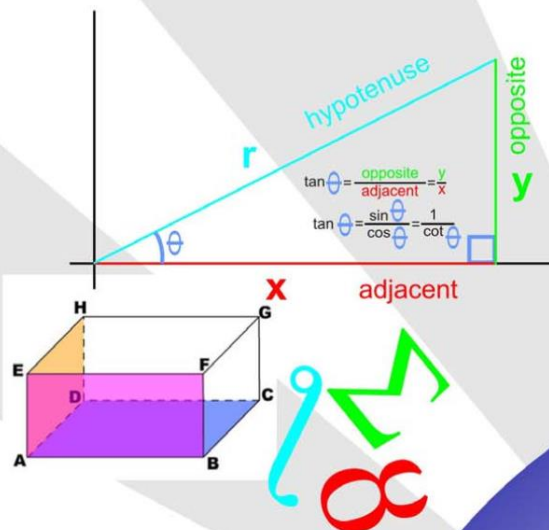


# Proceeding ICeMATH 2011

## The International Conference on Numerical Analysis & Optimization

No. ISBN : 978-602-98919-1-1



June 6 - 8, 2011



Hosted by :  
Departement of Mathematics  
Faculty of Mathematics and Natural Sciences

## The International Conference on Numerical Analysis and Optimization (ICeMATH2011)

The field of numerical analysis predates the invention of modern computers by many centuries. Numerical analysis is the area of mathematics and computer science that creates, analyzes, and implements algorithms for solving numerically the problems of continuous mathematics. Such problems originate generally from real-world applications of algebra, geometry, and calculus, and they involve variables which vary continuously.

On the other hand, Numerical Optimization is defined as a scientific approach in finding the finest solution of a particular problem that is interpreted in mathematical models. Hence, the combination of numerical analysis with numerical optimization is highly important for scientific efforts in the areas of developmental work as well as humanity in general.

Therefore, on the occasion of the 50th anniversary of its founding celebration, [Universitas Ahmad Dahlan](#) (UAD) with the collaboration of Journal KALAM has initiated **The International Conference on Numerical Analysis and Optimization (ICeMATH 2011)** to be held at Yogyakarta, Indonesia.

### Objectives:

- Provide a platform for researchers, professionals, and academicians to exchange ideas and discuss their research findings.
- Encourage future collaborations between participants.
- Provide room for researchers to discuss their thoughts and views on the development of this field that can contribute towards future works as well as being a very beneficial program for all participants.

### Topic of Discussions:

Numerical Analysis, Numerical Methods, Operations Research, Mathematics, Statistics, Numerical Optimization, Differential Equation, Applied Mathematics and Statistics, Interval Mathematics, Fuzzy, Computational Mathematics, Combinatory, Algebra, Engineering Mathematics, Mathematics Education

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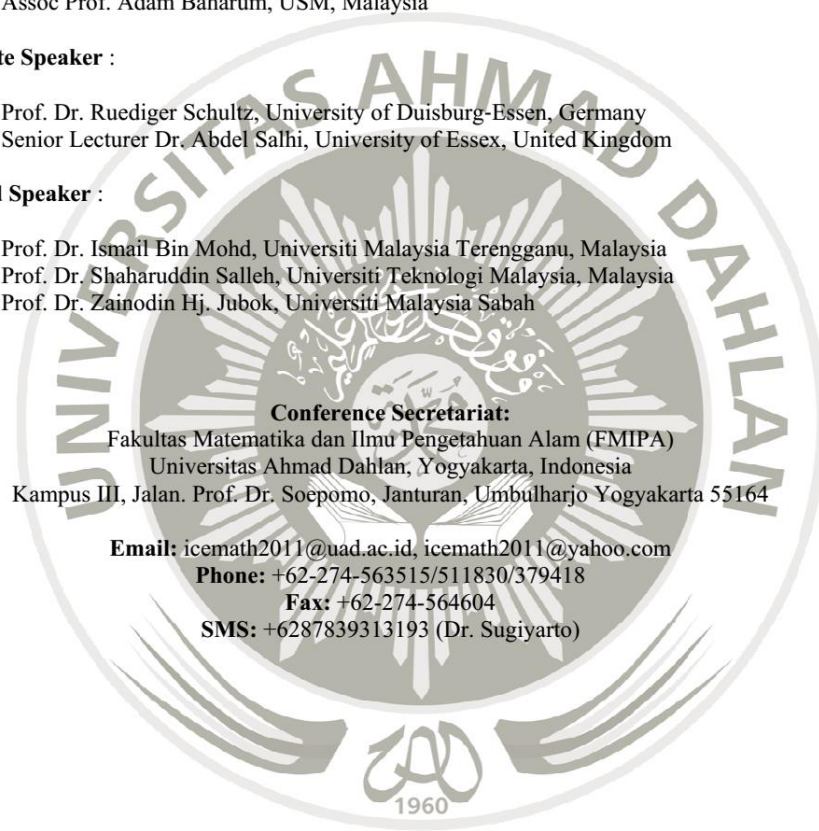
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The International Conference  
on Numerical Analysis and Optimization  
(ICeMATH 2011)

**Schedule of Parallel Session**  
(Short Communication, 15 minutes each)

*New update per May 31, 2011*

Numerical Optimization / Day 2: June 07, 2011 / Room 4

13.00 – 13.15	A 0-1 GOAL PROGRAMMING MODEL FOR FIREMAN SCHEDULING Asdalifah Talibe , Low Sieur Chuan Universiti Malaysia Sabah
13.15 – 13.30	MIXED GEOGRAPHICALLY WEIGHTED REGRESSION MODEL FOR LOCAL AND GLOBAL PREDICTOR VARIABLE Hasbi Yasin Diponegoro University, Semarang
13.30 – 13.45	STRATEGY FOR OPTIMAL CONTROL OF TUBERCULOSIS WITH EXOGENOUS REINFECTION Hasnan Nasrun, Subchan, M.Yunus Institut Teknologi Sepuluh Nopember, Surabaya
13.45 – 14.00	OPTIMIZATION OF LOWER LIMB SEGMENT DURING BACKPACK CARRIAGE Hasyatun Che Nan, Azmin Sham Rambely Universiti Kebangsaan Malaysia
14.00 – 14.15	IDENTIFYING AN UNKNOWN RADIATION TERM IN A HEAT EQUATION WITH NON-LINEAR BOUNDARY CONDITION Mortaza Abtahi, Reza Pourgholi Damghan University
14.15 – 14.30	WEAKLY REACHABILITY AND WEAKLY OBSERVABILITY OF LINEAR SYSTEM OVER MAX PLUS ALGEBRA Tri Siwi Nasrulyati*, Subiono, Erna Apriliani Institut Teknologi Sepuluh Nopember, Surabaya
14.30 – 14.45	CHARGING SCHEME OF INTERNET PRICING UNDER QOS NETWORK Fitri Maya Puspita, Kamaruzzaman Bin Seman, Bahrom Bin Sanugi. Sriwijaya University Sumatera Selatan Islamic Science University of Malaysia
14.45 – 15.00	NUMERICAL OPTIMIZATION BASED ON TRANSFORMATION OF DATA CHARACTERIZATION Zainodin H. J. Universiti Malaysia Sabah
15.00 – 15.15	THE ECCENTRIC DIGRAPH OF A FIRECRACKER GRAPH Tri Atmojo Kusmayadi* , Yayan Uji Utama
15.15 – 15.30	DIFFERENT EVOLUTIONARY ALGORITHMS (EAS) SUITE DIFFERENT OPTIMIZATION PROBLEMS Wali Khan(Mashwani) University Of Essex,Wivenhoe Park Colchester, Essex,Uk

# Internet Charging Scheme under Multiple QoS Networks

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April 18, 2011

## Abstract

As internet is becoming critical in economics life, Internet Service Providers (ISPs) now deal with high demand to promote good quality information. However, the knowledge to develop new pricing plans that serve both customers and supplier is known, but only a few pricing plans involve QoS networks. This paper will analyze the dynamical situation in network where new proposed pricing plans are offered with QoS networks involved. The plan begins from simple QoS network and tries to generalize into multiple QoS networks. For further research, optimal solution of the plan will be considered through maximizing the ISP's point of view.

*Keywords: charging scheme, QoS network*

## 1 Introduction

Pricing product or service is critical business decisions or core activity that will be focussed on this paper. There are many approaches to pricing involving scientific method or otherwise [2]. Internet has to provide the best QoS meaning that the mechanism that allows differentiation of network services based on their unique service requirements [1, 3, 4]. In the past several decades, researchers have been studying graph theory to understand problems related to communication networks and find appropriate solutions [5].

The customers, nowadays, have tendency to apply flat rate pricing since that scheme is simple by paying a subscription fee for each month and getting all the

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service. However, this pricing scheme actually has disadvantage since it does not solve congestion problem. Tragedy of commons [6] occurs in the overloaded use of flat rate pricing scheme. For telecommunication companies (telcos), flat rate scheme has bad effect on revenue maximization. They have deployed multi QoS networks to give customers more options in using the service. If customers want the quality of their service to be guaranteed, they will use the highest quality network, but a higher price. If they do not care about quality, then they can choose the flat rate which is the lowest quality. Telcos are having difficulties in coming out with the right pricing schemes with this multiple QoS networks.

Yang [11] and Yang et al [12, 13] formulate pricing strategy for differentiated service networks. In their discussion, they focus on auction algorithm to find the optimal solution. We apply their mathematical formulation and combine it with mathematical formulation discussed by Byun and Chatterjee [1].

Basically, our contribution can be described as follows:

- we modify the mathematical formulation of [1, 12] since it could also combine into simpler formulation by taking into consideration the utility function, base price, quality premium, index performance, capacity and also bandwidth required
- we consider the problem of internet charging scheme as Mixed Integer Nonlinear Programming (MINLP) that can be solved using LINGO version 12.0 [14] to obtain optimal solution.

## 2 Literature Review

Byun & Chatterjee [1] is basically one of the few studies about pricing which focuses on economic point of view. The results show that by designing proper pricing scheme with quality index is in pricing formula yields simpler formula but of course it is also dynamic. The possible changes in service pricing and revenue changes can also be made. Karp [7] explains problems related to congestion and how to control it. If, for instance, there is single flow which is sending packets from source to destination, if it transmits at certain rate, it get dropped packet, but if it chooses to send other rate, it can reach destination. How can the source A, for instance, know and manage its flow over continuing certain time, meaning that time is divided into duration length of time like explained in [8, 9].

Wu et al [10] described the optimal pricing schemes both in consumer's and supplier's perspectives by considering the homogenous and heterogeneous customers. In homogenous case, all customers have the same utility on consumption level per day while in heterogeneous case, customers have two segments according to their willingness to pay and level of usage.

Recent work on multiple service network is due to [11, 12, 13]. She described the pricing scheme based auction to allocate QoS and maximize ISP's revenue. According to her, the auction pricing scheme is scalability, efficiency and fairness in sharing resources. The solution of the optimization problem goes from

single bottleneck link in the network and then generalize into multiple bottleneck link using heuristic method. Although QoS mechanisms are available in some researches, there are few practical QoS networks.

### 3 Mathematical Formulation

The idea basically generates from [1, 11, 12, 13] for single QoS network and also we also use utility function adopted by [11, 12, 13].

#### 3.1 Assumptions

Assume that there is only one single network from source to destination, so we deal with single bottleneck link. This is because we only concentrate on service pricing scheme not service routing scheme. We can assume that the routing schemes are already set up by the ISP.

As [12] pointed out, we have 2 parts of utility function namely, base cost which does not depend on resource consumption and cost which depends on resource consumption. The utility function has characteristics as marginal profit as function of bandwidth decreasing with increasing bandwidth.

The Objective of ISP is to obtain maximized revenue subject to constraints based on system' available resources.

We have parameters

- $\alpha_j$  : base price for class  $j$
- $\beta_j$  : quality premium of class  $j$  that has  $I_q^j$  service performance
- $Q$  : total bandwidth
- $V_i$  : minimum bandwidth required by user  $i$
- $X_j$  : bandwidth for class  $j$
- $M$  : a very large positive number

Decision variables:

- $Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$
- $X_{ij}$  : final bandwidth obtained by user  $i$  for class  $j$
- $L_{mj}$  : minimum bandwidth for class  $j$
- $W_j$  : price for class  $j$
- $I_q^j$  : quality index of class  $j$

The mathematical model will be

$$\max P_{ij}.U_{ij} = \sum_{j=1} \sum_i (\alpha_j + \beta_j * I_q^j) W_j * \ln \frac{X_{ij}}{L_{mj}} * Z_{ij} \quad (1)$$

subject to

$$\sum_j \sum_i X_{ij} \leq Q \quad (2)$$

$$0 \leq I_q^j \leq 1 \quad (3)$$

$$X_{ij} \geq L_{mj} - (1 - Z_{ij}) * M \quad (4)$$

$$W_j \leq W_{ij} + (1 - Z_{ij}) * M \quad (5)$$

$$X_{ij} \geq V_i - (1 - Z_{ij}) * M \quad (6)$$

$$X_{ij} \geq X_j - (1 - Z_{ij}) * M \quad (7)$$

$$X_{ij} \geq 0 + Z_{ij} * M \quad (8)$$

$$X_{ij} \geq 0; L_{mj} \geq 0; W_j \geq 0 \quad (9)$$

$$X_{ij} \leq X_j \quad (10)$$

$$Z_{ij} = 0 \text{ or } 1 \quad (11)$$

Objective function (1) basically states that ISP wants to maximize its revenue from total sum of price and its utility function. Eq(2) tells as that total final bandwidth of all users cannot exceed the total bandwidth available. Quality index is the average of service quality that has value between 0 (meaning at base quality) or 1 (meaning that has best quality) as Eq(3) showed. Eq(4) states that bandwidth for user  $i$  has greater than the negative of minimum bandwidth for class  $j$  if user  $i$  is admitted to class  $j$  or otherwise. Eq(5) tells us about price for class  $j$  should be less than the price of user  $i$  willing to pay in class  $j$  if the user  $i$  will admit to class  $j$ . Next, Eq (6) basically shows that final bandwidth obtained by user  $i$  for class  $j$  will exceed negative of minimum bandwidth required by  $i$  if user  $i$  is admitted to class  $j$  or otherwise. Eq(7) states that final bandwidth obtained by user  $i$  at class  $j$  should be exceed bandwidth for class  $j$  if user  $i$  is admitted to class  $j$  or otherwise. Eq(8) tells us that final bandwidth obtained by user  $i$  should be greater than a very large positive number if user  $i$  is admitted to class  $j$  or not, if otherwise. Eq(9) state about the nonnegative requirements of the variables, Eq(10) shows that final bandwidth of user  $i$  to class  $j$  should not exceed the bandwidth of class  $j$  and lastly, Eq(11) tells us about decision if user  $i$  is admitted to class  $j$  or not.

## 4 Optimal Solution

The model above is the optimization problem; we are able to solve that problem by using integer programming. In this paper, we use LINGO 12.0 [14] to solve computation result.



## 4.1 Solution in One Class

### 4.1.1 Examples

Basically, we begin from assuming that there only exists one class, so we can omit the index  $j$ . For simplicity, we consider there exist 2 users that apply to use the service then the computation can be described below. First, we consider case when  $Q = M = X$ . We give numbers on parameters such as  $\alpha = \$0.1$ ,  $\beta = \$0.05/\text{bps}$ ,  $Q = 50 \text{ bps}$ ,  $X = 50 \text{ bps}$ ,  $M = 50$  with Generator Memory used(K) that shows the amount of memory LINGO's model generator is currently used from its memory part, and Elapsed Runtime(in sec) shows that the total time used to generate and solve the model and may be affected by other applications running in user's system [14]. The computation is summarized in Table 1.

Next case is when  $Q > X$ ,  $Q > M$  and  $X = M$ . Again, we put numbers on parameters like in case 1 with same  $\alpha$  and  $\beta$  but  $Q = 100 \text{ bps}$ ,  $X = 50 \text{ bps}$ ,  $M = 50$ . Table 2 shows the computation. Last case is when  $Q > X$ ,  $Q > M$  and  $X > M$ . When we put numbers on parameters with same  $\alpha$  and  $\beta$  but  $Q = 120 \text{ bps}$ ,  $X = 50 \text{ bps}$ ,  $M = 40$  the objective bound for each conditions gives different result (smaller value than objective value) with objective value which means that there is no feasible solution to the model since objective bound gives a bound on the best possible solution to the model [14].

Table 1. Case when  $Q=M=X$

	$V_1 < V_2$ $W_1 < W_2$	$V_1 > V_2$ $W_1 > W_2$	$V_1 = V_2$ $W_1 = W_2$
$V_1$	5	6	5
$V_2$	6	5	5
$W_1$	7	8	8
$W_2$	8	7	8
GMU	23	23	23
ER	0	0	0
OV	82.89	82.89	82.89
OB	82.89	82.89	82.89
ESS	3	3	3
TSI	229	212	239
$I_q$	1	1	1
$W$	8	8	8
$X_1$	0	50	0
$L_m$	0	0	0
$Z_1$	0	1	0
$X_2$	50	0	50
$Z_2$	1	0	1

where

- OV : Objective value
- OB : Objective bound
- ESS : Extended solver steps
- TSI : Total solver iterations

Table 2. Case when  $Q > X, Q > M$  and  $X = M$ 

	$V_1 < V_2$ $W_1 < W_2$	$V_1 > V_2$ $W_1 > W_2$	$V_1 = V_2$ $W_1 = W_2$
$V_1$	5	6	5
$V_2$	6	5	5
$W_1$	7	8	8
$W_2$	8	7	8
GMU	23	23	23
ER	0	0	0
OV	145.06	145.06	145.06
OB	145.06	145.06	145.06
ESS	3	3	3
TSI	550	524	554
$I_q$	1	1	1
$W$	7	7	7
$X_1$	50	50	50
$L_m$	0	0	0
$Z_1$	1	1	1
$X_2$	50	50	50
$Z_2$	1	1	1

We can see that from Table 1, when  $V_1 < V_2$  then  $Z_1 = 0, Z_2 = 1$ . It means that user 2 is admitted to the class since minimum bandwidth required by user 2 is larger than 1's. So between two users within one class, user that has larger minimum bandwidth required will be admitted to the class with price for that class is  $W = \max\{W_1, W_2\}$ . Table 2 explains different things. Since  $Q > X, Q > M$  and  $X = M$  then the value of  $X_1 = X_2 = X$  and all users are admitted to that class with price for that class  $W = \min\{W_1, W_2\}$ .  $I_q = 1$  means that  $\alpha + \beta$  is the upper bound price for perfect service [1].

## 4.2 Solutions in Multiple Classes

### 4.2.1 Examples

We begin with introducing two classes and two users. so  $J = 2$  and  $i = 2$ . We consider 2 cases that are when  $Q > M, X_1 = M, X_2 > M$  and  $Q > M, X_1 = X_2 = M$ . Put quantities to parameters for case 1 that is  $Q = 100$  bps,  $X_1 = 50$  bps,  $X_2 = 60$  bps and  $M = 50$ . The computations are summarized in Table 3.

Tabel 3.  $Q > M, X_1 = M, X_2 > M$

	$V_1 < V_2$ $X_1 < X_2$ $W_{11} < W_{21}$ $W_{12} < W_{22}$	$V_1 > V_2$ $X_1 > X_2$ $W_{11} > W_{21}$ $W_{12} > W_{22}$
$V_1, V_2$	5, 6	6, 5
$X_1, X_2$	50, 60	60, 50
$W_{11}, W_{21}$	7, 8	8, 7
$W_{12}, W_{22}$	7, 8	8, 7
GMU	29	29
ER	22	2
OV	1378.1	1402.27
OB	1378.1	1402.27
$I_q^1, I_q^2$	1, 1	0, 48, 1
$W_1, W_2$	37.2, 57	0.53, 58
$X_{11}, X_{21}$	19.8, 20.2	0, 0
$X_{12}, X_{22}$	0, 60	6.07, 50
$L_{m1}, L_{m2}$	0, 0	0.17, 0
$Z_{11}, Z_{21}$	0, 0	0, 0
$Z_{12}, Z_{22}$	0, 1	0, 1

For case 1, only one user is admitted to only one class  $j$ . In this case, user 2 is admitted to class 2 ( $Z_{22} = 1$ ) having  $I_q^2 = 1$  and minimum bandwidth for class 2 is 0. Final bandwidth obtained by user  $i$  for class  $j$  who is admitted to class  $j$ ,  $X_{ij} = \min\{X_j\}$ .

For case 2, if we put quantities on parameters that are  $Q = 100$  bps,  $X_1 = X_2 = M = 50$  bps,  $V_1 = V_2 = 5, W_{11} = 8, W_{21} = 7, W_{12} = 8, W_{22} = 7$  then we have the same results discussed in Table 3 ( $V_1 > V_2, X_1 > X_2, W_{11} > W_{21}, W_{12} > W_{22}$ ). But if we see the in QoS networks, each class must have different bandwidth. so it is not possible to have  $X_1 = X_2$ , it should be  $X_1 > X_2$  or  $X_1 < X_2$ .

## 5 Conclusion

For model in one class with 2 users, we can see from each class, different results have been obtained. It depends on the minimum bandwidth required by each user, total capacity, price for each user, total capacity, bandwidth for the class and also large number chosen. If service provider (SP) will admit each user to use the service, SP can set up price and minimum bandwidth required for each user or whether only choose some users to admit to the class.

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*By Fitri maya Puspita*

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Faculty of Science and Technology  
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Pricing product or service is critical business decisions or core activity that will be focussed on this paper. There are many approaches to pricing involving scientific method or otherwise [2]. Internet has to provide the best QoS meaning that the mechanism that allows differentiation of network services based on their unique service requirements [1, 3, 4]. In the past several decades, researchers have been studying graph theory to understand problems related to communication networks and find appropriate solutions [5].

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Basically, our contribution can be described as follows:

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#### 3.1 Assumptions

Assume that there is only one single network from source to destination, so we deal with single bottleneck link. This is because we only concentrate on service pricing scheme not service routing scheme. We can assume that the routing schemes are already set up by the ISP.

As [12] pointed out, we have 2 parts of utility function namely, base cost which does not depend on resource consumption and cost which depends on resource consumption. The utility function has characteristics as marginal profit as function of bandwidth decreasing with increasing bandwidth.

The Objective of ISP is to obtain maximized revenue subject to constraints based on system' available resources.

We have parameters

- $\alpha_j$  : base price for class  $j$
- $\beta_j$  : quality premium of class  $j$  that has  $I_q^j$  service performance
- $Q$  : total bandwidth
- $V_i$  : minimum bandwidth required by user  $i$
- $X_j$  : bandwidth for class  $j$
- $M$  : a very large positive number

Decision variables:

- $Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$
- $X_{ij}$  : final bandwidth obtained by user  $i$  for class  $j$
- $L_{mj}$  : minimum bandwidth for class  $j$
- $W_j$  : price for class  $j$
- $I_q^j$  : quality index of class  $j$

The mathematical model will be

$$\max P_{ij}.U_{ij} = \sum_{j=1} \sum_i (\alpha_j + \beta_j * I_q^j) W_j * \ln \frac{X_{ij}}{L_{mj}} * Z_{ij} \quad (1)$$

subject to

$$\sum_j \sum_i X_{ij} \leq Q \quad (2)$$



$$0 \leq I_q^j \leq 1 \quad (3)$$

$$X_{ij} \geq L_{mj} - (1 - Z_{ij}) * M \quad (4)$$

$$W_j \leq W_{ij} + (1 - Z_{ij}) * M \quad (5)$$

$$X_{ij} \geq V_i - (1 - Z_{ij}) * M \quad (6)$$

$$X_{ij} \geq X_j - (1 - Z_{ij}) * M \quad (7)$$

$$X_{ij} \geq 0 + Z_{ij} * M \quad (8)$$

$$X_{ij} \geq 0; L_{mj} \geq 0; W_j \geq 0 \quad (9)$$

$$X_{ij} \leq X_j \quad (10)$$

$$Z_{ij} = 0 \text{ or } 1 \quad (11)$$

Objective function (1) basically states that ISP wants to maximize its revenue from total sum of price and its utility function. Eq(2) tells as that total final bandwidth of all users cannot exceed the total bandwidth available. Quality index is the average of service quality that has value between 0 (meaning at base quality) or 1 (meaning that has best quality) as Eq(3) showed. Eq(4) states that bandwidth for user  $i$  has greater than the negative of minimum bandwidth for class  $j$  if user  $i$  is admitted to class  $j$  or otherwise. Eq(5) tells us about price for class  $j$  should be less than the price of user  $i$  willing to pay in class  $j$  if the user  $i$  will admit to class  $j$ . Next, Eq (6) basically shows that final bandwidth obtained by user  $i$  for class  $j$  will exceed negative of minimum bandwidth required by  $i$  if user  $i$  is admitted to class  $j$  or otherwise. Eq(7) states that final bandwidth obtained by user  $i$  at class  $j$  should be exceed bandwidth for class  $j$  if user  $i$  is admitted to class  $j$  or otherwise. Eq(8) tells us that final bandwidth obtained by user  $i$  should be greater than a very large positive number if user  $i$  is admitted to class  $j$  or not, if otherwise. Eq(9) state about the nonnegative requirements of the variables, Eq(10) shows that final bandwidth of user  $i$  to class  $j$  should not exceed the bandwidth of class  $j$  and lastly, Eq(11) tells us about decision if user  $i$  is admitted to class  $j$  or not.

## 4 Optimal Solution

The model above is the optimization problem; we are able to solve that problem by using integer programming. In this paper, we use LINGO 12.0 [14] to solve computation result.

## 4.1 Solution in One Class

### 4.1.1 Examples

Basically, we begin from assuming that there only exists one class, so we can omit the index  $j$ . For simplicity, we consider there exist 2 users that apply to use the service then the computation can be described below. First, we consider case when  $Q = M = X$ . We give numbers on parameters such as  $\alpha = \$0.1$ ,  $\beta = \$0.05/\text{bps}$ ,  $Q = 50 \text{ bps}$ ,  $X = 50 \text{ bps}$ ,  $M = 50$  with Generator Memory used(K) that shows the amount of memory LINGO's model generator is currently used from its memory part, and Elapsed Runtime(in sec) shows that the total time used to generate and solve the model and may be affected by other applications running in user's system [14]. The computation is summarized in Table 1.

Next case is when  $Q > X$ ,  $Q > M$  and  $X = M$ . Again, we put numbers on parameters like in case 1 with same  $\alpha$  and  $\beta$  but  $Q = 100 \text{ bps}$ ,  $X = 50 \text{ bps}$ ,  $M = 50$ . Table 2 shows the computation. Last case is when  $Q > X$ ,  $Q > M$  and  $X > M$ . When we put numbers on parameters with same  $\alpha$  and  $\beta$  but  $Q = 120 \text{ bps}$ ,  $X = 50 \text{ bps}$ ,  $M = 40$  the objective bound for each conditions gives different result (smaller value than objective value) with objective value which means that there is no feasible solution to the model since objective bound gives a bound on the best possible solution to the model [14].

Table 1. Case when  $Q=M=X$

	$V_1 < V_2$ $W_1 < W_2$	$V_1 > V_2$ $W_1 > W_2$	$V_1 = V_2$ $W_1 = W_2$
$V_1$	5	6	5
$V_2$	6	5	5
$W_1$	7	8	8
$W_2$	8	7	8
GMU	23	23	23
ER	0	0	0
OV	82.89	82.89	82.89
OB	82.89	82.89	82.89
ESS	3	3	3
TSI	229	212	239
$I_g$	1	1	1
$W$	8	8	8
$X_1$	0	50	0
$L_m$	0	0	0
$Z_1$	0	1	0
$X_2$	50	0	50
$Z_2$	1	0	1

where

- OV : Objective value
- OB : Objective bound
- ESS : Extended solver steps
- TSI : Total solver iterations

Table 2. Case when  $Q > X, Q > M$  and  $X = M$ 

	$V_1 < V_2$ $W_1 < W_2$	$V_1 > V_2$ $W_1 > W_2$	$V_1 = V_2$ $W_1 = W_2$
$V_1$	5	6	5
$V_2$	6	5	5
$W_1$	7	8	8
$W_2$	8	7	8
GMU	23	23	23
ER	0	0	0
OV	145.06	145.06	145.06
OB	145.06	145.06	145.06
ESS	3	3	3
TSI	550	524	554
$I_q$	1	1	1
$W$	7	7	7
$X_1$	50	50	50
$L_m$	0	0	0
$Z_1$	1	1	1
$X_2$	50	50	50
$Z_2$	1	1	1

We can see that from Table 1, when  $V_1 < V_2$  then  $Z_1 = 0, Z_2 = 1$ . It means that user 2 is admitted to the class since minimum bandwidth required by user 2 is larger than 1's. So between two users within one class, user that has larger minimum bandwidth required will be admitted to the class with price for that class is  $W = \max\{W_1, W_2\}$ . Table 2 explains different things. Since  $Q > X, Q > M$  and  $X = M$  then the value of  $X_1 = X_2 = X$  and all users are admitted to that class with price for that class  $W = \min\{W_1, W_2\}$ .  $I_q = 1$  means that  $\alpha + \beta$  is the upper bound price for perfect service [1].

## 4.2 Solutions in Multiple Classes

### 4.2.1 Examples

We begin with introducing two classes and two users. so  $J = 2$  and  $i = 2$ . We consider 2 cases that are when  $Q > M, X_1 = M, X_2 > M$  and  $Q > M, X_1 = X_2 = M$ . Put quantities to parameters for case 1 that is  $Q = 100$  bps,  $X_1 = 50$  bps,  $X_2 = 60$  bps and  $M = 50$ . The computations are summarized in Table 3.

Tabel 3.  $Q > M, X_1 = M, X_2 > M$

	$V_1 < V_2$ $X_1 < X_2$ $W_{11} < W_{21}$ $W_{12} < W_{22}$	$V_1 > V_2$ $X_1 > X_2$ $W_{11} > W_{21}$ $W_{12} > W_{22}$
$V_1, V_2$	5, 6	6, 5
$X_1, X_2$	50, 60	60, 50
$W_{11}, W_{21}$	7, 8	8, 7
$W_{12}, W_{22}$	7, 8	8, 7
GMU	29	29
ER	22	2
OV	1378.1	1402.27
OB	1378.1	1402.27
$I_q^1, I_q^2$	1, 1	0.48, 1
$W_1, W_2$	37.2, 57	0.53, 58
$X_{11}, X_{21}$	19.8, 20.2	0, 0
$X_{12}, X_{22}$	0, 60	6.07, 50
$L_{m1}, L_{m2}$	0, 0	0.17, 0
$Z_{11}, Z_{21}$	0, 0	0, 0
$Z_{12}, Z_{22}$	0, 1	0, 1

For case 1, only one user is admitted to only one class  $j$ . In this case, user 2 is admitted to class 2 ( $Z_{22} = 1$ ) having  $I_q^2 = 1$  and minimum bandwidth for class 2 is 0. Final bandwidth obtained by user  $i$  for class  $j$  who is admitted to class  $j$ ,  $X_{ij} = \min\{X_j\}$ .

For case 2, if we put quantities on parameters that are  $Q = 100$  bps,  $X_1 = X_2 = M = 50$  bps,  $V_1 = V_2 = 5, W_{11} = 8, W_{21} = 7, W_{12} = 8, W_{22} = 7$  then we have the same results discussed in Table 3 ( $V_1 > V_2, X_1 > X_2, W_{11} > W_{21}, W_{12} > W_{22}$ ). But if we see the in QoS networks, each class must have different bandwidth. so it is not possible to have  $X_1 = X_2$ , it should be  $X_1 > X_2$  or  $X_1 < X_2$ .

## 5 Conclusion

For model in one class with 2 users, we can see from each class, different results have been obtained. It depends on the minimum bandwidth required by each user, total capacity, price for each user, total capacity, bandwidth for the class and also large number chosen. If service provider (SP) will admit each user to use the service, SP can set up price and minimum bandwidth required for each user or whether only choose some users to admit to the class.

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