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Contents

PLANNING THE ZONE TARIFF OF BRT TRANS MUSI USING SEQUENTIAL AGGLOMERATIVE HIERARCHICAL NON-OVERLAPPING (SAHN) ALGORITHM

Sisca Octarina[1], Putra BJ Bangun[2], Eddy Roflin[3], and Ahmad Khoiri[4]

[1,2,3,4] Departement of Mathematics FMIPA Universitas Sriwijaya
Jalan Raya Palembang Prabumulih KM.32 Kampus Indralaya
Indralaya 30662 South Sumatera-Indonesia
tosca_0511@yahoo.co.id

Abstract. Bus Rapid Transit (BRT) Trans Musi is one kind of public transport in Palembang, Indonesia. It uses a fixed tariff for the trip. This tariff system is annoying for the respective customers. The customer who traveled in short distance must pay the same tariff with the customer who traveled in long distance. A new tariff system, zone tariff was developed to solve this unfair condition. This study aims to obtain a model of zone tariff for corridors of BRT Trans Musi that operate in Palembang using Sequential Agglomerative Hierarchical Non-Overlapping (SAHN) algorithm. The complete network is partitioned into zones, and the tariff depends on the number of passed zones, from the origin to the destination of the trip. Based on results and analysis, there are 3 zones and 3 models of zone tariff.
Keywords: Model, Zone, Zone Tariff

1 Introduction

Transportation is a tool for people to travel from one place to another. Transportation includes public and private transport. Bus Rapid Transit (BRT) Trans Musi is one of public transportation in Palembang. BRT Trans Musi uses fixed rates, which apply the same ticket price for both near and far distances.

Fixed tariff system is quite transparent and easily understood by the passengers [5]. This system allows the company to determine the tariff to be paid by the passenger. This system is considered unfair for some passengers because passengers who travel short distance required to pay the same rate as other passengers who travel farther.

Previous studies have determined the BRT zone tariff system based on Trans Musi transit stop [6]. The purpose of the planning system is to establish zones tariff and tariff so that the new ticket prices were very close to the current price. As an objective function, noted deviations between the new price and some reference price given. This deviation can be interpreted as a justice to the new tariff system or as a change from the current prices.

Zone development using heuristic algorithms such as Sequential Agglomerative Hierarchical Non-overlapping (SAHN) and Greedy algorithm have also been carried out by [3]. SAHN is an algorithm with different sequences that can be done depends on the definition of the distance between the two zones with each zone there are at least one stop and combine any two closest zone into the new zone starting from $n = | V |$ zone. Greedy algorithm performed by counting all bow or the distance between the zone and take a bow with the smallest weight. The iterations required in the zone determination using SAHN is shorter than Greedy algorithm [3]. Based on this background, the planning of BRT zone tariff system uses SAHN were done.

2 Research Methods

The research methods in this study are as follows:

1. Get a complete route map of BRT Trans Musi from PT. Sarana Pembangunan Palembang Jaya.
2. Review to the location to check the BRT Trans Musi route network.
3. Revise the route map of BRT Trans Musi and redraw these maps only for the active route.
4. Describe the data in the form of:
   a. Data of the corridors name, the inital and ending bus stop's name of each corridor, and the name of transit stops operating in Palembang. Data obtained by reviewing directly to the location.
   b. Data of the corridors names passing through the transit stop.
   c. Defines the bus stop's names into variables ($v_i,\ i = 1, 2, 3, \ldots, 13$). The total bus stops are 13 based on the initial, ending and transit stops at the end of each corridor.
5. Determining zones using SAHN algorithm, as follows:
   a. Make a simple map of BRT Trans Musi.
   b. Measure the distance between bus stops. Let $d(Z_u, Z_v) = d_{uv}$ for all zones $(Z_u, Z_v) \in \mathcal{Z}$. The distance measurement using Global Positioning System (GPS).
   c. Presenting the distance between bus stops v_i and v_j into matrix **V**.
   d. Specifies the number of zones (L) where each zone has a minimum of three main stops.
   e. Starting the calculation where the number of initial zones | V | = 13 in accordance with the number of existing bus stops and each zone has a single stop.
   f. Determine two zones such that $Z_u \neq Z_v$, $(Z_u, Z_v) \in \mathcal{Z}$ with a minimum distance $d(Z_u, Z_v)$.
   $\mathcal{Z}$ is a collection of all the starting zone.
   g. Combining Z_u and Z_v into new zones Z_k in order to obtain a new $\mathcal{Z}$.
   h. Calculating a new distance to all parts Z_k using $d(Z_k, Z) = \frac{1}{2}(d(Z_u, Z) + d(Z_v, Z) + c|d(Z_u, Z) - d(Z_v, Z)|)$
   i. If the desired amount is reached, the zone and the incorporation zone calculation is completed.
6. Determining the zone tariff of BRT Trans Musi, as follows:
   a. Forming the zone matrix.
   b. Calculating the value $z^*(p)$ using $z^*(p) = \underset{\substack{(v_{i1}, v_{j1}), (v_{i2}, v_{j2}) \in V, \\ n_{v_{i1}v_{ji}} = n_{v_{i2}v_{j2}} = p}}{\text{maks}} \frac{W_{i_1 j_1} W_{i_2 j_2}}{W_{i_1 j_1} + W_{i_2 j_2}} \left(d_{i_1 j_1} - d_{i_2 j_2}\right)$
   c. Calculating the value of $c^*(p)$ using $z_{ij} = c^*_{maks}(p) = \underset{\substack{(v_i, v_j) \in V, v_i \neq v_j \\ n_{ij} = p}}{\text{maks}} d_{ij} - \frac{z^*(p)}{W_{ij}}$ based on $z^*(p)$.
7. Interpretation of the results.

3 Result and Discussion

3.1 Data Description

The design of the zone tariff of BRT Trans Musi require the names of corridors, the names of transit stops, and the distance from the initial and ending stop to the transit stop on each corridor. The names of the corridors can be seen in Table 1.

Table 1. The Name of Corridors of BRT Trans Musi

Corridors	Direction
I	Alang-Alang Lebar – Ampera
II	PIM – Sako
III	Plaju – PS Mall
IV	Karya Jaya – Jakabaring
V	Alang-Alang Lebar – Bandara SMB II
VI	Pusri – PS Mall

Based on Table 1, BRT Trans Musi has 6 corridors, corridor I Terminal Alang-Alang Lebar – Ampera, corridor II PIM – Sako, corridor III Plaju – PS Mall, corridor IV Terminal Karyajaya – Jakabaring, corridor V Terminal Alang-Alang Lebar – Bandara Sultan Mahmud Badaruddin (SMB) II, dan corridor VI PUSRI – PS Mall. The name of the initial and ending stops were according to the name of the direction in each corridor. For example, the initial bus stop for corridor I is Terminal Alang-Alang Lebar and the ending bus stop is Ampera. The name of transit stops can be seen in Table 2.

Table 2. The Name of Transit Stops

Corridor	The name of transit stops						
	Sim Polda	Masjid Agung	Cinde	Pasar Gubah	Jakabaring	Term AAL	MONPERA
I	✓	✓	✓				✓
II	✓			✓			
III		✓		✓	✓		✓
IV					✓		
V						✓	
VI		✓	✓				

BRT Trans Musi has 7 transit stops, there are Simpang Polda, Masjid Agung, Bank BNI Syariah/Cinde, Pasar Gubah, Jakabaring, Terminal Alang-Alang Lebar, and MONPERA transit stops.

3.2 Defining The Variable

The bus stop's names which use in planning the zone tariff of BRT Trans Musi were defining to the variable $v_i, i = 1, 2, 3, \dots, 13$.

Table 3. Defining The Bus Stop's Name into Variable

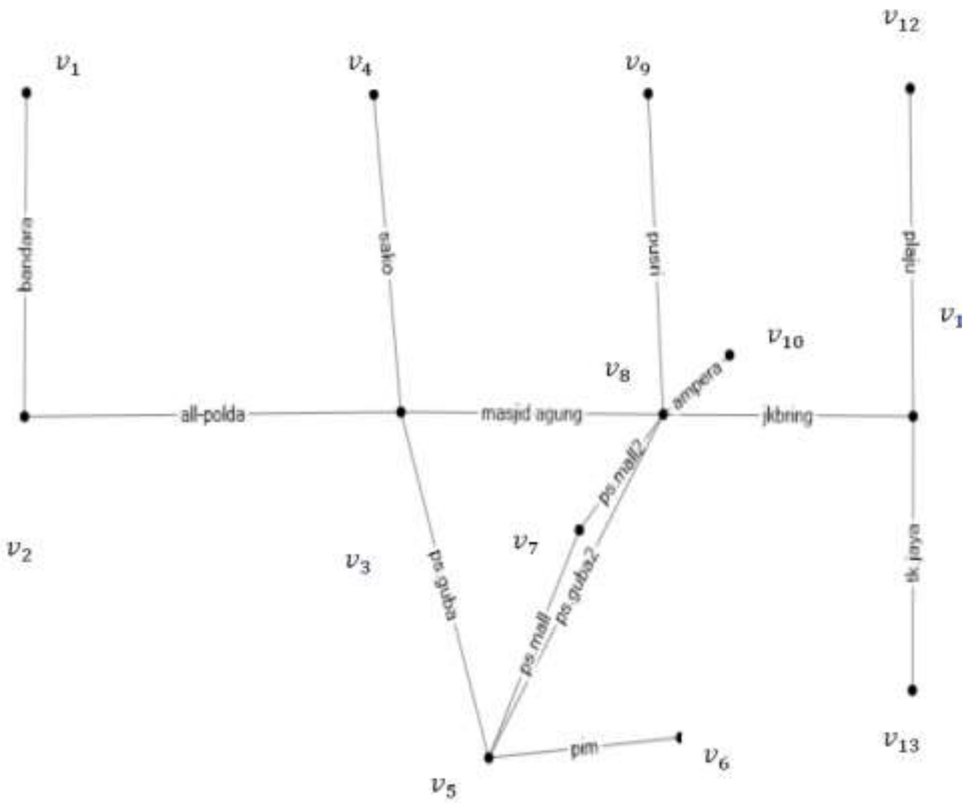
No.	The Bus Stop's Name	Variable
1.	Bandara SMB II	v_1
2.	Terminal Alang-Alang Lebar	v_2
3.	Simpang Polda	v_3
4.	Terminal Sako	v_4
5.	Pasar Gubah	v_5
6.	PIM	v_6
7.	PS Mall	v_7
8.	Masjid Agung	v_8
9.	Pusri	v_9
10.	Jembatan Ampera	v_{10}
11.	Jakabaring	v_{11}
12.	Plaju	v_{12}
13	Terminal Karya Jaya	v_{13}

Bandara SMB II was defined to v_1, Terminal Alang-Alang Lebar was defined to v_2, Simpang Polda to v_3, and so on.

3.3 Determining The Zone of BRT Trans Musi Using Sequential Agglomerative Hierarchical Non-overlapping (SAHN) Algorithm

BRT Trans Musi which operates in Palembang has 6 corridors. On each corridor, there are some bus stops where passengers can go down or up. In addition, it also provided transit stops where passengers can switch buses from one corridor to another corridor. The simple route map of BRT Trans Musi can be seen in Fig. 1.

Fig. 1. Simple Route Map of BRT Trans Musi



Based on Picture 1, the distance between each bus stops can be seen in Table 4.

Table 4. The Distance between v_i to v_j in km

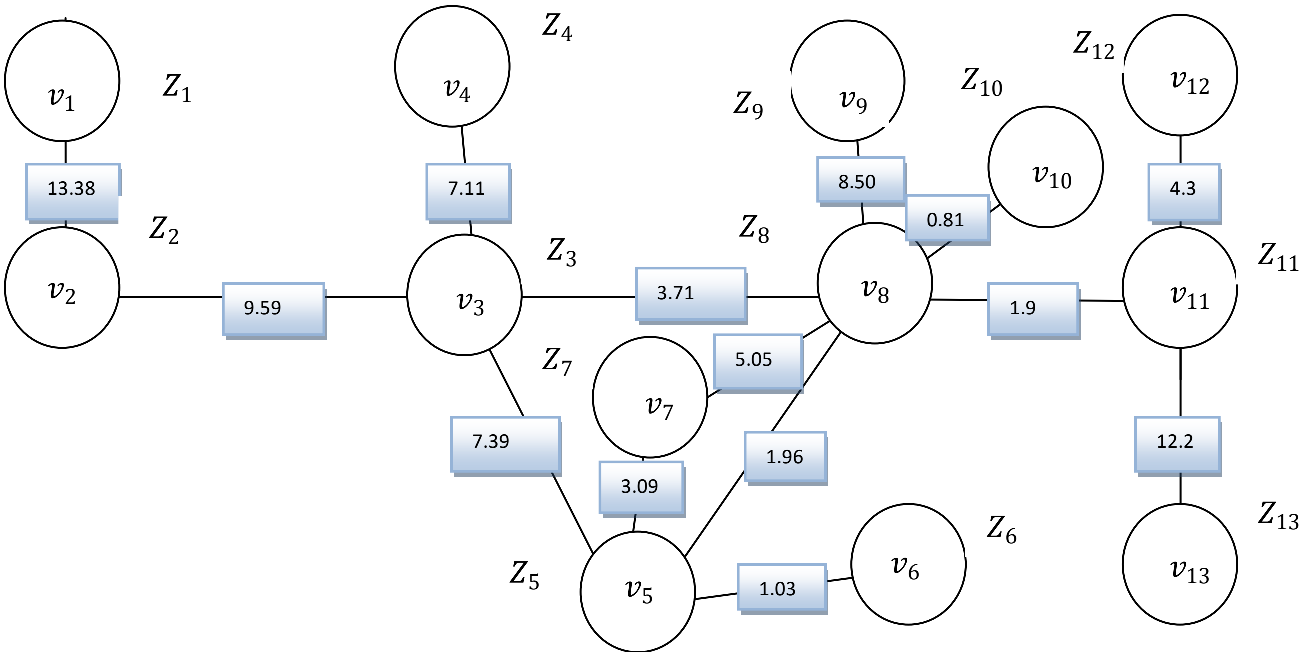
	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}
v_1	0	13.38	22.97	30.08	30.36	31.39	33.45	26.68	35.18	27.49	28.58	32.88	40.78
v_2	13.38	0	9.59	16.70	16.98	18.01	20.07	13.30	21.80	14.11	15.20	19.50	27.40
v_3	22.97	9.59	0	7.11	7.39	8.42	10.48	3.71	12.21	4.52	5.61	9.91	17.81
v_4	30.08	16.70	7.11	0	14.50	15.53	17.59	10.82	19.32	11.63	12.72	17.02	24.92
v_5	30.36	16.98	7.39	14.50	0	1.03	3.09	1.96	10.46	2.77	3.86	8.16	16.06
v_6	31.39	18.01	8.42	15.53	1.03	0	4.12	2.99	11.49	3.80	4.89	9.19	17.09
v_7	33.45	20.07	10.48	17.59	3.09	4.12	0	5.05	13.55	5.86	6.95	11.25	19.15
v_8	26.68	13.30	3.71	10.82	1.96	2.99	5.05	0	8.50	0.81	1.90	6.20	14.10
v_9	35.18	21.80	12.21	19.32	10.46	11.49	13.55	8.50	0	9.31	10.40	14.70	22.60
v_{10}	27.49	14.11	4.52	11.63	2.77	3.80	5.86	0.81	9.31	0	2.71	7.01	14.91
v_{11}	28.58	15.20	5.61	12.72	3.86	4.89	6.95	1.90	10.40	2.71	0	4.30	12.20
v_{12}	32.88	19.50	9.91	17.02	8.16	9.19	11.25	6.20	14.70	7.01	4.30	0	16.50
v_{13}	40.78	27.40	17.81	24.92	16.06	17.09	19.15	14.10	22.60	14.91	12.20	16.50	0

The data of the distance between v_i to v_j in Table 4 were made into matrix **V**. The size of matrix **V** is 13x13. That size is the numbers of initial, ending and transit bus stops.

$$V = \begin{bmatrix} 0 & 13.38 & 22.97 & 30.08 & 30.36 & 31.39 & 33.45 & 26.68 & 35.18 & 27.49 & 28.58 & 32.88 & 40.78 \\ 13.38 & 0 & 9.59 & 16.70 & 16.98 & 18.01 & 20.07 & 13.30 & 21.80 & 14.11 & 15.20 & 19.50 & 27.40 \\ 22.97 & 9.59 & 0 & 7.11 & 7.39 & 8.42 & 10.48 & 3.71 & 12.21 & 4.52 & 5.61 & 9.91 & 17.81 \\ 30.08 & 16.70 & 7.11 & 0 & 14.50 & 15.53 & 17.59 & 10.82 & 19.32 & 11.63 & 12.72 & 17.02 & 24.92 \\ 30.36 & 16.98 & 7.39 & 14.50 & 0 & 1.03 & 3.09 & 1.96 & 10.46 & 2.77 & 3.86 & 8.16 & 16.06 \\ 31.39 & 18.01 & 8.42 & 15.53 & 1.03 & 0 & 4.12 & 2.99 & 11.49 & 3.80 & 4.89 & 9.19 & 17.09 \\ 33.45 & 20.07 & 10.48 & 17.59 & 3.09 & 4.12 & 0 & 5.05 & 13.55 & 5.86 & 6.95 & 11.25 & 19.15 \\ 26.68 & 13.30 & 3.71 & 10.82 & 1.96 & 2.99 & 5.05 & 0 & 8.50 & 0.81 & 1.90 & 6.20 & 14.10 \\ 35.18 & 21.80 & 12.21 & 19.32 & 10.46 & 11.49 & 13.55 & 8.50 & 0 & 9.31 & 10.40 & 14.70 & 22.60 \\ 27.49 & 14.11 & 4.52 & 11.63 & 2.77 & 3.80 & 5.86 & 0.81 & 9.31 & 0 & 2.71 & 7.01 & 14.91 \\ 28.58 & 15.20 & 5.61 & 12.72 & 3.86 & 4.89 & 6.95 & 1.90 & 10.40 & 2.71 & 0 & 4.30 & 12.20 \\ 32.88 & 19.50 & 9.91 & 17.02 & 8.16 & 9.19 & 11.25 & 6.20 & 14.70 & 7.01 & 4.30 & 0 & 16.50 \\ 40.78 & 27.40 & 17.81 & 24.92 & 16.06 & 17.09 & 19.15 & 14.10 & 22.60 & 14.91 & 12.20 & 16.50 & 0 \end{bmatrix}$$

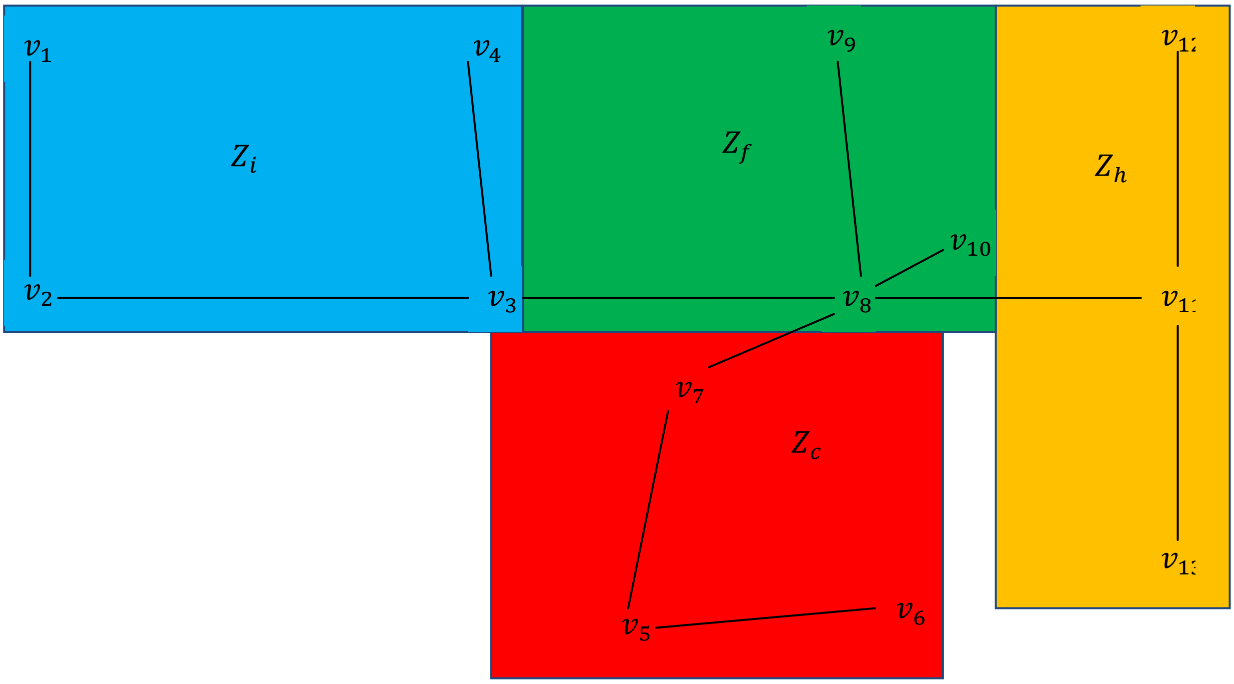
Let L is the number of zones. The number of zones in this case are four zones where the zones are defined as the point of the main zone. The zone is a collection of several small zones around it. The calculation is begun where the number of initial zones in accordance with the number of stops as much as 13 zones $(Z_1, Z_2, \dots, Z_{13})$. Each zone has single stops. All of the 13 initial zones can be seen in Fig. 2.

Fig. 2. Initial Zone Map of BRT Trans Musi (13 Zones)



The calculation of zone continued to 4 zone. It can be seen in Fig. 3.

Fig. 3. Last Zone Map of BRT Trans Musi (4 Zones)



Based on Fig. 3. there are 4 zones ($Z_{i,}, Z_c, Z_f$, and Z_h). The calculation process stopped when the number of zones reached. Furthermore, let Z_i be Z_I, Z_c be Z_{II}, Z_f be Z_{III}, and Z_h be Z_{IV}. Zone Z_i has 4 main stops (v_1, v_2, v_3, and v_4), zone Z_c has 3 main stops (v_5, v_6, and v_7), zone Z_f has 3 main stops (v_8, v_9, and v_{10}), and zone Z_h has 3 main stops (v_{11}, v_{12}, and v_{13}).

3.4 Determining The Zone Tariff of BRT Trans Musi

If the trip was done from v_i to v_j and in one zone, so $c(0) = 0$. If the trip was done from v_i to v_j and overs one zone, so $c(1) = 1$. $c(2) = 2$ if the trip overs two zones and c(3) = 3 if the trip overs three zones. The number of zone that passed from v_i to v_j (n_{ij}) was entried to matrix D.

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Let x_1, x_2, x_3, x_4 and x_5 be the tariff of BRT Trans Musi. x_1 is the tariff for 0 km - 10 km, x_2 is the tariff for 10.01 km - 20 km, x_3 is the tariff for 20.01 km - 30 km, x_4 is the tariff for 30.01 km - 40 km, and x_5 is the tariff for 40.01 km - 50 km. The further trip means the increase tariff, so $x_1 < x_2 < x_3 < x_4 < x_5$.

Zone tariff was the combination of distance and fixed tariff. The bus stops classified to the zones. The tariff for this system depends on the the number of zones that passed from departure to arrival zone. Based on the calculation $c^*_{maks}(0) = 0.758256x_4 + 0.241744x_1$, $c^*_1(0) = x_1$ and $c^*_2(0) = 0.29971x_1 + 0.36809x_2 + 0.14384x_3 + 0.18837x_4$ for the trips in one zone ($p = 0$), $c^*_{maks}(1) = 0.650019x_4 + 0.349981x_2$, $c^*_1(1) = x_2$ and $c^*_2(1) = 0.14891x_1 + 0.4851x_2 + 0.1133x_3 + 0.25278x_4$ for the trips over one zone ($p = 1$), $c^*_{maks}(2) = 0.645505x_4 + 0.3210x_2$, $c^*_1(2) = x_2$ and $c^*_2(2) = 0.13609x_1 + 0.49748 + 0.25045x_3 + 0.11598x_4$ for ($p = 2$), and $c^*_{maks}(3) = 0.67651x_5 + 0.32349x_2$, $c^*_1(3) = x_3$ and $c^*_2(3) = 0.12199x_1 + 0.30374x_2 + 0.24797x_3 + 0.14565x_4 + 0.18064x_5$ for the trips over three zones ($p = 3$).

4 CONCLUSION AND SUGGESTION

4.1 Conclusion

The zone tariff of BRT Trans Musi was the transition from the fixed tariff to the tariff based on the number of the zones. The model is as follows :

1. Minimum $c(0) \in \{(0.758256x_4 + 0.241744x_1),\ x_1, (0.29971x_1 + 0.36809x_2 + 0.14384x_3 + 0.18837x_4)\}$for the trip in one zone.
2. Minimum $c(1) \in \{(0.650019x_4 + 0.349981x_2),\ x_2, (0.14891x_1 + 0.4851x_2 + 0.1133x_3 + 0.25278x_4)\}$ for the trip over one zone.
3. Minimum $c(2) \in \{(0.645505x_4 + 0.3210x_2),\ x_2, (0.13609x_1 + 0.49748 + 0.25045x_3 + 0.11598x_4)\}$ for the trip over two zones.
4. Minimum $c(3) \in \{(0.67651x_5 + 0.32349x_2),\ x_3, (0.12199x_1 + 0.30374x_2 + 0.24797x_3 + 0.14565x_4 + 0.18064x_5)\}$ for the trip over three zones.

4.2 Suggestion

For the further research, the zone tariff can be interpreted to programming software such that the tariff can be implemented effectively.

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