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Preface: The 6th National Conference on Mathematics and Mathematics Education (SENATIK)

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Preface: The 6th National Conference on Mathematics and Mathematics Education (SENATIK)

The 6th National Conference on Mathematics and Mathematics Education (SENATIK) was held by Mathematics Education Study Program, Universitas PGRI Semarang, Indonesia, in 11 August 2021. The seminar theme is Numerize and Digitaze of Mathematics Toward Freedom of Learning. According to the theme, this seminar aims to improve mathematics teaching, solve mathematics problems, and expand mathematics contribution to society.

Freedom of learning is a policy implemented by the Indonesian Ministry of Education and Culture. Freedom learning encourages students to master literacy, numeracy, and character. Numeration is one of the ways to make mathematics easy. At the same time, it provides opportunities for students to collaborate, has critical thinking, creative thinking, communication, good character, and face the challenges of an increasingly global world with advances in science and technology. Having numeracy skills will impact good thinking patterns and habits associated with numbers or calculations with existing problems.

Along with the freedom learning program development during the COVID-19 pandemic, it is very clear that technological developments have a high impact on the education world. This impact also occurs in the learning process, especially in accessing information as a learning resource, both online and offline learning. The availability of abundant information and easily accessible also causes learning to experience a digitization process. The era of digitalization brings challenges as well as opportunities in the world of education. There is an opportunity to integrate technology into the learning process so that learning outcomes are more effective. The integration of technology in the learning process results in digitization in the education world, especially in the learning process. The findings that were discussed in the seminar: In mathematics learning and problem-solving, teachers and students need technology. Integration of mathematics and technology is a crucial process.

There are 151 manuscripts through the peer-review and end up with 76 papers which are published in this AIP Conference Proceeding. Together with the keynote speakers and the presenters, they shared their research results on different fields in the plenary and parallel sessions attended by more than 300 participants.

We want to thank the keynote speakers; 1) Prof. Helia Jacinto, Ph. D. (University of Lisbon, Portugal); 2) Dr. Rully Charitas Indra Prahmana, S.Si., M.Pd. (Universitas Ahmad Dahlan, Indonesia), and; 3) Dr. Muhtarom, M.Pd. (Universitas PGRI Semarang, Indonesia). Many thanks go as well to the speakers in the workshop session that are Sutrisno, S.Pd., M.Pd. (Universitas PGRI Semarang, Indonesia) and Dr. Muhtarom, M.Pd (Universitas PGRI Semarang, Indonesia). We also would like to thank all the committee for arranging this conference.

The conference's success is achieved due to the support and commitment of many people, and we acknowledge their contribution, especially all the participants and presenters. For all participants and presenters, we hope they enjoy the seminar, so they are valuable, rewarding and improving their knowledge and experiences.

Thank you,

Dr. Widya Kusumaningsih, M. Pd.
Chairman The 6th National Conference on Mathematics and Mathematics Education
SENATIK
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Committee: The 6th National Conference on Mathematics and Mathematics Education (SENATIK)

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PRELIMINARY

MATHEMATICS

Mathematical model of information service pricing scheme based on utility functions of constant elasticity of substitution

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Fitri Maya Puspita, Melisa Ulina, Evi Yuliza, et al.

Mathematical Model of Information Service Pricing Scheme Based on Utility Functions of Constant Elasticity of Substitution

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Abstract. Today, the internet has become an essential thing, so it becomes a challenge for service providers to provide the best service and gain maximum benefits by maintaining the utility functions of Constant Elasticity of Substitution (CES). This study seeks to formulate an optimal pricing scheme model analytically with three pricing schemes, namely flat-fee, usage-based, and two-part tariff on homogeneous consumer issues, heterogeneous consumers (high-end and low-end), and heterogeneous consumers (high-demand and low-demand). The models designed analytically through lemmas will be compared and analyzed to determine which pricing scheme models achieve the most optimal results based on CES utility functions. The optimal pricing scheme model for each type of consumer is applied to the mail traffic data obtained from one of the local servers in Palembang. The optimal profit obtained by ISPs for homogeneous consumers is if ISPs implement flat-fee and two-part tariff pricing schemes. The optimal advantage is obtained by ISPs applying heterogeneous (high-end and low-end) pricing schemes, namely two-part tariffs. For heterogeneous consumers (high-demand and low-demand), optimal profit is obtained by ISPs if applying the flat-fee scheme.

INTRODUCTION

The rapid development of the times makes people use the internet more. The more people who use the internet, the Internet Service Provider (ISP) must also create internet services that attract many consumers. The use of the internet emerges rapidly then becomes a challenge for ISP in satisfying consumers [1][2][3]. Interconnected networking or often called the internet, is a global system of all computer networks that are interconnected using the standard Internet Protocol Suite (TCP/IP) to serve many internet users, and the internet is a global provider of information [4][5][6][7].

ISP is an internet service company or manufacturer that provides internet access services or online-based communication and information media. This network has a wide reach so that consumers can connect to the global internet network. QoS (Quality of Service) is the way of measurement of how good the network and network quality is an effort to interpret the characteristics and properties of the service [7][8].

A utility function is a function to measure the level of satisfaction with what consumers get for a specific purpose [9][10][11][12]. This study uses the utility function of Constant Elasticity of Substitution (CES), which is still rarely used in internet pricing schemes. Fortunately, this function has the advantage of the elasticity of substitution between inputs is not fixed [13]. It is important to analyze the pricing strategy used in the CES utility function because of its advantage. Then, three pricing schemes used in this study, namely flat-fee, usage-based, and two-part tariff [12][14][15], will be used to analyze the strategies for homogeneous consumers, high-end and low-

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end heterogeneous consumers, and heterogeneous high demand and low demand consumers. So, the new study concerning the information service pricing scheme based-CES utility function becomes critical to be developed due to the CES utility function's ability to be elastic in substituting between inputs.

The contribution of the study then is to get a new comparison among pricing schemes in mathematical model analytically through lemmas to get new formulate a maximum pricing scheme model for service users by using CES utility functions and for three pricing schemes for heterogeneous consumer problems (high-end and low-end) and heterogeneous consumers (high and low demand).

METHOD

In this study, the data used was mail traffic data taken from January 1, 2021-February 28, 2021, and used to validate the model designed analytically, obtained from the local server in Palembang.

The steps taken in conducting the research are described as follows.

1. Describe secondary data consisting of inbound and outbound data, categorized into 2 groups: rush hour (07.00 AM-5.00 PM) and non-peak hours (07.00 PM-05.00 AM), Indonesian time.
2. This data described in Step 1 is used to elaborate the analytical results to show exact results.
3. Describe parameters and decision variables that will be used in modeling internet pricing schemes.
4. Establish the models of internet pricing schemes based on the function of the CES utility with three pricing schemes, namely flat-fee, usage-based, and two-part tariff, which will be compared on consumer-type by designed lemmas per pricing schemes.
5. Complete the internet pricing scheme model by analyzing using the differential method.
6. Compare the optimal revenue of each internet scheme model using the secondary data to validate the model designed.

RESULT AND DISCUSSION

From this study, optimization is divided into two categories, namely consumer problems and provider problems which are stated and adopted [15].

Consumer Problem Optimization

$$\max_{F.G.A} \ln F^c - \ln G^y - J_f F - J_g G - JA \tag{1}$$

Subject to:

$$F \le \overline{F} A$$
$$G \le \overline{G} A$$
$$\ln F^c + \ln G^y - J_f F - J_g G - JA \ge 0$$
$$A = 0 \text{ or } 1$$

Optimization of Provider Problems

$$\max_{J.J_F.J_G} \sum i\left(J_f F^* + J_g G^* + JA\right) \tag{2}$$

With $\left(F_{X*}, G_{X*}, A_{X*}\right) \arg\max U_X\left(F_X, G_X\right) J_f F_X - J_g G_X - J_\alpha A_X$

Subject to:

$$F \le \overline{\overline{F}} A$$
$$G \le \overline{G} A$$

$$\ln F^c - \ln G^y - J_f F - J_g G - JA \geq 0$$
$$A_i = 0 \text{ or } 1$$

Equation (1) explains the optimization of consumer problems using CES utility function. Meanwhile, Equation (2) shows the optimization of the provider problem.

The parameters stated are as follows.

J	:	Subscription fees incurred by consumers to follow the service.
J_F	:	The unit price of the service set by the service provider during peak hours.
J_G	:	Service unit price set by the off-peak service provider.
$U_i\ (J_i, G_i)$	:	The consumer utility function i on the level of consumption at peak hours and non-peak hours Where J_i is the maximum consumption level of consumer i in peak hour service.
G_i	:	The maximum consumption level of consumers in off-peak hours.
J_i	:	Consumer consumption rate i of services during peak hours.
G_i	:	Consumer consumption level i of the service during off-peak hours.
A_i	:	The variable that is worth 1 if the consumer chooses to join the program and if not join is worth 0.
$\overline{G}_i$	:	The highest level of consumption of consumer i of services during off-peak hours.

For Provider

$J_{i*} = J_i\left(J_F J_G J\right)$	:	Consumer service consumption rate i during peak hours.
$G_{i*} = G_i\left(J_F J_G J\right)$	:	Consumer service consumption level i during off-peak hours.
$A_{i*} = A_i\left(J_F J_G J\right)$	:	Consumer decision variable i about participants.
$U_i\left(J_i, G_i\right)$	:	The consumer utility function i on the level of consumption at peak hours and not busy hours
J_i	:	maximum consumption level of consumer i in peak hour services and G_i is the maximum consumption level of consumers in off-peak hours.
$\overline{F}_i$	:	The maximum consumption level of consumer i during peak hours of service.
$\overline{G}_i$	:	The maximum consumption level of consumer i during off-peak hours.

The decision variables are stated as follows.

J	:	The fees charged for consumers who subscribe to the service program
J_F	:	Prices charged by service providers during off-peak hours
J_G	:	Prices charged by service providers during peak hours.

Constant Elasticity of Substitution (CES) Utility Function

According to Balasko [16], the function of the CES form is as follows.

$$U\left(c, y\right) = \ln F^c + \ln G^y : c, y > 0 \tag{3}$$

Where c and y are constants and Equation (3) shows the utility function to be utilized in the proposed model.

Homogeneous Consumers

Optimization of Consumer Problems is as follows:

$$\max_{F.G.A} \ln F^c - \ln G^y - J_f F - J_g G - JA \tag{4}$$

Subject to:

$$F \le \overline{\overline{F}}A \tag{5}$$

$$G \le \overline{G}A \tag{6}$$

$$\ln F^c - \ln G^y - J_f F - J_g G - JA \ge 0 \tag{7}$$

$$A = 0 \text{ or } 1$$

Where Equation (4) explains the objective function for maximizing the consumer problem subject to Equations (5)-(7), and the decision values are set as binary values.

Optimization of Provider Problems:

$$\max_{J.J_F.J_G} \sum x\left(J_f F^* - J_g G^* + JA^*\right) \tag{8}$$

With $\left(F^* G^* A^*\right)$ = arg max $F^c G^y - J_f F - J_g G - JA$
Subject to:

$$F \le \overline{F}A$$
$$G \le \overline{G}A$$
$$\ln F^c + \ln G^y - J_f F - J_g G - JA \ge 0$$
$$A_X = 0 \text{ or } 1$$

Where Equation (8) informs that ISP intend to maximize the objective function to get revenue from the consumer subject to the constraints.

Next, the discussions of optimal benefits on each pricing scheme applying the CES utility function that will be used by ISP, are conducted analytically by establishing lemmas for each pricing strategy.

Case 1: If the ISP uses a flat-fee price, it will be determined as $J_F = 0$, $J_G = 0$, and $J > 0$. The specified value set that will be used by the ISP, does not affect the time of use when (peak hours and non-busy hours). So, optimization of consumer problems becomes this following Expression (9).

$$\max_{X,Y,Z} \ln F^c + \ln G^y - J \tag{9}$$

where Expression (9) shows the result for objective function value after the first manipulation of the computation. For example, using constraints on Equation (7), then we obtain Expression (10).

$$\ln F^C + \ln G^Y - J_f F - J_g G - JA \ge 0 \Leftrightarrow J \le \ln F^c + \ln G^y \tag{10}$$

Then the Provider's problem is expressed in Equation (11).

$$\max_{J,J_F,J_G} \sum i\left(J_f F^* + J_g G^* + JA\right) = \max_{J,J_F,J_G} \sum i\left[\ln F^c + \ln G^y\right] \tag{11}$$

Equations (10) and (11) show the process to get objective function for case 1 by setting a flat fee scheme for homogeneous users. Then, produced consumer prices are $\left[\ln \overline{F}^c + \ln \overline{G}^y\right]$ from ISP. The optimal value that ISP obtained from its consumers is $\overline{F}^c \overline{G}^y$, and optimal profit will be $\sum i\left[\ln \overline{F}^c + \ln \overline{G}^y\right], \forall i$. Thus, Lemma 1 is obtained.

Lemma 1: If the ISP chooses to use a flat-fee pricing package, it will be given a value of $\left[\ln \overline{F}^{c} \ln \overline{G}^{y}\right]$ with the optimal advantage that ISP gets is $\sum i\left[\ln \overline{F}^{c} + \ln \overline{G}^{y}\right], \forall i$.

Case 2: If the ISP uses usage-based price, it will be determined $J_F > 0, J_G > 0$ and $J = 0$. Then, ISP hereby distinguishes the value during peak hours and not busy hours. According to the provisions in Equation (4), then the known function is expressed by Equation (12).

$$\max_{F,G,A} F^{c} + \ln G^{y} - J_f F - J_g G \tag{12}$$

To optimize the function by using the necessary conditions and sufficient conditions as follows
The necessary condition:

$$\frac{\partial\left(\ln F^{c} + \ln G^{y} - J_f F - J_g G\right)}{\partial F} > 0 \text{ Then it will be } C\frac{1}{F} - J_F = 0 \Leftrightarrow J_f = \frac{C}{F} \Leftrightarrow F^* = \left(\frac{c}{J_F}\right) \tag{13}$$

The sufficient condition:

$$\frac{\partial^2\left(\ln F^{c} + \ln G^{y} - J_f F - J_g G\right)}{\partial F^2} > 0 \Leftrightarrow \frac{\partial\left(cF^{-1}\right)}{\partial F} > 0 \Leftrightarrow \frac{-c}{F^2} > 0. \text{ So } J_F = \frac{c}{f} \text{ is the optimal price, and}$$

The necessary condition:

$$\frac{\partial\left(\ln F^{c} + \ln G^{y} - J_f F - J_g G\right)}{\partial G} = 0 \text{. So, } y\frac{1}{G} - J_G = 0 \Leftrightarrow J_G = \frac{y}{F} \Leftrightarrow G^* = \left(\frac{y}{J_G}\right) \tag{14}$$

The sufficient condition:

$$\frac{\partial^2\left(\ln F^{c} + \ln G^{y} - J_F F - J_G G\right)}{\partial G^2} > 0 \Leftrightarrow \frac{\partial\left(YG^{-1}\right)}{\partial G} > 0 \Leftrightarrow \frac{-y}{G^2} > 0. \text{ So } J_G = \frac{y}{G} \text{ is the optimal price.}$$

Then, the optimization provider problems are displayed in Expression (15).

$$\max_{J,J_F,J_G} \sum i\left[\left(c+y\right)\right] \tag{15}$$

with Expressions (12)-(15) explain the steps taken in proving using necessary and sufficient conditions for both variables F and G. While J = 0, the optimal consumer value is $J_F = \frac{-c}{F^2}$ and $J_G = \frac{-y}{G^2}$ and optimal advantages $\sum i\left[\left(c+y\right)\right]; \forall i$. Thus Lemma 2 is obtained.

Lemma 2: If the ISP chooses to use a usage-based pricing scheme, it will be given the values $J_F = \frac{c}{F^2}$ and $J_G = \frac{y}{G^2}$ with the optimal profit obtained by the ISP is $\sum i\left[\left(c+y\right)\right]; \forall i$.

Case 3: If the ISP uses the two-part tariff price, it will be determined $J_F > 0, J_G > 0$, and $J > 0$ then the ISP will charge a joining fee during peak hours and not busy hours. Equation (10) and Equation (5) were used and then substituted to Equation (7). Then the constraint becomes $P \le \ln F^c + \ln G^y - c - y$. So, optimization of provider problems:

$$\max_{J, J_F, J_G} \sum i \left[\ln F^C + \ln G^y \right] \tag{16}$$

Where Expression (16) describes that objective function for a two-part tariff scheme for homogeneous users. Then, the value of consumers that are introduced is J_F and J_G will be $J_F = \dfrac{c}{F}$ and $J_G = \dfrac{Y}{F}$.

Lemma 3: If the ISP chooses to use a two-part tariff pricing package*,* it will be given a value of $J_F = \dfrac{c}{\overline{F}}$ and $J_G = \dfrac{y}{\overline{G}}$ with the optimal advantage that ISP obtained is $\sum i \left[\ln \overline{F}^C + \ln \overline{G}^y \right]; \forall i$

High-end and Low-end Heterogeneous Consumers

Suppose there are consumers high-end (i = 1) and n consumers low-end (i = 1). Then the level of consumption at peak hours and non-peak hours is stated as $c_1 > c_2$ and $y_1 > y_2$.
Optimization Problem of Consumers will be $\max_{F_i, G_i, A_i} \ln F_i^{c_i} + \ln G_i^{y_i} - J_F F_i - J_G G_i - J A_i$
Subject to:

$$F_i \le \overline{F} A_i$$
$$G_i \le \overline{G} A_i$$
$$\ln F_i^{c_i} + \ln G_i^{y_i} - J_F F_i + J_G G_i - J A_i \ge 0$$
$$A_i = 0 \text{ or } 1$$

Optimization Problems of Provider:

$$\max_{J, J_F, J_G} m\left(J_F F_1 * + J_G G_1 * + J A_1 *\right) + n\left(J_F F_1 * + J_G G_2 * + J A_2 *\right)$$

With $F_1*, G_1*, A_1* = \arg\max F_i c_i G_i y_i - J F_i - J_G G_i - J A_i$
Subject to:

$$F_i \le \overline{F} A_i$$
$$G_i \le \overline{G} A_i$$
$$\ln F_i^{c_i} + \ln G_i^{y_i} - J_F F_i + J_G G_i - J A_i \ge 0$$
$$A_i = 0 \text{ or } 1$$

The following discusses the maximum profit determinants in the pricing scheme used by service providers.
Case 4: If the ISP uses a flat-fee pricing scheme, then it is specified by $J_F = 0, J_G = 0$ and $J > 0$. Thus, every high-end consumer will be charged a fee of $J \le \ln \overline{F}_1^c + \ln \overline{G}_1^y$ and low-end consumers are of $J \le \ln \overline{F}_2^{c_2} + \ln \overline{G}_2^{y_2}$. To maximize profits, the service provider will use the $J = \ln \overline{F}_2^{c_2} + \ln \overline{G}_2^{y_2}$. Optimization Problems of Providers will be

$\max_{P} m\left(JA_1 *\right)+n\left(JA_2 *\right)=\left(m+n\right)\left(\ln \overline{F}^{c_2}+\ln \overline{G}^{y_2}\right)$. The maximum profit of the providers is $(m+n)(\ln\bar{F}^{c_2}+\ln\bar{G}^{y_2})$. Therefore, from this case, Lemma 4 was obtained.

Lemma 4: If the ISP chooses to use a flat-fee pricing scheme. Then the value given is $\ln \overline{F}^{c_2}+\ln \overline{G}^{y_2}$ with the optimal advantage that ISP achieved is $\left(m+n\right)\left(\ln \overline{F}^{c_2}+\ln \overline{G}^{y_2}\right)$.

Case 5: If the ISP usage-based price is specified $J_F>0, J_G>0$ and $J=0$.

Therefore, optimization of high-end heterogeneous consumer problems is as follows:

$$\max_{F,G,A} \ln F_1^{c_1}+\ln G_1^{y_1}-J_F F_1-J_G G_1$$

To optimize price will be used necessary and sufficient conditions as follows.
The necessary condition:

$$\frac{\partial\left(\ln F_1^{c_1}+\ln G_1^{y_1}-J_F F_1-J_G G_1\right)}{\partial F_1}=0 \text{ then is obtained } c_1\frac{1}{F_1}-J_F=0 \Leftrightarrow J_F=\frac{c_1}{F_1} \Leftrightarrow F_1*=\left(\frac{c_1}{J_F}\right)$$

The sufficient condition:

$$\frac{\partial^2\left(\ln F_1^{c_1}+\ln G_1^{y_1}-J_F F_1-J_G G_1\right)}{\partial F_1^2}>0 \text{ then is obtained } \frac{\partial\left(c_1 F_1^{-1}\right)}{\partial F_1}>0 \Leftrightarrow \frac{-c_1}{F_1^2}>0$$

The necessary condition:

$$\frac{\partial\left(\ln F_1^{c_1}+\ln G_1^{y_1}-J_F F_1-J_G G_1\right)}{\partial G_1}=0 \text{ then is obtained } y_1\frac{1}{G_1}-J_G=0 \Leftrightarrow J_G=\frac{y_1}{G_1} \Leftrightarrow G_1*=\left(\frac{y_1}{J_G}\right)$$

The sufficient condition:

$$\frac{\partial^2\left(\ln F_1^{c_1}+\ln G_1^{y_1}-J_F F_1-J_G G_1\right)}{\partial G_1^2}>0 \text{ then is obtained } \frac{\partial\left(c_1 G_1^{-2}\right)}{\partial G_1}>0 \Leftrightarrow \frac{-c_1}{G_1^2}>0$$

Optimization of heterogeneous consumer problems of the low-end: $\max_{J,J_F,J_G} \ln F_2^{c_2}+\ln G_2^{y_2}-J_F F_2-J_G G_2$.
To optimize the price, the necessary terms and conditions are used:
The necessary condition:

$$\frac{\partial\left(\ln F_2^{c_2}+\ln G_2^{y_2}-J_F F_2-J_G G_2\right)}{\partial F_2}=0 \text{ then is obtained } c_2\frac{1}{F_2}-J_F=0 \Leftrightarrow F_2*=\left(\frac{c_2}{J_F}\right)$$

The sufficient condition:

$$\frac{\partial^2\left(\ln F_2^{c_2}+\ln G_2^{y_2}-J_F F_2-J_G G_2\right)}{\partial F_2{}^2}>0 \text{ then is obtained } \frac{\partial\left(c_2 F_2^{-2}\right)}{\partial F_2}>0 \Leftrightarrow -\frac{c_2}{F_2^2}>0$$

This analysis is then applied during peak hours and during non-peak hours

1. Peak hour issues; ISP optimizes $J_F, J_F \le \frac{c_1}{F_1}$ to maximize the objective function in Equation (11). When ISP sets the price of $J_F \le \frac{c_1}{F_1}$ then the profit is not maximum if $F_1{}^* \le \overline{F}$ or $F_2{}^* \le \overline{F}$. Then to make J_F the optimal price it must be $\frac{c_2}{F_2} \le J_F \le \frac{c_1}{F_1}$.
2. Problems during non-peak hours; ISP should optimize $J_G; J_G \le \frac{y_1}{G_1}$ to maximize Equation (11). When ISP settles the $J_G \le \frac{c_1}{G_1}$ then the profit is not maximum if $G_1{}^* \le \overline{G}$ or $G_2{}^* \le \overline{G}$. Then to maximize J_G it has to be $\frac{c_2}{G_2} \le J_G \le \frac{c_1}{G_1}$.

Optimization Problems of Providers: $\max\limits_{J,J_F,J_G} m\left(J_F F_1{}^* + J_G G_1{}^*\right) + n\left(J_F F_1{}^* + J_G G_2{}^*\right) = \max\limits_{J,J_F,J_G} (m+n)\left(c_2+y_2\right)$

So, for both consumers (high-end and low-end) for the optimal price charged for peak hours is $J_F = \frac{c_2}{F_2}$ dan for not busy hours is $J_G = \frac{y_2}{G_2}$, and for optimal advantages is $(m+n)\left(c_2+y_2\right)$. Then Lemma 5 is obtained.

Lemma 5: If the ISP chooses to use a usage-based pricing scheme. Then the value given for peak hours is $J_F = \frac{c_2}{F_2}$ and hours are not busy $J_G = \frac{y_2}{G_2}$ with the optimal advantage that ISP got will be $(m+n)\left(c_2+y_2\right)$.

Case 6: If the ISP uses the two-part tariff price, then it is determined that $J_F > 0; J_G > 0$ and $J > 0$. For optimization of consumer problems of high-end and low-end, Equations (12)-(15). Equation (12) and Equation (13) will be used to represent the demand curve of high-end and low-end consumers during peak hours. Equations (14)-(15) represent the high-end and low-end consumer demand curves during peak hours. If it is specified that $c_1 > c_2$ then the determination of the high-end costs will follow the value for the low-end costs so: $c_1(m) < c_2(m+n) \Leftrightarrow c\frac{c_1(m+n)}{m}$. If the consumer is charged as much $J_F = \frac{c_1}{F_1}; J_G = \frac{y_1}{G_1}$ and $J = \ln F_1^{(c_1-1)} + \ln G_1^{(y_1-1)} - \left(c_1+y_1\right)\left(\ln F_1^{c_1} + \ln G_1^{y_1}\right)$. Then, only high-end consumers who can join this service because the low-end has constraints, that is $J_F = \frac{c_2}{F_2}; J_G = \frac{y_2}{G_2}$ and $J = \ln F_2^{(c_2-1)} + \ln G_2^{(y_2-1)} - \left(c_2+y_2\right)\left(\ln F_2^{c_2} + \ln G_2^{y_2}\right)$. The optimal advantage that ISPs got is $J_F = \frac{c_2}{F_2}; J_G = \frac{y_2}{G_2}$ and $J = \ln F_2^{(c_2-1)} + \ln G_2^{(y_2-1)} - \left(c_2+y_2\right)\left(\ln F_2^{c_2} + \ln G_2^{y_2}\right)$. ISP problems will be $\max\limits_{J,J_F,J_G} m\left(J_F F_1{}^* + J_G G_1{}^* + JA_1{}^*\right) + n\left(J_F F_1{}^* + J_G G_2{}^* + JA_2{}^*\right) = (m+n)\left[\ln \overline{F}^{c_2} + \ln \overline{G}^{y_2}\right]$

Lemma 6: If the ISP chooses to use a two-part tariff pricing scheme. Then the value given sequentially is $J_F = \frac{c_2}{F_2}; J_G = \frac{y_2}{G_2}$ and $J = \ln F_2^{c_2} + \ln G_2^{y_2} - (c_2 + y_2)$ with the optimal profit obtained by ISP, namely $(m+n)\left(\ln \overline{F}^{c_2} + \ln \overline{G}^{y_2}\right)$.

High-demand and Low-demand Heterogeneous Consumers

There are two consumers with high demand (version 1) and optimal consumption levels and low demand consumers with optimal consumer levels $\overline{G}_2$. There are *m* version 1 consumer and *n* version 2 consumer with $c_1 = c_2 = c$ and $y_1 = y_2 = y$. Then it will be discussed for the determination of optimal profit in the pricing scheme used by ISP.

Case 7: If the ISP uses a flat-fee pricing scheme, then it is set $J_F = 0; J_G = 0$ and $J > 0$. If the consumer chooses to join then it will choose the consumer level $F_1 = \overline{F}_1, G_1 = \overline{G}_1$ for consumers of high demand or $F_2 = \overline{F}_2, G_2 = \overline{G}_2$ for low demand. For the value that ISP gives to consumers, namely $\leq \ln \overline{F}_1^c + \ln \overline{G}_1^y$ (low demand) and $J \leq \ln \overline{F}_2^c + \ln \overline{G}_2^y$ (low demand), ISP cannot distinguish between high demand consumer and ISP assigns high demand $J = \ln \overline{F}_1^c + \ln \overline{G}_1^y$ by serving only consumers with the high demand of usage or setting prices $J = \ln \overline{F}_2^c + \ln \overline{G}_2^y$. If $m\left(\ln \overline{F}_1^c + \ln \overline{G}_1^y\right) < (m+n)\left(\ln \overline{F}_2^c + \ln \overline{G}_2^y\right)$, so the ISP sets the value of the $P = \ln \overline{F}_2^c + \ln \overline{G}_2^y$ for high and low usage levels with the optimal advantages that ISP achieved is $(m+n)\left(\ln \overline{F}_2^c + \ln \overline{G}_2^y\right)$, Therefore, from this case, Lemma 7 was obtained.

Lemma 7: If the ISP chooses to use a flat-fee pricing scheme, then the value given is $J = \ln \overline{F}_2^c + \ln \overline{G}_2^y$ with the optimal advantages gained by ISP are $(m+n)\left(\ln \overline{F}_2^c + \ln \overline{G}_2^y\right)$.

Case 8: If the ISP uses a usage-based pricing scheme set $J_F > 0; J_G > 0$ and $J > 0$. The condition of the first order for optimization problem of consumers high and low demand are: For heterogeneous consumers high demand of strategy, then $\max\limits_{J, J_F, J_G} \ln F_1^{c_1} + \ln G_1^{y_1} - J_F F_1 - J_G G_1$. So, to optimize the function by using the necessary conditions and sufficient conditions is as follows.

The necessary condition:

$$\frac{\partial\left(\ln F_1^c + \ln G_1^y - J_F F_1 - J_G G_1\right)}{\partial F_1} = 0 \text{ then is obtained } c\frac{1}{F_1} - J_F = 0 \Leftrightarrow J_F = \frac{c}{F_1} \Leftrightarrow F_1{*} = \left(\frac{c}{J_F}\right)$$

The sufficient condition:

$$\frac{\partial^2\left(\ln F_1^c + \ln G_1^y - J_F F_1 - J_G G_1\right)}{\partial F_1^2} > 0 \text{ then is obtained } \frac{\partial\left(cF_1^{-1}\right)}{\partial F_1} > 0 \Leftrightarrow \frac{-c}{F_1^2} > 0$$

The necessary condition:

$$\frac{\partial\left(\ln F_1^c + \ln G_1^y - J_F F_1 - J_G G_1\right)}{\partial G_1} = 0 \text{ then } \ln F_1^c + y\frac{1}{G_1} - J_G = 0 \Leftrightarrow J_G = \frac{y}{G_1} \Leftrightarrow G_1{*} = \left(\frac{y}{J_G}\right)$$

The sufficient condition:

$$\frac{\partial^2\left(\ln F_1^c + \ln G_1^y - J_F F_1 - J_G G_1\right)}{\partial G_1^{\,2}} > 0 \text{ then } \frac{\partial\left(cG_1^{-2}\right)}{\partial G_1} > 0 \Leftrightarrow \frac{-c}{G_1^2} > 0$$

Low demand heterogeneous consumer is as follows $\max\limits_{F,G,A} \ln F_2^a + \ln G_2^b - J_F F_2 - J_G G_2$

To optimize the price by using the necessary and sufficient conditions as follows.

The necessary condition:

$$\frac{\partial\left(\ln F_2^c + \ln G_2^y - J_F F_2 - J_G G_2\right)}{\partial F_2} = 0 \text{ then } c\frac{1}{F_2} - J_F = 0 \Leftrightarrow F_2{}^* = \left(\frac{c}{J_F}\right)$$

The sufficient condition:

$$\frac{\partial^2\left(\ln F_2^c + \ln G_2^y - J_F F_2 - J_G G_2\right)}{\partial F_2^{\,2}} > 0 \text{ then is obtained } \frac{\partial\left(cF_2^{-2}\right)}{\partial F_2} > 0 \Leftrightarrow -\frac{c}{F_2^2} > 0$$

Foreseeable $m\left(\overline{F}_1\right) < (m+n)\left(\overline{F}_2\right)$ so that the ISP can be set $J_F = \frac{c}{F_2}$ and $J_G = \frac{y}{G_2}$ to serve consumers with high and low demand. Optimization problems of providers will be

$$\max_{J_F, J_G} m\left(J_F F_1{}^* + J_G G_1{}^*\right) + n\left(J_F F_1{}^* + J_G G_2{}^*\right) = \max_{J_F, J_G}(m+n)\left(c_2 + y_2\right)$$

For example, ISP uses usage-based then the optimal value that is allowed in busy hours is $J_F = \frac{c}{F_2}$ and in non-busy hours will be $J_G = \frac{y}{G_2}$.

Lemma 8: If ISP chooses to use a usage-based pricing scheme then the value given at peak hours is $J_F = \frac{c}{F_2}$ and the non-busy hours is $J_G = \frac{y}{G_2}$ with the optimal advantages gained by ISP are $(m+n)\left(c_2 + y_2\right)$.

Case 9: If the ISP uses a two-part tariff pricing scheme, then it is set $J_F > 0; J_G > 0$ and $J > 0$. By using Equations (15)-(18) for provider problems high and low demand, then $J_F = \frac{c}{F_2}$, ISP only attract consumers with high demand (peak hours) and $J_G = \frac{y}{G_2}$ low demand (not busy hour). With Equation (12), it is obtained: $\ln F_2^c + \ln G_2^y - J_F F_2 - J_G G_2 - JA_2 \geq 0$, $J_F = \frac{c}{F_2}$, $J_G = \frac{y}{G_2}$ and $P \leq \ln F_2^a + \ln G_2^b - c - y$

Optimization Problems of ISP are $m\left(\ln \overline{F}_1^c + \ln \overline{G}_1^y\right) + n\left(\ln \overline{F}_2^c + \ln \overline{G}_2^y\right)$.

Lemma 9: If the ISP chooses to use a two-part tariff pricing scheme then the optimal value $J_F = \frac{c}{F_2}$ and $J_G = \frac{y}{G_2}$ and $P \leq \ln F_2^c + \ln G_2^y - c - y$ with the optimal advantages gained by ISP are $m\left(\ln \overline{F}_1^c + \ln \overline{G}_1^y\right) + n\left(\ln \overline{F}_2^c + \ln \overline{G}_2^y\right)$.

The mail traffic data was classified as the parameter values for each type of consumer stated in Table 1 until Table 4. The constants used are based on the arbitrary constants chosen for comparing the strategies stated in each lemma.

TABLE 1. Parameter values for three pricing schemes.

Parameter	Pricing Scheme		
	Flat-fee	**Usage-Based**	**Two-Part Tariff**
$\bar{F}_1$	316.20	316.20	316.20
$\bar{F}_2$	310.56	310.56	310.56
$\bar{G}_1$	381.29	381.29	381.29
$\bar{G}_2$	358.08	358.08	358.08

Table 1 states the values for parameters used in three pricing schemes: flat-fee, usage-based, and two-part tariff. Table 2 describes the values of the parameter for Homogeneous consumers, and Table 3 and Table 4 explain the parameter values for high-end and low-end heterogeneous consumers and high and low demand heterogeneous consumers, respectively.

TABLE 2. Parameter values for homogeneous consumers.

Parameter	Pricing Scheme		
	Flat-fee	**Usage-Based**	**Two-Part Tariff**
c	4	4	4
y	3	3	4
$\bar{F}$	316.20	316.20	316.20
$\bar{G}$	381.29	381.29	381.29

TABLE 3. Parameter values for high-end and low-end heterogeneous consumers.

Parameter	Pricing Scheme		
	Flat-fee	**Usage-Based**	**Two-Part Tariff**
c_1	4	4	4
c_2	3	3	4
y_1	3	3	4
y_2	2	2	2

TABLE 4. Parameter values for high-demand and low-demand heterogeneous consumers.

Parameter	Pricing Scheme		
	Flat-fee	**Usage-Based**	**Two-Part Tariff**
c_1	3	3	3
c_2	3	3	3
y_1	2	2	2
y_2	2	2	2

with

$\overline{F} = \overline{F}_1$ is the highest level of consumption at peak hours in kbps

$\overline{F}_2$ is the second-highest level of consumption are during peak hours in kbps

$\overline{G} = \overline{G}_1$ is the consumption levels are the highest when hours are not busy in kbps

$\overline{G}_2$ is the consumption levels are lower during non-peak hours in kbps

Based on the analysis of cases 1 to 9 and Lemma 1-9 to compare the most optimal profit obtained by each model of pricing scheme, and by using the parameter values stated in Table 2 to Table 4, the result is shown as follows.

TABLE 5. The comparisons among three pricing schemes.

Pricing Strategy	Homogeneous	High-End and Low-End	High Demand and Low Demand
Flat-fee	$\sum i\left(\ln \overline{F}^{c} + \ln \overline{G}^{y}\right)$ $= \sum i\left(\ln 316.20^{4} + \ln 381.29^{3}\right)$ $= \sum 40.85i$	$(m+n)\left(\ln \overline{F}^{c_2} + \ln \overline{G}^{y_2}\right)$ $= (1+1)\left(\ln 316.20^{3} + \ln 381.29^{2}\right)$ $= 58.312$	$(m+n)\left(\ln \overline{F_2}^{c} + \ln \overline{G_2}^{y}\right)$ $= (1+1)\left(\ln 310.56^{3} + \ln 358.08^{b}\right)$ $= 58.204$
Usage-based	$\sum i(c+y)$ $= \sum i(4+3)$ $= \sum 7i$	$(m+n)(c_2+y_2)$ $= (1+1)(3+2)$ $= 10$	$(m+n)(c_2+y_2)$ $= (1+1)(3+2)$ $= 10$
Two-Part Tariff	$\sum i\left(\ln \overline{F}^{c} + \ln \overline{G}^{y}\right)$ $= \sum i\left(\ln 316.20^{4} + \ln 381.29^{3}\right)$ $= \sum 40.85i$	$(m+n)\left(\ln \overline{F}^{c} + \ln \overline{G}^{y}\right)$ $= (1+1)\left(\ln 316.20^{4} + \ln 381.29^{3}\right)$ $= 81.712$	$m\left(\ln \overline{F_1}^{c} + \ln \overline{G_1}^{y}\right)$ $+n\left(\ln \overline{F_2}^{c} + \ln \overline{G_2}^{y}\right)$ $= 1\left(\ln 316.20^{3} + \ln 318.29^{2}\right)$ $+1\left(\ln 310.56^{3} + \ln 358.08^{2}\right)$ $= 51.8245$

Based on Table 5, it can be seen that the maximum value for the homogeneous consumer with the utility function of CES are obtained by the ISPs by applying flat-fees and two-part tariff schemes, then for high-end and low-end consumers, the optimal benefits obtained by the ISP are applying two-part tariffs and lastly, for high demand and low demand consumers obtained the optimal benefits obtained by ISP is by applying flat-fee scheme.

CONCLUSION

This research can be sought for the optimal revenues for each pricing scheme by utilizing the CES function, which is stated in Lemma 1- Lemma 9. The comparison among the three pricing schemes also is conducted using secondary local data. For homogeneous consumers, the most optimal benefits obtained by ISP are a flat-fee and a two-part tariff. For consumers high-end and low-end profit heterogeneous consumes, ISP achieves maximum profit by choosing two-part tariff schemes, and for high demand low demand optimal heterogeneous consumers, the profit obtained by ISP is by choosing a flat-fee scheme. For further research, the optimization model needs to be developed to have other views in solving the present models presented analytically. Optimization models also seek to include other parameters dealing with consumers' behavior, like preferences in gender in consuming the network.

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