to complete iterations usually deal with the saving resources in real problem. That is why the reason that we conclude that improved model of two part tariff pricing scheme is the best choice that ISP can adopt to maximize the revenue.

## V. CONCLUSION

From the comparison between the original model with the modified model in 3-pricing schemes, the service provider will get maximum benefit when applying the improved model on the pricing scheme of usage based by first determining the cost of making bundles and the total cost of the reservation for each bundle. Other improved models actually shows the same benefit to ISP, but resources are fully utilized in quite long time, it will cause the wasting time and energy.

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