

# Mixed Integer Nonlinear Programming Model of Wireless Pricing Scheme with QoS Attribute of Bandwidth and End-to-End Delay

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**Abstract.** The pricing for wireless networks is developed by considering linearity factors, elasticity price, price factors. Mixed Integer Nonlinear Programming of wireless pricing model are proposed as the nonlinear programming problem that can be solved optimally using LINGO 13.0. The solutions are expected to give some information about the connections between the acceptance factor and the price. Previous model worked on the model that focus on bandwidth as the QoS attribute. The models attempt to maximize the total price for a connection based on QoS parameter. The QoS attributes used will be the bandwidth and the end to end delay that affect the traffic. The maximum goal to maximum price is achieved when the provider determine the requirement for the increment or decrement of price change due to QoS change and amount of QoS value.

## 1 Introduction

In terms of networking, QoS (Quality of Service) refers to the ability to provide different services to network traffic by different classes. QoS itself is useful as a measure of how well the network and is an attempt to define the characteristics and properties of a service.

Yang [1-4] described the pricing scheme based on internet auctions to allocate QoS and then she attempted to maximize the revenues using QoS parameters in multi-class QoS network. While on the network there are many parameters that affect the QoS that can be considered. Besides that, the models can also be extended into this current framework focusing on wireless network.

Based on that idea, this paper will introduce the improved models on wireless network that extend the idea of pricing model of [1-4] and discussed in wired network of [5, 6] and the models of [7-9] that works on the 3G networks. The model is designed by searching for information on the parameters and variables. The objective function of [7] is  $\sum_j^m \sum_i^n (PR_{ij} \pm PQ_{ij})$ . This means that we intend to maximize the total amount comprises the cost to connect to the available QoS ( $PR_{ij}$ ), changes in the cost of all the changes in QoS ( $PQ_{ij}$ ).

Since the objective function proposed by [2] is also powerful in that sense by giving full information on utility function, price sensitivity for user and for

the class in network, the adoption of the objective function together with the objective function of 3g network will gain more benefit.

So, basically the contribution of the paper is as follows. The improved models are designed to fit in current network situation that works on wireless network [10]. The models have the ability to detect the price sensitivity of the user, the price sensitivity of the class, the cost dealing with QoS, how much changes in QoS, the users admitted to each class of network offered while we also can examine the changes of increment or decrement of cost in connecting the available QoS and the cost of changes in QoS.

## 2 Research Methodology

The model used in this study using wireless internet pricing scheme developed by [7], with only two attributes QoS applied namely bandwidth and end-to-end delay. Those QoS attributes is the common attributes used in the network where the model will be modified by adding an original model of the [2] into the objective function and constraint functions. The data used to test this model in the form of secondary data obtained from one of the local server in Palembang, which consists of the data traffic and traffic digilib mail. The data can then be completed with the help of the program LINGO 11.0 to obtain the optimal solution.

## 3 Model Formulation

This study aims to provide the maximum benefit for the internet service provider. The model provided by [7] will be combined with the original model of Yang [2]. The model was formed in order to obtain information about the parameters and related variables.

So, the objective function will be to maximize

$$\sum_j^m \sum_i^n (PR_{ij} \pm PQ_{ij} + (\alpha_j + W_j \log \frac{\tilde{x}_{ij}}{L_{mj}}) Z_{ij}) \quad (1)$$

The objective function is useful to maximize the total amount comprises the cost to connect to the available QoS ( $PR_{ij}$ ), changes in the cost of all the changes in QoS ( $PQ_{ij}$ ), the base cost per class  $\alpha_j$  and the utility function as the measurement of customers's satisfaction. As well as the set of constraints that act as a barrier function of the objective to be met in the aim of obtaining optimal results.

Then, the set of constraints are as follows.

$$PQ_{ij} = \left(1 \pm \frac{x}{Q_{bij}}\right) PB_{ij} Lx \quad (2)$$

This constraint explains the changes in costs depending on the cost factors for each QoS attribute bandwidth and end-to-end delay, the basic cost with the user  $i$  and  $j$  class, as well as the linearity factor.

$PB_{ij}$  is defined with

$$PB_{ij} = a_{ij}(e - e^{-xB})T_l/100 \quad (3)$$

which is a base cost for a connection with the user  $i$  and  $j$  classes that depend on linear cost factor in the user  $i$  and  $j$ , the linear factor  $(e - e^{-xB})$ , and amount of traffic load.

$Lx$  is the linearity factor that depends on parameter  $a$  and  $(e - e^{-xB})$ . Then,

$$L_x = a(e - e^{-xB}) \quad (4)$$

with the assumption of  $0 \leq x \leq 1$ .

The linearity factor  $a_{ij}$  lies between the prescribed value provided by the provider, say  $f$  and  $g$  then we have

$$f \leq a_{ij} \leq g. \quad (5)$$

Range of the allowed traffic load  $t_l$  is also predetermined by the service provider, say  $h$  and  $k$ , then

$$h \leq t_l \leq k. \quad (6)$$

The next constraint explains a number of increases or decreases in the value of QoS, which is set to 0 and 1 that indicates implicitly that if 0 means to be in a condition best effort and 1 is in a state of perfect service.

$$0 \leq x \leq 1. \quad (7)$$

Value  $B$  is set to be between 0.8 and 1.07, because in this range the best quality services occur.

$$0.8 \leq B \leq 1.07 \quad (8)$$

Value of  $a$  is a linear parameter to be determined, by a factor sets level of base cost, so that

$$a = 1. \quad (9)$$

Next constraints are as follows.

$$\sum_{j=1}^2 \sum_i X_{ij} \leq Q, i = 1, \dots, n \quad (10)$$

where  $Q$  is total bandwidth of 100MBps or 102400Kbps.

$$X_{ij} \geq L_{m_j} - (1 - Z_{ij}), i = 1, \dots, n; j = 1, 2, \dots \quad (11)$$

$$W_j \leq W_{ij} + (1 - Z_{ij}), i = 1, \dots, n; j = 1, 2, \dots \quad (12)$$

$W_{ij}$  is the user  $i$  price sensitivity in class  $j$ .

$$X_{ij} \geq V_i - (1 - Z_{ij}), i = 1, \dots, n; j = 1, 2, \dots \quad (13)$$

$V_i$  is minimum bandwidth for each user of  $V_1 = 6Kbps$  for user 1 and  $V_2 = 5Kbps$  for user 2.

$$X_{ij} \geq X_j - (1 - Z_{ij}), i = 1, \dots, n; j = 1, 2, \dots \quad (14)$$

$$X_{ij} \geq Z_{ij}, i = 1, \dots, n; j = 1, 2, \dots \quad (15)$$

$$X_{ij} \geq 0, i = 1, \dots, n; j = 1, 2, \dots \quad (16)$$

$$L_{m_j} \geq 0.01, j = 1, 2, \dots \quad (17)$$

$$W_j \geq 0, j = 1, 2, \dots \quad (18)$$

$$X_{ij} \leq X_j, i = 1, \dots, n; j = 1, 2, \dots \quad (19)$$

$$Z_{ij} = \begin{cases} 1, & \text{user } i \text{ in admtd to class } j \\ 0, & \text{otherwise} \end{cases} \quad (20)$$

Followings are the decision variables and parameters involved in the model.

- $PR_{ij}$  : Cost to connect to the available QoS.
- $PQ_{ij}$  : Changes in the cost of all the changes QoS.
- $x$  : The amount of increase or decrease in the value of QoS.
- $Q_{bij}$  : Nominal value of QoS attribute in the network provider.
- $PB_{ij}$  : The base cost for a connection with user  $i$  and class  $j$ .
- $Lx$  : Linearity factor.
- $a_{ij}$  : Factor of linearity cost of user  $i$  and class  $j$ .
- $T_l$  : Traffic load.
- $a, B$  : Predetermined linearity parameter.
- $f$  and  $g$  : Lower and upper bound value of  $a_{ij}$
- $h$  and  $k$  : Lower and upper bound value of  $T_l$
- $\alpha_j$  : Base cost for class  $j$ .
- $Z_{ij}$  :  $\begin{cases} 1, & \text{user } i \text{ in admitted to class } j \\ 0, & \text{otherwise} \end{cases}$
- $W_j$  : Sensitivity price for class  $j$ .
- $\tilde{X}_{ij}$  : Final bandwidth obtained by user  $i$  and class  $j$
- $L_{mj}$  : Minimum bandwidth for class  $j$ .
- $Q$  : Total bandwidth.
- $W_{ij}$  : Sensitivity price for user  $i$  in class  $j$ .
- $V_i$  : Minimum bandwidth needed by user  $i$ .
- $X_j$  : Bandwidth for each user in class  $j$ .

## 4 Results and Discussion

The models, with the objective function (1) and constraints (2) - (20) are solved by using LINGO 11.0. to obtain the optimal solution for the 4 cases in each QoS attribute that involves an increase or decrease in costs due to changes in QoS and a decrease or increase in the value of QoS.

Results obtained from the above model with the help of the program LINGO 11.0, can be seen in the table and explanations as follows.

The objective function (1) with constraints (2) - (20) then are solved by using LINGO 11.0. to obtain the optimal solution for the 4 cases in each QoS attribute that involves an increase or decrease in costs due to changes in QoS and a decrease or increase in the value of QoS.

Results obtained from the above model with the help of the program LINGO 11.0, can be seen in the table and explanations as follows.

#### 4.1 Bandwidth QoS Attribute

Table 1 and Table 2 show the solver status for each case and the value of decision variables, respectively.

**Table 1.** The Solver Status of Wireless Pricing Scheme Model with Bandwidth QoS Attribute

Variables	$PQ_{ij}$ increase $x$ increase	$PQ_{ij}$ increase $x$ decrease	$PQ_{ij}$ decrease $x$ increase	$PQ_{ij}$ decrease $x$ decrease
Model Class	INLP	INLP	INLP	INLP
State	Local Optimal	Local Optimal	Local Optimal	Local Optimal
Objective	257.76	257.775	252.99	252.99
Infeasibility	0	$3.63798 \cdot 10^{-12}$	0	0
Iterations	113	111	103	103
GMU	35K	35K	35K	35K
ER	0s	0s	0s	0s

In Table 1, the solver status of the model with bandwidth QoS attribute are shown for each case. The optimal solution can be viewed on objective line. Thus for QoS bandwidth available in four cases, the value will achieve the most optimal results in the second case of 257.775. The results will be obtained by iterating as many as 111 times with infeasibility of  $3.63798 \cdot 10^{-12}$ . Generated Memory Used (GMU) shows the amount of used memory allocation of 35K and Elapsed Runtime (ER) shows the total time used to produce and terminate the model which is 0.

The values of the variables obtained in bandwidth QoS attribute for each case to achieve the optimal solution is presented in Table 2.

Based on Table 2, it can be seen that the values of variables for case 1 and case 2 is almost the same, but far different from the values of the variables for cases 3 and 4 where the value of the variable values for cases 3 and 4 is equal value. Differences in values for cases 1 and 2 of the cases 3 and 4 can be examined at the changes in the cost of all the changes in QoS, for cases 1 and 2  $PQ_{ij}$  value close to 1, while the value of  $PQ_{ij}$  cases 3 and 4 even approaching 0.1. For cases 1 and 2, increase or decrease in the value of QoS is 1 that shows the services that are in perfect condition, as well as for cases 3 and 4, increase or decrease in the value of QoS is 0 which indicates the service is in best effort state.

The value of  $Z_{ij}$  indicates the admittance of the user  $i$  in class  $j$ .  $Z_{ij} = 1$  states that the user is in in class  $j$ , while the value  $Z_{ij} = 0$  states otherwise.  $Z_{11} = 1$ ,  $Z_{12} = 0$ ,  $Z_{21} = 0$ , dan  $Z_{22} = 1$ , showed that user 1 is admitted to class 1 and user 2 is in class 2.

**Table 2.** The Decision Values of Wireless Pricing Scheme Model with Bandwidth QoS Attribute

Variables	$PQ_{ij}$ increase $x$ increase	$PQ_{ij}$ increase $x$ decrease	$PQ_{ij}$ decrease $x$ increase	$PQ_{ij}$ decrease $x$ decrease
$PQ_{11}$	1.218333	1.217116	0.07381231	0.07381231
$PQ_{12}$	1.137111	1.135975	0.08857477	0.08857477
$PQ_{21}$	1.055889	1.054834	0.1033372	0.1033372
$PQ_{22}$	0.9746667	0.9736925	0.1180997	0.1180997
$x$	1	1	0	0
$PB_{11}$	0.5126671	0.5126671	0.04295705	0.04295705
$PB_{12}$	0.4784893	0.4784893	0.05154845	0.05154845
$PB_{21}$	0.4443115	0.4443115	0.06013986	0.06013986
$PB_{22}$	0.4101337	0.4101337	0.06873127	0.06873127
$a_{11}$	0.15	0.15	0.05	0.05
$a_{12}$	0.14	0.14	0.06	0.06
$a_{21}$	0.13	0.13	0.07	0.07
$a_{22}$	0.12	0.12	0.08	0.08
$L_x$	2.375273	2.375273	1.718282	1.718282
$T_1$	143.89	143.89	50	50
$B$	1.07	1.07	0.8	0.8
$Z_{11}$	1	1	1	1
$Z_{12}$	0	0	0	0
$Z_{21}$	0	0	0	0
$Z_{22}$	1	1	1	1
$W_1$	8	8	8	8
$W_2$	9	9	9	9
$\tilde{X}_{11}$	24094.59	24094.59	24094.59	24094.59
$\tilde{X}_{12}$	27105.41	27105.41	27105.41	27105.41
$\tilde{X}_{21}$	24093.59	24093.59	24093.59	24093.59
$\tilde{X}_{22}$	27106.41	27106.41	27106.41	27106.41
$L_{m1}$	0.01	0.01	0.01	0.01
$L_{m2}$	0.01	0.01	0.01	0.01
$W_{11}$	8	8	8	8
$W_{12}$	8	8	8	8
$W_{21}$	7	7	7	7
$W_{22}$	9	9	9	9
$X_1$	24094.59	24094.59	24094.59	24094.59
$X_2$	27106.41	27106.41	27106.41	27106.41

#### 4.2 End-to-End Delay QoS Attribute

Table 3 and Table 4 explain the solver status for each case and the value of decision variables, respectively.

**Table 3.** The Solver Status of Wireless Pricing Scheme Model with End-to-End Delay QoS Attribute

Variables	$PQ_{ij}$ increase $x$ increase	$PQ_{ij}$ increase $x$ decrease	$PQ_{ij}$ decrease $x$ increase	$PQ_{ij}$ decrease $x$ decrease
Model Class	INLP	INLP	INLP	INLP
State	Local Optimal	Local Optimal	Local Optimal	Local Optimal
Objective	257.77	257.745	252.99	252.99
Infeasibility	$3.63798 \cdot 10^{-12}$	$3.63798 \cdot 10^{-12}$	0	0
Iterations	113	113	103	103
GMU	35K	35K	35K	35K
ER	0s	0s	0s	0s

Table 3 shows the optimal solutions obtained in QoS end-to-end delay of each case. The optimal solution can be viewed on objective line. Thus, for QoS end-to-end delay of four cases available, the value will achieve the most optimal results in the first case which amounted to 257.77. The results to be obtained by doing as much as 113 times of iteration with infeasibility of  $3.63798 \cdot 10^{-12}$ . Generated Memory Used (GMU is equal to 35K and Elapsed Runtime (ER) is 0 seconds.

The values of the variables obtained in end-to-end delay QoS attribute for each case to achieve the optimal solution is presented in Table 4.

Based on Table 4, it can be seen that the values of variables for case 1 and case 2 is not much different, but very much different from the case 3 and case 4 where the case 3 and case 4 have the values of the same variable. Differences in values for cases 1 and 2 of the cases 3 and 4 looks at the changes in the cost of all the changes in QoS, for cases 1 and 2  $PQ_{ij}$  value close to 1, while the value of  $PQ_{ij}$  for cases 3 and 4 even approaching 0. 1. Besides that, for case 1 and case 2, the increase or decrease in value of 1 indicates that the QoS for services that are in perfect condition, as well as for cases 3 and 4, cases of increase or decrease in the value of QoS is equal to 0 that indicates the service is in a state best effort.

Value of  $Z_{ij}$  indicates the admittance of the user  $i$  in class  $j$ . If the value  $Z_{ij}= 1$  then the user  $i$  is in class  $j$ , whereas for  $Z_{ij}= 0$  states otherwise. So for all four cases, it can be seen that user 1 is at class 1 and user 2 is in the class 2.

The comparison table of each attribute QoS for each case are explained in Table 5.

**Table 4.** The Decision Values of Wireless Pricing Scheme Model with End-to-End Delay QoS Attribute

Variables	$PQ_{ij}$ increase $x$ increase	$PQ_{ij}$ increase $x$ decrease	$PQ_{ij}$ decrease $x$ increase	$PQ_{ij}$ decrease $x$ decrease
$PQ_{11}$	1.221204	1.214245	0.07381231	0.07381231
$PQ_{12}$	1.139790	1.133296	0.08857477	0.08857477
$PQ_{21}$	1.058377	1.052346	0.1033372	0.1033372
$PQ_{22}$	0.9769630	0.9713962	0.1180997	0.1180997
$x$	1	1	0	0
$PB_{11}$	0.5126671	0.5126671	0.04295705	0.04295705
$PB_{12}$	0.4784893	0.4784893	0.05154845	0.05154845
$PB_{21}$	0.4443115	0.4443115	0.06013986	0.06013986
$PB_{22}$	0.4101337	0.4101337	0.06873127	0.06873127
$a_{11}$	0.15	0.15	0.05	0.05
$a_{12}$	0.14	0.14	0.06	0.06
$a_{21}$	0.13	0.13	0.07	0.07
$a_{22}$	0.12	0.12	0.08	0.08
$L_x$	2.375273	2.375273	1.718282	1.718282
$T_l$	143.89	143.89	50	50
$B$	1.07	1.07	0.8	0.8
$Z_{11}$	1	1	1	1
$Z_{12}$	0	0	0	0
$Z_{21}$	0	0	0	0
$Z_{22}$	1	1	1	1
$W_1$	8	8	8	8
$W_2$	9	9	9	9
$\tilde{X}_{11}$	24094.59	24094.59	24094.59	24094.59
$\tilde{X}_{12}$	27105.41	27105.41	27105.41	27105.41
$\tilde{X}_{21}$	24093.59	24093.59	24093.59	24093.59
$\tilde{X}_{22}$	27106.41	27106.41	27106.41	27106.41
$L_{m1}$	0.01	0.01	0.01	0.01
$L_{m2}$	0.01	0.01	0.01	0.01
$W_{11}$	8	8	8	8
$W_{12}$	8	8	8	8
$W_{21}$	7	7	7	7
$W_{22}$	9	9	9	9
$X_1$	24094.59	24094.59	24094.59	24094.59
$X_2$	27106.41	27106.41	27106.41	27106.41



**Table 5.** The Result Comparison Between Two QoS Attributes

Variables	$PQ_{ij}$ increase $x$ increase		$PQ_{ij}$ increase $x$ decrease		$PQ_{ij}$ decrease $x$ increase		$PQ_{ij}$ decrease $x$ decrease	
	Bandw	End to End delay	Bandw	End to End delay	Bandw	End to End delay	Bandw	End to End delay
Model Class	INLP	INLP	INLP	INLP	INLP	INLP	INLP	INLP
State	Local Opt	Local Opt	Local Opt	Local Opt	Local Opt	Local Opt	Local Opt	Local Opt
Objective	257.76	257.77	257.775	257.745	252.99	252.99	252.99	252.99
Infeasibility	0	$3.6379 \cdot 10^{-12}$	$3.63798 \cdot 10^{-12}$	$3.63798 \cdot 10^{-12}$	0	0	0	0
Iterations	113	113	111	113	103	103	103	103
GMU	35K	35K	35K	35K	35K	35K	35K	35K
ER	0s	0s	0s	0s	0s	0s	0s	0s

Table 5 shows a comparison of the optimal solution based QoS attributes of each case. In the first case it appears that the optimal solution lies in end-to-end delay QoS that is equal to 257.77, with as many as 113 iterations. In the second case, the optimal solution instead lies in the bandwidth QoS that is equal to 257.775, with as many as 111 iterations iteration which iteration number less than the number of iterations on QoS –end-to-end delay. In the case of the third and fourth cases the optimal solution both QoS same value is 252.99, with the same number of iterations as many as 103 iterations.

## 5 Conclusion

From the two QoS attributes discussed in the form of bandwidth and end-to-end delay, with each attribute of four cases, it is showed that for bandwidth QoS attribute will be optimal if it is the case with the increase of  $PQ_{ij}$  and  $x$  decrease, while for end-to-end delay QoS if the optimal solution would be the first case which is  $PQ_{ij}$  increase and increase of  $x$ .

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