15.icets 2016

By Fitri maya Puspita

COBB-DOUGLAS UTILITY FUNCTION OF INFORMATION SERVICE PRICING SCHEME BASED ON MONITORING AND MARGINAL COSTS

Robinson Sitepu [1], Fitri Maya Puspita[1*], Hadi Tanuji[1], Icha Puspita Novyasti [1]

[1]Mathematics Department, Faculty of Mathematics and Natural Sciences, Sriwijaya University, Inderalaya 30662

[*]email: robinson.sitepu@rocketmail.com

ABSTRACT

ISP is a provider of services to access the internet either for personal, offices, schools, and for the public. Past research has focused only on the selection of the Cobb-Douglas utility function and regardless the application of monitoring and marginal cost. In this paper the Cobb-Douglass utility function on information service pricing schemes with monitoring and marginal costs will be discussed with the objective function for three pricing schemes namely flat fee, usage based and two-part tariff pricing strategies. This paper seeks to optimize the schemes for the ISP. The results of this paper is expected to optimize the pricing schemes for so that these can be adopted by ISP. The results show that by applying the monitoring and marginal costs into the scheme based on Cobb-Douglas utility function, ISPs gain less price to be spent rather than only applying the utility function without adopting the monitoring and marginal costs.

Keywords: Cobb-Douglass utility function, information service pricing scheme, monitoring cost, marginal cost.

INTRODUCTION

Development of the internet is growing rapidly so that Internet service providers or Internet Service Provider (ISP) are motivated to provide the best service at an affordable price for consumers (Puspita et al., 2015). According to Wang and Schulzrinne (2001) utility function usually was associated with the level of satisfaction that users get on the consumption of services that can be maximized by ISPs to obtain benefits to achieve certain goals. Therefore, it takes the best utility functions that can not only be beneficial for ISPs but also for consumers by providing the best service to consumers.

There are many utility functions that are often used, including Cobb-Douglas (Indrawati et al., 2014a), quasi-linear(Indrawati et al., 2014b), perfect-substitutes (Indrawati et al., 2014), perfect complement and bandwidth diminished with increasing bandwidth (Irmeilyana et al., 2015; Puspita et al., 2013; Yang et al., 2005). Previously, studies to obtain an optimum solution by using a utility function have been performed by Wu and Banker (2010). In their research, the results of the analysis of financing with a flat fee and a two-part tariff are more optimal than the usage-based scheme. Their research just compare these three financing strategies for the Cobb-Douglass utility function to maximize profits for ISPs to terms with

paying attention to the satisfaction of the customers with a utility function that has been modified.

Research conducted by Indrawati et al (2014a, 2014b; 2015; 2014) and Wu and Banker (2010) at the election of utility functions that can maximize the benefits for service providers and ignore the monitoring and marginal costs. In fact marginal and monitoring costs are to be important in the development of information services for the three schemes of financing (flat-fee, usage-based, and two-part-tariff). Because the marginal cost and expense controls may affect the price of the optimal financing schemes. For that, it needs to be assessed on monitoring and marginal costs for financing schemes involving four information services utility function such as Cobb-Douglas, quasi-linear, perfect-substitutes, perfect complements (Hutchinson, 2011).

In this paper, the utility function Cobb-Douglas will be applied to the financing scheme information services with the addition of the cost of monitoring and marginal costs with three financing schemes namely flat fee, usage-based, and the two-part tariff financing schemes. we refer to the homogeneous and heterogeneous consumers (high end and low end), consumers are heterogeneous (high demand and low demand).

METHODS

The steps to be taken in this study, namely:
1. Determine the parameters and decision variables to be used on any financing schemes
2. Formulate theories of Cobb-Douglass utility functions on financing scheme information services by expanding the model with the addition of monitoring costs and marginal costs with the type of financing of flat fee, usage-based, and the two-part tariff for homogeneous and heterogeneous consumers.
3. Analyze utility functions obtained under Step 1 and Step 2.
4. Apply financing schemes that have been obtained on the local server data.
5. Compare the utility function Cobb-Douglass without the costs of monitoring and the marginal cost and with those costs which is most optimal.

RESULTS AND DISCUSSION

This study aims to maximize profits by using a flat-fee, usage-base, and two-part tariff financing schemes for consumers homogeneous and heterogeneous consumers.
The consumer optimization problem will be as following (Indrawati *et al*, 2014):

$$\max_{X,Y,Z} X^a Y^b - P_X X - P_Y Y - PZ - (X + Y)c \tag{1}$$

Subject to

$$X \leq \bar{X} Z \tag{2}$$
$$Y \leq \bar{Y} Z \tag{3}$$
$$X^a Y^b - P_X X - P_Y Y - PZ - (X + Y)c \geq 0 \tag{4}$$
$$Z = 0 \text{ atau } 1 \tag{5}$$

And the provider optimization problem will be as follows.

$$\max_{P,P_X,P_Y} \sum_i (P_X X^* + P_Y Y^* + PZ^*) \tag{6}$$

dengan $(X^*, Y^*, Z^*) = arg\ max\ X^a Y^b - P_X X - P_Y Y - PZ - (X + Y)c$
dengan kendala :
$$X \leq \bar{X} Z$$

$$Y \leq \bar{Y}Z$$
$$X^a Y^b - P_X X - P_Y Y - PZ - (X + Y)c \geq 0$$
$$Z_i = 0 \text{ or } 1$$

The following discussion of determining the maximum profit on any financing scheme providers use.

For Homogeneous Case
Lemma 1:
If a service provider is using flat-fee charges, the prices charged by $\bar{X}^a \bar{Y}^b - (\bar{X} + \bar{Y})c$ and maximum benefits are achieved will be $\sum_i [\bar{X}^a \bar{Y}^b - (\bar{X} + \bar{Y})c]$; i declares consumers.

Lemma 2:
If a service provider is using a usage-based, then the optimal price becomes $P_X = a\bar{X}^{a-1}\bar{Y}^b - (c+t)$ and $P_Y = b\bar{X}^a \bar{Y}^{b-1} - (c+t)$ with maximum profit $\sum_i \{(a+b)[\bar{X}^a \bar{Y}^b] - (c+t)\bar{X} - (c+t)\bar{Y}\}$; i denote the number of consumers.

Lemma 3:
If the service provider is using the price of two-part tariff, then best price P_X and P_Y will be $P_X = a\bar{X}^{a-1}\bar{Y}^b - (c+t)\bar{X}$, $P_Y = b\bar{X}^a \bar{Y}^{b-1} - (c+t)\bar{Y}$ and a fixed fee of $(a+b)\bar{X}^a\bar{Y}^b$. Therefore, the maximum profit is achieved by the service providers is $\sum_i [\bar{X}^a \bar{Y}^b - (c+t)\bar{X} - (c+t)\bar{Y}]$; i declare consumers.

Heterogeneous (High end dan Low end)
In the case of consumers analyzed are consumer high-end and low-end heterogeneous. Suppose that there are m consumers upper class ($i = 1$) and the lower class consumer n = 2 It is assumed that every consumer has the same upper limit heterogeneous $\bar{X}$ and $\bar{Y}$ is the level of consumption during peak hours and during off-peak hours, $a_1 > a_2$ and $b_1 > b_2$.
The consumer optimization problem will be:
$$\max_{X_i, Y_i, Z_i} a_i X_i + b_i Y_i - P_x X_i - P_y Y_i - PZ_i \tag{7}$$
Subject to
$$X_i \leq \bar{X}_\iota Z_i \tag{8}$$
$$Y_i \leq \bar{Y}_\iota Z_i \tag{9}$$
$$a_i X_i + b_i Y_i - P_x X_i - P_y Y_i - PZ_i \geq 0 \tag{10}$$
$$Z_i = 0 \text{ atau } 1 \tag{11}$$
And the provider optimization problem will be
$$\max_{P, P_X, P_Y} m(P_X X_1^* + P_Y Y_1^* + PZ_1^*) + n(P_X X_2^* + P_Y Y_2^* + PZ_2^*) \tag{12}$$
where $(X_i^*, Y_i^*, Z_i^*) = \text{argmax } a_i X_i + b_i Y_i - P_x X_i - P_y Y_i - PZ_i$ \tag{13}
subject to
$$X_i \leq \bar{X}_\iota Z_i$$
$$Y_i \leq \bar{Y}_\iota Z_i$$
$$a_i X_i + b_i Y_i - P_x X_i - P_y Y_i - PZ_i - (X + Y)c \geq 0$$
$$Z_i = 0 \text{ or } 1$$

Lemma 4:
If ISPs use flat fee financing scheme, the price charged to consumers will be $\bar{X}^{a_2} \bar{Y}^{b_2} - (\bar{X} + \bar{Y})c$ and the maximum profit obtained is for $(m + n)[\bar{X}^{a_2} \bar{Y}^{b_2} - (\bar{X} + \bar{Y})c]$.

Lemma 5:
If a service provider using a usage-based price, then the optimal price is given for the rush hour is $P_X = a_2 \bar{X}^{a_2-1}\bar{Y}^{b_2} - (c+t)$ and optimal prices in off-peak hours is $P_Y = b_2 \bar{X}^{a_2}\bar{Y}^{b_2-1} - (c+t)$ with a maximum gain is $(m+n)[(a_2 + b_2)(\bar{X}^{a_2} \bar{Y}^{b_2}) - (c+t)\bar{X} - (c+t)\bar{Y}]$

Lemma 6:

If the service provider is using the price of a two-part tariff, then sequentially P_X and P_Y optimal be $a_2 X_2{}^{a_2-1} Y_2{}^{b_2} - (c+t)$, $b_2 Y_2{}^{b_2-1} X_2{}^{a_2} - (c+t)$, and $P = X_2{}^{a_2} Y_2{}^{b_2} - (a_2 + b_2)(X_2{}^{a_2} Y_2{}^{b_2}) - (c+t)$, with profits the maximum reached is $(m+n)(\bar{X}^{a_2} \bar{Y}^{b_2} - (c+t)\bar{X} - (c+t)\bar{Y})$.

Heterogeneous Consumer : *High-demand* dan *Low-demand*

Lemma 7:

If the service provider is using flat-fee cost, then the price is set at $P = \bar{X}^{a_2} \bar{Y}^{b_2} - (X+Y)c$ with maximum profit achieved $(m+n)\left(\bar{X}_2{}^{a}\bar{Y}_2{}^{b} - (\bar{X}_2 + \bar{Y}_2)c\right)$.

Lemma 8:

If a service provider using a usage-based price, then the optimal price at rush hour is $P_X = a\bar{X}^{a-1}\bar{Y}^{b} - (c+t)$, the optimal price at off-peak hours is $P_Y = b\bar{Y}_2{}^{b-1}\bar{X}_2{}^{a} - (c+t)$. $(m+n)\left[(a+b)\left(\bar{X}_2{}^{a}\bar{Y}_2{}^{b}\right) - (c+t)\bar{X}_2 - (c+t)\bar{Y}_2\right]$ is the maximum profit.

Lemma 9:

If the service provider is using the price of a two-part tariff, then P_X and P_Y optimal sequentially into $a\bar{X}_2{}^{a-1}\bar{Y}_2{}^{b} - (c+t)$ and $b\bar{X}_2{}^{a}\bar{Y}_2{}^{b-1} - (c+t)$, so $P = \bar{X}_2{}^{a}\bar{Y}_2{}^{b} - a\bar{X}_2{}^{a}\bar{Y}_2{}^{b} - b\bar{X}_2{}^{a}\bar{Y}_2{}^{b} - (c+t)\bar{X}_2 - (c+t)\bar{Y}_2$ the maximum profit is achieved is

$$m\left[\left(a\bar{X}_2{}^{a-1}\bar{Y}_2{}^{b}\right)(\bar{X}_1 - \bar{X}_2) + \left(b\bar{X}_2{}^{a}\bar{Y}_2{}^{b-1}\right)(\bar{Y}_1 - \bar{Y}_2) - (\bar{X}_1 + \bar{Y}_1)(c+t)\right]$$
$$+ n[(\bar{X}_2 + \bar{Y}_2)(c+t)] + (m+n)\left(\bar{X}_2{}^{a}\bar{Y}_2{}^{b}\right)$$

greater than what can be achieved either with the cost prices flat -Fee or usage- based.

The Optimal financing scheme is processed using data traffic on digilib, files, and mail applications obtained from local server in Palembang within 30 days, ie February 27, 2016 until March 27, 2016. The data is shown in Table 1.

Table 1 Local Server Data

Notation	Digilib(Megabyte)	Files(Megabyte)	Mail(Megabyte)
$\bar{X}_2$	1.7236	309.94	0.2231
$\bar{Y}_1 = \bar{Y}$	2.3460	277.52	0.9822
$\bar{Y}_2$	2.2230	260.35	0.3357
$\bar{X}_1 = \bar{X}$	2.0775	429.15	1.1253

where
1. $\bar{X}_1$ or $\bar{X}$ is the maximum level of consumption during peak hours, in Mega bytes.
2. $\bar{X}_2$ is the maximum level of consumption during peak hours, without taking the data $\bar{X}_1$, so $X_1 > \bar{X}_2$, units of Mega bytes.
3. $\bar{Y}_1$ or $\bar{Y}$ is the maximum level of consumption during off-peak hours, in Mega bytes.
4. $\bar{Y}_2$ is the maximum level of consumption during off-peak hours, without taking the data $\bar{Y}_1$, so $\bar{Y}_1 > \bar{Y}_2$, units of Mega byte.

The parameter will be
1. $a > 0, b > 0, a_1 > a_2$, and $b_1 > b_2$
   suppose : $a = 2, b = 1, a_1 = 2, a_2 = 1, b_1 = 2$, and $b_2 = 1$.
2. Cobb Douglas utility function is a non-linear function. The general function: $X^a + Y^b$.

Table 2 displays the solution of pricing scheme by using Cobb-Douglas utility function without applying marginal and monitoring costs. The table describes the price needed to use the scheme and the profit that ISP obtained by applying the scheme. In Table 3, our proposed schemes with marginal and monitoring cost are applied.

Table 2 The solution of the Optimal Financing Schemes by Using Cobb-Douglass Utility Function without Monitoring an Marginal Cost Proposed by Indrawati et al. (2014a)

Consumer Type		Cobb-Douglass		
		Digilib	files	mails
Homogeneous	Price	P_X=9.74763 P_Y=2.0875	P_X=238195.4 P_Y=184169.7	P_X=2.2105 P_Y=1.2663
High end vs low end	Price	P_X=2.346 P_Y=2.0775	P_X=277.52 P_Y=429.15	P_X=0.9822 P_Y=1.1253
High demand vs low demand	Price	P_X=7.6631 P_Y=3.8315	P_X=16135.758 P_Y=96062.8036	P_X=0.1497 P_Y=0.0497

Table 3 The solution of the Optimal Financing Schemes by Using Cobb-Douglass Utility Function with Monitoring an Marginal Cost

Consumer Type		Cobb-Douglas		
		Digilib	files	mails
Homogeneous	Price	P_X=7.7471 P_Y=2.3155	P_X=238.522 P_Y=184.039	P_X=0.2064 P_Y=-0.7456
High end vs low end	Price	P_X=0.346 P_Y=0.0774	P_X=276 P_Y=119.26	P_X=-1.015 P_Y=-0.88
High demand vs low demand	Price	P_X=5.6626 P_Y=0.9704	P_X=161.198 P_Y=96.098	P_X=-1.8501 P_Y=-1.9502

As explained in Table 3, the price value obtained in our models yield better value rather than without applying monitoring cost and marginal cost. It means that ISP spends less money as operational cost setting up the service. It is due to marginal and monitoring cost applied in each pricing scheme.

CONCLUSION

From the results and discussion, we can conclude as follows.
1. Results of Cobb-Douglass utility function and monitoring costs with marginal yield financing schemes are different from Cobb-Douglass utility function alone.
2. optimal financing scheme models for heterogeneous consumers based on the willingness to pay is on usage-based financing schemes whereas for heterogeneous consumers based on usage levels are on two-part tariff financing scheme.
3. Based on the application of the model on any traffic data, found that the use of a utility function Cobb-Douglas by adding and monitoring costs marginal yield optimal financing price set up than the utility function Cobb-Douglas alone.

ACKNOWLEDGEMNT

REFERENCES

Hutchinson, E. (2011). *Economics*.

Indrawati, Irmeilyana, Puspita, F. M., & Lestari, M. P. (2014a). Cobb-Douglass Utility Function in Optimizing the Internet Pricing Scheme Model. *TELKOMNIKA, 12*(1).

Indrawati, Irmeilyana, Puspita, F. M., & Lestari, M. P. (2014b). *Perbandingan Fungsi Utilitas Cobb-Douglass Dan Quasi-Linear Dalam Menentukan Solusi Optimal Masalah Pembiayaan Layanan Informasi*. Paper presented at the Seminar Nasional Matematika dan Statistika 2014.

Indrawati, Irmeilyana, Puspita, F. M., & Sanjaya, O. (2015). Internet pricing on bandwidth function diminished with increasing bandwidth utility function. *TELKOMNIKA, 13*(1), 299-304.

Indrawati, Irmeilyana, Puspita, F. M., Susanti, E., Yuliza, E., & Sanjaya, O. (2014). *Numerical Solution of Internet Pricing Scheme Based on PErfect Substitute Utility Function*. Paper presented at the 1st International Conference on Computer Science and Engineering, Palembang, South Sumatera Indonesia.

Irmeilyana, Indrawati, Puspita, F. M., & Herdayana, L. (2015). Improving the Models of Internet Charging in Single Link Multiple Class QoS Networks. In H. A. Sulaiman, M. A. Othman, M. F. I. Othman, Y. A. Rahim & N. C. Pee (Eds.), *Advanced Computer and Communication Engineering Technology* (Vol. 315). Switzerland: Springer Publishing International.

Puspita, F. M., Seman, K., & Taib, B. M. (2015). The Improved Models of Internet Pricing Scheme of Multi Service Multi Link Networks with Various Capacity Links. In H. A. Sulaiman, M. A. Othman, M. F. I. Othman, Y. A. Rahim & N. C. Pee (Eds.), *Advanced Computer and Communication Engineering Technology* (Vol. 315). Switzeland: Springer International Publishing.

Puspita, F. M., Seman, K., Taib, B. M., & Shafii, Z. (2013). Improved Models of Internet Charging Scheme of Single Bottleneck Link in Multi QoS Networks. *Journal of Applied Sciences, 13*(4), 572-579.

Wang, X., & Schulzrinne, H. (2001). *Pricing network resources for adaptive applications in a differentiated services network*. Paper presented at the Proceedings of IEEE INFOCOM 2001, Anchorage, AK, April 2001.

Wu, S.-y., & Banker, R. D. (2010). Best Pricing Strategy for Information Services. *Journal of the Association for Information Systems, 11*(6), 339-366.

Yang, W., Owen, H. L., & Blough, D. M. (2005). *Determining Differentiated Services Network Pricing Through Auctions*. Paper presented at the Networking-ICN 2005, 4th International Conference on Networking April 2005 Proceedings, Part I, Reunion Island, France.

15.icets 2016

9%

SIMILARITY INDEX

PRIMARY SOURCES

1   eprints.unsri.ac.id
    Internet   115 words — 5%

2   www.sispress.org
    Internet   25 words — 1%

3   iieng.org
    Internet   20 words — 1%

4   docplayer.net
    Internet   13 words — 1%

5   Indrawati, Fitri Maya Puspita, Evi Yuliza, Oki   11 words — 1%
    Dwipurwani, Yossy Eka Putri, Affriyanti. "Improved
    cloud computing model of internet pricing schemes based on
    Cobb-Douglas utility function", Journal of Physics: Conference
    Series, 2019
    Crossref

EXCLUDE QUOTES   ON   EXCLUDE MATCHES   < 1%
EXCLUDE
BIBLIOGRAPHY   ON