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By Sisca Octarina

Research Article

The Application to Find Cutting Patterns in Two Dimensional Cutting Stock Problem

Sisca Octarina*, Putra B. J. Bangun and Samuel Hutapea

Department of Mathematics, Faculty of Mathematics and Natural Sciences, Sriwijaya University, Indralaya, Indonesia
Corresponding author: s.octarina@gmail.com

Abstract. Two dimensional Cutting Stock Problem (CSP) is a problem tofind the appropriate patterns that fulfilled the demand with different length and cut from two sides, the length and width. Two dimensional CSP aims to minimize the cutting waste that called Trim Loss. This research designed and made the application of finding cutting patterns in two dimensional CSP. Based on the results, it found that Modified Branch and Bound Algorithm makes the pattern searching become easier than manual searching. This application also yields the optimal patterns with minimum Trim Loss.

Keywords. Cutting Stock Problem; Trim Loss; Modified Branch and Bound Algorithm

MSC. 31-XX

1. Introduction

Raw material is one of the most important ingredients in the production process. Many types of raw materials are used in the paper industry. Paper materials should be cut into some pieces before being used. Cutting problems in Optimization better known as the Cutting Stock Problem (CSP). CSP on paper cuts often result in residual cuts which can not be used so-called trim loss. Trim loss is the part that should be minimized because it will minimize the profits [2].

CSP is divided into three sections based on the trim loss. There are one-dimensional, two-dimensional and three dimensional CSP. Many researches on CSP have been done.

[3] didoptimization CSP with Integer Linear Programming (ILP) methods. This method produces minimumtrim loss with solutions offered in different conditions.

Research on two-dimensional CSP also has a lot to do. Two-dimensional CSP always regards to the width and the length of cutting. All researches on the CSP either one-dimensional or two-dimensional still use manual patterns searching.The weakness of this pattern searching has its own difficulties, takes a long time and still missed some of the possiblecutting patterns. [4] and [6] have minimized trim loss in one-dimensional CSP without using pattern searching method. So it could not found all patterns and influenced the model. [5] havedeveloped Modified Branch and Bound Algorithm to find two-dimensional paper-cutting patterns in large amounts. But the algorithm is still used manually, so it takes a long time and high accuracy to complete.

Based on this background, this study made the application to find cutting patternsin two-dimensional CSP. Research was conducted on paper cutting with the data derived from the CV PRODA. CV PRODA is one of the printing in Palembang since 1997.

2. Methodology

This research method is a case study at CV Proda. The procedures are carried out as follows.

a. Describing the secondary data including the product name, size and product demand in June, 2015.

b. Ordering products based on order size.

c. Defining the required variables.

d. Creating Modified Branch and Bound Algorithm to find the cutting patterns in the 2-dimensional CSP.

e. Creating an application using Modified Branch and Bound Algorithm in Javascript language.

f. Testing the application into the 2-dimensional CSP.

g. Finding the cutting patterns using the application.

h. Forming the ILP models which consists of the objective function and constraint functions.

i. Solving the model of ILP.

j. Interpreting and analyzing the solution.

k. Making conclusions from the results and discussion.

3. Modified Branch and Bound Algorithm

Let the number of main sheets being cut according to the jth pattern is denoted by x_j and the cutting loss for each number of each jth pattern is denoted by c_j. The demand for the ith item is

denoted by d_i, so the number of occurences of the ith item in the jth pattern is denoted by p_{ij}, with m is the number of items and n is the number of patterns.

The 2-dimensional CSP model is:

Minimize $Z = \displaystyle\sum_{j=1}^{n} c_j x_j$

Subject to

$$\sum_{j=1}^{n} p_{ij} x_j \geq d_i, \quad \text{for all } i = 1, 2, \ldots, m \tag{1}$$

$$x_j, \; p_{ij} \geq 0 \text{ and integer for all } x_j, \; p_{ij}$$

In this case, the main sheet is viewed in two dimensions, consider the size of length and width. The length and width of main sheets are denoted by L and W, respectively. Furthermore l_i and w_i are the length and width of each ith product respectively.

The algorithm of *Modified Branch and Bound Algorithm* which was proposed by [5] is as follows:

1.   a. Arrange required lengths l_i, $i = 1, 2, \ldots, m$ in decreasing order, $l_1 > l_2 > \ldots l_m$ where m is number of items.

   b. Arrange required widths w_i, $i = 1, 2, \ldots, m$ according to the corresponding length l_i, $i = 1, 2, \ldots, m$.

2. For $i = 1, 2, \ldots, m$ and $j = 1$ do steps 3 to 5.

3. Set

$$a_{11} = \left\lfloor \left\lfloor \frac{L}{l_1} \right\rfloor \right\rfloor \quad \text{and} \quad a_{ij} = \left\lfloor \left\lfloor \frac{\left(L - \sum_{z=1}^{i-1} a_{zj} l_z\right)}{l_i} \right\rfloor \right\rfloor, \tag{2}$$

   where L is the length of main sheets.

4. If $a_{ij} > 0$ then set

$$b_{ij} = \lfloor \lfloor W / w_i \rfloor \rfloor \tag{3}$$

   Else set $b_{ij} = 0$, where W is the width of the main sheet.

5. Set

$$p_{ij} = a_{ij} b_{ij} \tag{4}$$

6. *Cutting loss*

   (i) Cut loss along the length of the main sheet:

$$c_u = \left(L - \sum_{i=1}^{m} a_{ij} l_i\right) \times W \tag{5}$$

   For $i = 1, 2, \ldots, m$.

   If $\left(L - \sum_{i=1}^{m} a_{ij} l_i\right) \geq w_i$ and $W \geq l_i$, considering $90°$ rotation.

Set

$$A_{ij} = \left\lfloor \left\lfloor \frac{\left(L - \sum_{i=1}^{m} a_{ij} l_i\right)}{w_i} \right\rfloor \right\rfloor \tag{6}$$

$$B_{ij} = \begin{cases} \lfloor \lfloor W/l_i \rfloor \rfloor \\ 0, \ \text{otherwise} \end{cases} , \ \text{if } A_{ij} > 0 \tag{7}$$

$$p_{ij} = p_{ij} + A_{ij} B_{ij} \tag{8}$$

Else set

$$\left. \begin{array}{l} A_{ij} = 0 \\ B_{ij} = 0 \\ P_{ij} = P_{ij} \end{array} \right\} \tag{9}$$

If $A_{ij} > 0$, then set

$$C_u = \left[\left(L - \sum_{i=1}^{m} a_{ij} l_i \right) - A_{ij} W_i \right] \times B_{ij} l_i \tag{10}$$

$$C_v = \left[\left(L - \sum_{i=1}^{m} a_{ij} l_i \right) \right] \times \left(W - B_{ij} l_i \right) \tag{11}$$

else

$$C_u = \left(L - \sum_{i=1}^{m} a_{ij} l_i \right) \times W \tag{12}$$

(ii) Cut loss along the width of the main sheet:

$$C_v = \left(a_{ij} l_i \right) \times k_{ij} \tag{13}$$

$$k_{ij} = W - \left(b_{ij} w_i \right). \tag{14}$$

If $\left(b_{ij} w_i \right) = 0$ then set $k_{ij} = 0$, where k_{ij} is the remaining width of each item in each pattern.

For $z \neq i$, if $\left(a_{ij} l_i \right) \geq l_z$ and $k_{ij} \geq w_z$ then set

$$A_{zj} = \left\lfloor \left\lfloor \frac{a_{ij} l_i}{l_z} \right\rfloor \right\rfloor \tag{15}$$

$$B_{zj} = \begin{cases} \lfloor \lfloor W/l_i \rfloor \rfloor \\ 0, \ \text{otherwise} \end{cases} , \ \text{if } A_{zj} > 0 \tag{16}$$

$$p_{zj} = p_{zj} + A_{zj} B_{zj} \tag{17}$$

Else set

$$\left. \begin{array}{l} A_{ij} = 0 \\ B_{ij} = 0 \\ P_{ij} = P_{ij} \end{array} \right\} \tag{18}$$

If $A_{zj} > 0$, then set

$$C_u = \left[a_{ij} l_i - A_{zj} l_z \right] \times B_{zj} w_{zi}; \tag{19}$$

$$C_v = a_{ij} l_i \times (k_{ij} - B_{zj} l_z) \tag{20}$$

Else

$$C_v = (a_{ij}l_i) \times k_{ij} \tag{21}$$

7. Set $r = m - 1$, while $r > 0$ do step 8

8. While $a_{rj} > 0$, set $j = j + 1$ and do step 9

9. If $a_{rj} \geq b_{rj}$, then generate a new pattern according to the following conditions :

   For $z = 1, 2, \ldots, r - 1$, set

   $$a_{zj} = a_{zj-1} \quad \text{and} \quad b_{zj} = b_{zj-1} \tag{22}$$

   For $z = r$, set

   $$a_{zj} = a_{zj-1} - 1 \tag{23}$$

   If $a_{zj} > 0$ then set

   $$b_{zj} = \lfloor W/w_z \rfloor \tag{24}$$

   Else set

   $$b_{zj} = 0 \tag{25}$$

   For $z = r + 1, \ldots, m$, calculate a_{zj} and b_{zj} using Equations (2) and (3), and go to step 5

   Else generate a new pattern according to the following conditions: For $z = 1, 2, \ldots, r - 1$, set

   $$a_{zj} = a_{zj-1} \quad \text{and} \quad b_{zj} = b_{zj-1} \tag{26}$$

   For $z = r$, set

   $$a_{zj} = a_{zj-1} - 1 \quad \text{and} \quad b_{zj} = b_{zj-1} - 1 \tag{27}$$

   For $z = r + 1, \ldots, m$, calculate a_{zj} and b_{zj} using Equations (2) and (3), go to step 5

10. Set $r = r - 1$

11. STOP

This application uses Javascript and the initial view can be seen in Figure 1. Furthermore, this application was used to find the cutting patterns in 2-dimensional CSP at CV PRODA. The size of the main sheet is 1090 mm × 970 mm. Each kind of the product and the number of order in each month can be seen in Table 1.

Table 1. Kinds of product and the number of orders

No.	Product name	Length (mm)	Width (mm)	Number of order (pieces)
1	Invitation card	325	225	300
2	Name card	90	60	1000
3	Brochure	210	150	3000
4	Book cover	230	160	500

Based on the data from Table 1 and from the application, there were 30.644 cutting patterns. We chose 24 cutting patterns with minimum trim loss that can be seen in Table 2. The model of 2-dimensional CSP of this case is as follows:

$$\text{Minimize } Z = \sum_{j=1}^{24} c_j x_j \tag{28}$$

$$\text{Subject to } \sum_{j=1}^{24} p_{ij} x_j \geq d_i, \quad \text{for all } i = 1, 2, 3, 4$$

$$x_j \geq 0 \text{ and integer} \tag{29}$$

By solving the model in Equations (28), it found that $Z = 31.650$, x_1, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, $x_{24} = 0$ $x_2 = 1$ and $x_{17} = 1$. It means that the optimum cutting patterns are the 2nd and the 17th. From Table 2, the 2nd pattern yields 4 pieces invitation cards, 4 pieces book covers and 33 pieces name cards whereas the 17th pattern only yields 31 pieces of brochure.
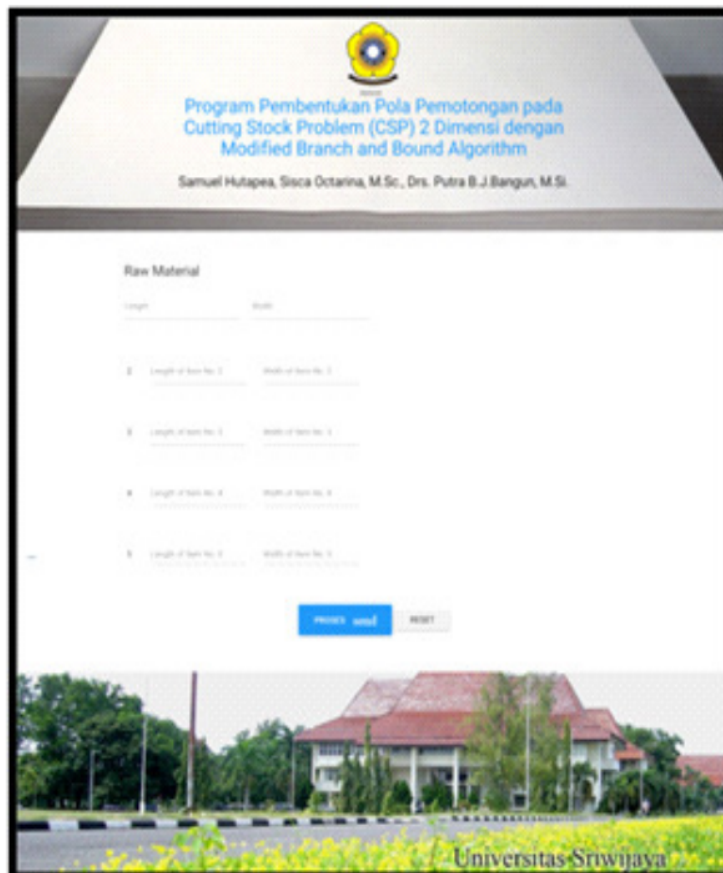

Figure 1. The application to find cutting patterns

Table 2. Cutting patterns with minimum trim loss

The jth pattern	Number of Items				Trim Loss(mm²)
	325×225	230×160	210×150	90×60	
1	12	0	0	26	72.750
2	4	4	0	33	13.250
3	6	1	5	17	25.750
4	3	11	5	8	36.325
5	2	15	0	8	61.350
6	2	8	8	10	57.550
7	8	0	0	0	472.300
8	0	0	0	180	75.600
9	2	8	8	2	45.600
10	2	6	1	50	32.650
11	4	3	5	15	136.800
12	0	28	0	0	18.400
13	1	10	10	13	60.800
14	3	5	5	10	136.000
15	1	4	1	40	47.850
16	1	8	1	64	41.950
17	0	0	31	0	18.400
18	3	4	1	48	152.000
19	2	12	3	18	12.450
20	2	12	5	16	86.550
21	0	3	4	112	121.700
22	8	0	0	5	385.000
23	0	4	3	112	8.000
24	4	3	5	20	165.600

4. Conclusions

Based on the results, it can be concluded that this application can make the cutting pattern searching become easier, especially for the complex problems.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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