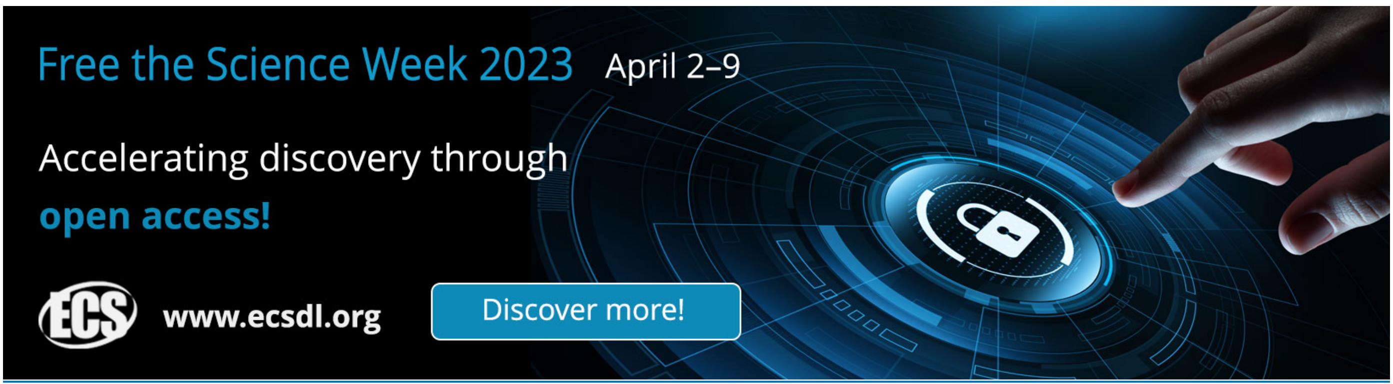
PAPER • **OPEN ACCESS**

Solving capacitated vehicle routing problem using of Clarke and Wright algorithm and LINGO in LPG distribution

To cite this article: E Yuliza *et al* 2020 *J. Phys.: Conf. Ser.* **1663** 012027



Solving capacitated vehicle routing problem using of Clarke and Wright algorithm and LINGO in LPG distribution

E Yuliza*, F M Puspita, S Yahdin and R Emiliya

Department of Mathematics, Universitas Sriwijaya, Indonesia

*Corresponding author e-mail: eviyuliza@mipa.unsri.ac.id

Abstract. Capacitated vehicle routing problem is a vehicle routing problem that has constraints in the form of vehicle capacity. It is a matter of optimization to find the optimal route for some vehicles with a specific capacity and homogeneous vehicles that serve some agents with several known requests before the distribution process takes place. LPG distribution is the application of capacitated vehicle routing problems. This research aims to determine the LPG distribution route based on the Clarke and Wright Algorithm and LINGO. The capacitated vehicle routing problem model will be solved in two phases. The first stage completes the capacitated vehicle routing problem model using the Clarke and Wright algorithm. The second stage is completing the results of the first stage in the form of LINGO-based sub tours to obtain the optimal solution. The solution of the capacitated vehicle routing problem in distribution LPG using the Clarke and Wright algorithm has a total distance of 151.94 km. The solution of the capacitated vehicle routing problem in distribution LPG using LINGO has a total distance of 161.59 km.

1. Introduction

The distribution of products from sources (depots) to several destinations is a complex problem. The existence of several destinations for product delivery will cause several distribution channels that consider the distance and travel time. The distribution of LPG gas products at PT Lebong Terang to several customers in the Palembang city area is expected to be able to create reliable delivery performance in the distribution of gas products. The problem under investigation is how homogeneous vehicles can serve oil demand from some gas stations from the depot and minimize mileage. The purpose of this optimization is to find a series of routes that include n customers with a minimum overall distance [1]. A series of vehicle routes that aim at the minimum cost of several customer locations so that each route starts and ends at the same location, and several obstacles are met [2]. Laporte [2] summarizes the results for the classic vehicle routing problem (VRP) on existing vehicle capacity constraints. Classic VRP aims to find a set of vehicle routes that begin and end at a depot, some vehicles of the same capacity, and every customer is visited exactly once. The solution of the classic VRP is a set of routes that all start and end at the depot and meet the obstacle that all customers are served only once. Real-life VRP problems are often far more complex than classic VRP. In 1959, VRP on the issue of modeling truck deliveries was introduced by Dantzig and Ramser [3]. In 1964, Clarke & Wright generalized the VRP problem for linear optimization problems in logistics and transportation problems. The problem studied is how to serve a set of customers scattered around a central depot, using a fleet of trucks of various capacities [4]. VRP addresses logistical and transportation problems. VRP is a generalization of the traveling salesman problem (TSP). Unlike TSP, VRP has a goal to make a total trip, total distance, or total time for all trips with the total amount used [5]. The VRP problem has evolved in accordance with real-world problems [6].

The process of distributing goods will run smoothly if an optimal vehicle route can be determined. CVRP is a vehicle route problem to determine the optimal vehicle route located in a depot in delivery commodities to several customer locations, which combines requests from each customer related to vehicle capacity. The purpose of CVRP is to minimize transportation costs and overall time. CVRP is a VRP which has constraints in the form of vehicle capacity. CVRP is an optimization problem to find the optimal route for some vehicles with a certain capacity and homogeneous (having the same capacity), which serves some agents with the number of requests known before the distribution process takes place. The distribution in each vehicle can only be carried out once, namely from the depot to each agent and then back to the depot. It aims to service systems in determining distribution routes to be more effective, efficient and can increase the ability of companies to be able to meet product demand more quickly so that consumer trust and satisfaction increases. Capacitated vehicle routing problem is also known as the variants of the vehicle routing problem [7]. Ibrahim, Abdulaziz and Ishaya [7] solve the CVRP using The exact method and column generation then compare their solutions. Akhand, Peya, Sultana, and Al-Mahmud [8] solve CVRP in finding the optimal route of a vehicle using swarm intelligence. Mamat, Jaaman, and Ahmad [9] discussed two methods commonly used in logistics and transportation, VRP and CVRP to solve investment allocation problems. CVRP designs an optimal delivery route where each vehicle only takes one route, each vehicle has the same characteristics, each customer has a request, and there is only one central depot. Azad and Hasin [10] have researched to find a solution to the vehicle routing problem using a genetic algorithm that is able to determine the optimal route for the vehicles.

There are various models of Integer Linear Programming (ILP) from CVRP [11][12]. The CVRP model formed in this study uses the CVRP model proposed by Borcinova [12]. One of the main differences lies in how to eliminate sub-tours, namely the cycle that does not go through the depot. Solutions from VRP can be solved by exact methods, classic heuristic methods, and metaheuristic methods. Achutha, Caccetta, and Hill [13] developed new cutting planes on CVRP problems and used them in the branch and cut algorithms.

CVRP on LPG distribution is completed in two stages. In the first phase, CVRP on LPG distribution was completed using Clarke and Wright Algorithm. Clarke and Wright algorithm is a heuristic approach to solving CVRP. The principle of Clarke and Wright algorithm is sub-routes formed related to vehicle capacity. If the sub-routes formed exceed the capacity of the vehicle, then create a new sub-route [14]. Caccetta [14] has improved the Clarke and Wright algorithm to search the capacitated vehicle routing problem. The CVRP model in this study aims to minimize the total distance of LPG gas distribution. For the second stage, sub-routes of the results using Clarke and Wright Algorithm are solved using LINGO software. LINGO software is a tool to solve non-linear programming problems and has a strong function of operational research software [15][16]. The purpose of this stage will be investigated whether the sub-routes formed are optimal. The CVRP model in this research was completed using Clarke and Wright algorithm and software LINGO.

2. Methods

The Clarke and Wright algorithm is used to solve the LPQ gas distribution problem so that the LPG gas distribution route is optimal. CVRP mathematical models can be solved using LINGO software. LINGO software is built for optimization problems such as integer programming and linear programming [16][17]. The gas-based data collection was carried out at 24 gas base locations.

There are five steps taken in this study. They are collecting data in the form of demand of LPG gas base and the distance of each gas base from the agent, determining the distance matrices, solving CVRP with the Clarke and Wright algorithm, formulating CVRP models, and solving the CVRP model based on LINGO 13.0.

3. Result and discussion

3.1. Data

Here, we have the distance of the agent to each gas base data and the gas demand for each gas data. The agent distributes LPG gas to n locations, which $n = 24$. The type of vehicle used is a truck with a capacity Q of 4000 kg. $\mathbf{A}$ is a distance matrix (km) of the agent to each gas base, which 0-agent and *i*–gas base to *i* with *i* = 1, 2, 3, 4, 5, 6, 7, 8, 9, …, 24.

A =

–	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0	–	8.7	1.5	12	9	1.1	2.5	2	1.6	0.3	3.2	1.5	7.9	2.9	0.85	16	5.3	2.5	4.2	3	3.4	0.24	0.5	0.7	3.3
1	8.7	–	12	21	4.7	15	16	10	11	12	8.9	12	6.6	15	12	24	8	15	9.4	12	18	12	13	13	18
2	1.5	12	–	13	10	3.3	3.2	3.1	2.6	1	4.5	0.05	9.2	3.6	1.9	16	6.6	3.3	5.5	1.9	4.2	0.9	1.1	0.45	4.1
3	12	21	13	–	15	8.1	8.3	11	10	9.6	12	10	13	7	9.6	9.5	14	6.6	13	9.5	10	9.6	9.6	10	9.8
4	9	4.7	10	15	–	11	13	8.2	8.5	10	6.9	10	3.7	13	10	22	3.5	13	7.5	10	14	10	11	11	13
5	1.1	15	3.3	8.1	11	–	1.4	2.7	2.2	1.5	3.9	2.2	8.6	1.8	1.5	14	6	1.4	4.9	0.7	2.4	1.5	1.6	1.9	2.2
6	2.5	16	3.2	8.3	13	1.4	–	4.5	4.1	3.4	5.7	4.1	11	0.75	3.4	13	7.9	0.4	6.7	3.3	3.8	3.3	3.4	3.8	3.6
7	2	10	3.1	11	8.2	2.7	4.5	–	0.45	3.5	1.6	4.7	7.5	6	1.3	19	5	5.7	2.4	2.7	6.6	3.4	3.7	3.8	6.4
8	1.6	11	2.6	10	8.5	2.2	4.1	0.45	–	3	2.4	4.2	7.1	5.6	1	18	4.5	5.2	3.4	2.9	6.1	3	3.2	3.4	6
9	0.3	12	1	9.6	10	1.5	3.4	3.5	3	–	3.3	1	8	2.9	0.95	16	5.4	2.6	4.2	0.85	3.5	0.23	0.21	0.6	3.4
10	3.2	8.9	4.5	12	6.9	3.9	5.7	1.6	2.4	3.3	–	4.3	6.3	5.6	3	18	3.7	5.2	1.2	3	6.2	3	3.2	3.4	6
11	1.5	12	0.05	10	10	2.2	4.1	4.7	4.2	1	4.3	–	9.2	3.7	1.9	16	6.6	3.3	5.4	1.9	4.3	0.9	0.9	0.45	4.1
12	7.9	6.6	9.2	13	3.7	8.6	11	7.5	7.1	8	6.3	9.2	–	10	13	19	3	10	9.3	13	13	13	13	13	13
13	2.9	15	3.6	7	13	1.8	0.75	6	5.6	2.9	5.6	3.7	10	–	3.4	13	7.8	0.4	6.7	3.3	1.7	3.3	3.4	3.8	1.8
14	0.85	12	1.9	9.6	10	1.5	3.4	1.3	1	0.95	3	1.9	13	3.4	–	16	4.9	3.1	2.2	0.35	4	0.8	1.1	1.2	3.8
15	16	24	16	9.5	22	14	13	19	18	16	18	16	19	13	16	–	20	13	19	16	16	16	16	16	16
16	5.3	8	6.6	14	3.5	6	7.9	5	4.5	5.4	3.7	6.6	3	7.8	4.9	20	–	13	6.8	9.3	13	9.3	9.6	9.8	12
17	2.5	15	3.3	6.6	13	1.4	0.4	5.7	5.2	2.6	5.2	3.3	10	0.4	3.1	13	13	–	6.3	2.9	3.3	2.9	3	3	32
18	4.2	9.4	5.5	13	7.5	4.9	6.7	2.4	3.4	4.2	1.2	5.4	9.3	6.7	2.2	19	6.8	6.3	–	3.8	7	3.8	4.1	4.2	6.8
19	3	12	1.9	9.5	10	0.7	3.3	2.7	2.9	0.85	3	1.9	13	3.3	0.35	16	9.3	2.9	3.8	–	4.1	0.95	1.2	1.4	4
20	3.4	18	4.2	10	14	2.4	3.8	6.6	6.1	3.5	6.2	4.3	13	1.7	4	16	13	3.3	7	4.1	–	3.9	3.9	4.3	1
21	0.24	12	0.9	9.6	10	1.5	3.3	3.4	3	0.23	3	0.9	13	3.3	0.8	16	9.3	2.9	3.8	0.95	3.9	–	0.45	0.45	3.5
22	0.5	13	1.1	9.6	11	1.6	3.4	3.7	3.2	0.21	3.2	0.9	13	3.4	1.1	16	9.6	3	4.1	1.2	3.9	0.45	–	0.45	3.4
23	0.7	13	0.45	10	11	1.9	3.8	3.8	3.4	0.6	3.4	0.45	13	3.8	1.2	16	9.8	3	4.2	1.4	4.3	0.45	0.45	–	3.9
24	3.3	18	4.1	9.8	13	2.2	3.6	6.4	6	3.4	6	4.1	13	1.8	3.8	16	12	32	6.8	4	1	3.5	3.4	3.9	–

The gas demand for each gas base is shown in table 1.

Table 1. Gas demand.

	Tube/month	Kg/month
*d*1	1600	4800
*d*2	450	1350
*d*3	910	2730
*d*4	840	2520
*d*5	400	1200
*d*6	1300	3900
*d*7	1080	3240
*d*8	480	1440
*d*9	250	750
*d*10	520	1560
*d*11	350	1050

	Tube/month	Kg/month
*d*12	720	2160
*d*13	150	450
*d*14	650	1950
*d*15	4480	13440
*d*16	480	1440
*d*17	780	2340
*d*18	960	2880
*d*19	900	2700
*d*20	500	1500
*d*21	300	900
*d*22	500	1500
*d*23	200	600
*d*24	2100	6300

Which *di* is the number of requests from gas base *i* for $i = 1, 2, 3, \ldots, 24$.

3.2. Clarke and Wright algorithm

CVRP in LPG gas distribution can be determined as a graph $G = (V, E)$ with $V = (0, 1, 2, 3, \ldots, 23)$ as the set of vertices and $E = \{(i,j) \mid i, j \in V, i \neq j\}$ as the set of arcs, where vertex represents the depot for a fleet of vehicles with the same capacity, and the remaining n vertices represent customers.

Based on table 2, the route for distributing gas to PT Terang Lebong uses Clarke and Wright algorithm as follows: agent – 5th gas base – 11th gas base – agent, agent – 3rd gas base – 13th gas base – agent, agent – 17thgas base – 15thgas base – agent, agent – 2nd gas base – 20thgas base 20 – agent, agent – 8thgas base – 10thgas base – agent, agent – 14thgas base – 18th gas base - agent, agent –19thgas base – 21stgas base - agent, agent – 22nd gas base – 23rd gas base – agent, agent – 14th gas base – 16th gas base – agent, agent – 7th gas base – 9thgas base - agent, agent – 12th gas base–1stgas base – agent, agent – 24th gas base – agent, and agent – 6thgas base – agent with a total distance of 151.94 km 14 sub tours.

Table 2. Distribution of LPG vehicle routes using Clarke and Wright Algorithm.

Sub Tours	Distribution routes	Total gas (tube)	Distance traveled (km)
1	0-4-16-0	1320	17.8
2	0-3-13-9-0	1310	22.2
3	0-10-12-0	1240	17.4
4	0-5-19-0	1300	4.8
5	0-2-11-23-21-0	1300	2.69
6	0-8-14-0	1130	3.45
7	0-17-20-0	1280	9.2
8	0-1-0	1600	17.4
9	0-6-0	1300	5
10	0-7-0	1080	4
11	0-15-0	4480	32
12	0-18-0	960	8.4
13	0-22-0	500	1
14	0-24-0	2100	6.6
Total distance			151.94

Next, the CVRP mathematical model formulation is formed, which is solved using LINGO.

3.3. Solution of the CVRP model using LINGO

The CVRP model for LPG gas distribution is the CVRP model proposed by Borcinova [12]. CVRP Model for route 2 is as follows.

Minimize:

$$12x_{03} + 2.9x_{013} + 0.3x_{09} + 12x_{30} + 7x_{313} + 9.6x_{39} + 2.9x_{130} + 7x_{133} + 2.9x_{139} + 0.3x_{90} + 9.6x_{93} + 2.9x_{913}$$

Subject to:

$$x_{03} + x_{133} + x_{93} \leq 1$$
$$x_{013} + x_{313} + x_{913} = 1$$
$$x_{09} + x_{39} + x_{139} = 1$$
$$x_{03} + x_{013} + x_{09} = 1$$
$$x_{03} + x_{133} + x_{93} - x_{30} - x_{313} - x_{39} = 0$$
$$x_{013} + x_{313} + x_{913} - x_{130} - x_{133} - x_{139} = 0$$
$$x_{09} + x_{39} + x_{139} - x_{90} - x_{93} - x_{913} = 0$$
$$2730x_{03} + 2730x_{133} + 2730x_{93} + 450x_{013} + 450x_{313} + 450x_{913} + 750x_{09} + 750x_{39} + 750x_{139} \leq 4000$$
$$x_{133} + x_{93} \leq 2$$
$$x_{313} + x_{913} \leq 2$$
$$x_{39} + x_{139} \leq 2$$

The results of calculations from the CVRP model using LINGO, as shown in table 3.

Table 3. Distribution of LPG vehicle routes using LINGO.

Sub Tours	Distribution Routes	Total gas (tube)	Distance traveled (km)
1	0-16-4-0	1320	17.8
2	0-13-9-0	400	6.1
3	0-3-0	910	24
4	0-12-10-0	1240	17.4
5	0-5-19-0	1300	4.8
6	0-2-11-0	800	3.05
7	0-23-21-0	500	1.39
8	0-14-8-0	1130	3.45
9	0-17-20-0	1280	9.2
10	0-6-0	1300	5
11	0-1-0	1600	17.4
12	0-7-0	1080	4
13	0-15-0	4480	32
14	0-18-0	960	8.4
15	0-22-0	500	1
16	0-24-0	2100	6.6
Total distance			161.59

The route for distributing gas to PT Terang Lebong uses the LINGO as follows: agent –16th gas base – 4th gas base – agent, agent –13th gas base – 9th gas base - agent, agent – 3rd gas base – agent, agent – 12thgas base – 10th gas base – agent, agent – 5thgas base – 19thgas base – agent, agent – 2nd gas base – 11th gas base – agent, agent – 23rd gas base – 21st gas base – agent, agent – 14th gas base – 8th gas base - agent, agent –17thgas base –20thgas base - agent, agent – 6thgas base - agent, agent –1stgas base - agent, agent –7thgas base - agent, agent –15thgas base – agent, agent – 18thgas base – agents, agents – 22nd gas base – agents and agents – 24th gas base – agents with a total distance of 161.59 km and 16 sub tours.

The formulation of CVRP solved using LINGO produces more efficient results than using Clarke and Wright algorithm. Borcinova [12] discussed that the formulation of CVRP has more efficient results when it is computationally solved. LPG gas distribution route using LINGO is more optimal than the Clarke and Wright algorithm.

4. Conclusion

In this research, VRP is solved using Clarke and Wright algorithm so that sub-routes are obtained. Next, the sub-routes formed are completed using LINGO software. There are differences in sub-routes formed by using Clarke and Wright algorithm with LINGO software. This is because the sub-routes completed with LINGO must be related to several inequalities. The solution of capacitated vehicle routing problem in distribution LPG using Clarke and Wrigth algorithm has a total distance of 151.94 km. The solution of the capacitated vehicle routing problem in distribution LPG using LINGO has a total distance of 161.59 km. For further research, the CVRP model can pay attention to time windows, and the CVRP model can be solved with other non-linear programming problem software such as CPLEX.

Acknowledgments

This research is supported by the Faculty of Mathematics and Natural Sciences, Universitas Sriwijaya through Science and Technology Research Grant Scheme, the year 2019.

References

[1] Jepsen M K and Pisinger D 2011 *Branch-and-cut and branch-and-cut-and-price algorithms for solving vehicle routing problems* Thesis Technical University of Denmark
[2] Laporte G 2007 *Res. Logist.* **54** 811–9
[3] Clarke G and Wright J W 1964 *Oper. Res.* **12** 568–81
[4] Braekers K, Ramaekers K and Van Nieuwenhuyse I 2016 *Comput. Ind. Eng.* **99** 300–13
[5] Beresneva E N and Avdoshin S 2018 *Proc. Inst. Syst. Program. RAS* **30** 235–50
[6] Atefi R, Salari M, C. Coelho L and Renaud J 2018 *Eur. J. Oper. Res.* **265** 316–27
[7] Ibrahim A A, Lo N, Abdulaziz R O and Ishaya J A 2019 *Int. J. Res.* **7** 310–27
[8] Akhand M A H, Peya Z J, Sultana T and Al-Mahmud 2015 *Proc. Int. Conf. Electr. Inf. Commun. Technol.* (Khulna, Bangladesh: IEEE)
[9] Mamat N J Z, Jaaman S H and Ahmad R 2016 *AIP Conf. Proc.* **1784** pp 50001-1-050001–7
[10] Azad T and Hasin M A A 2019 *Int. J. Logist. Syst. Manag.* **32** 132–46
[11] Alipour M M 2012 *J. Comput. Sci. Issue* **9** 138–45
[12] Borcinova Z 2017 *Croat. Oper. Res. Rev.* **8** 463–9
[13] Achuthan N R, Caccetta L and Hill S 2003 *Transp. Sci.* **37** 153–69
[14] Caccetta L, Alameen M and Abdul-niby M 2013 A *Eng. Technol. Appl. Sci. Res.* **3** 413–5
[15] Liangjun M, Peilin Z and Kechao S 2010 *Int. Conf. Mach. Vis. Hum-Mach Interface* (Kaifeng, China: *IEEE*)
[16] Liu B and Jin N 2015 *IEEE Adv. Inf. Technol. Electron. Autom. Control Conf.* (Chongqing, China: IEEE)
[17] Zhang Y Z and Xie S Y 2017 *2nd Asia-Pacific Conf. Intell. Robot Syst.* (Wuhan, China: IEEE)

SERTIFIKAT

No. 152/Pan.SENATIK/P.Mat/VIII/2020

diberikan Kepada

Evi Yuliza, M.Si

sebagai

PEMAKALAH

dalam kegiatan Seminar Nasional Matematika dan Pendidikan Matematika Tahun 2020 dengan tema "Merdeka Belajar: Integrasi Teknologi dalam Pembelajaran Matematika" pada tanggal 12 - 13 Agustus 2020 yang diselenggarakan oleh Program Studi Pendidikan Matematika FPMIPATI Universitas PGRI Semarang.

Semarang, 13 Agustus 2020

Dekan FPMIPATI	Ketua Panitia
Dr. Nur Khoiri, S.Pd., M.T., M.Pd.	Noviana Dini Rahmawati, S.Pd., M.Pd.
NPP. 047801165	NPP. 118701355