Generalized Model and Optimal Solution of Internet Pricing Scheme in Single Link under Multiservice Networks

Irmeilyana[1], Indrawati, Fitri Maya Puspita, Rahma Tantia Amelia
Department of Mathematics, Faculty of Mathematics and Natural Sciences, Sriwijaya University South Sumatera Indonesia
[1]imel_unsri@yahoo.co.id

***Abstract*—In this paper, we will analyze the internet pricing scheme under multi service network by generalizing the model into 9 services. The scheme is determined from the base price, quality premium and number of links to aid the internet service provider to maximize the profit and to serve better service to the customers. The objective function is generated by setting up the base price and quality premium as a constant or variable. We use nonlinear optimization model and solve it by using LINGO 11.0 to obtain the optimal solution. The results show that for each case by generalizing the model, the ISP obtains better solution by fixing the base price and fixing and varying the quality premium. ISP has a choice to adopt the model when ISP fixes the base price and also fix or vary the quality premium with maximum profit adopted by ISP is when fixing the base price and varying the premium quality.**

***Keywords*— multi service network, internet pricing scheme, generalized model, service quality, base price, quality premium.**

I. INTRODUCTION

The service quality of the network is determined by the user satisfaction utilizing the network. The ISPs have a task to serve better and different service quality (QoS) to all users in achieving the best information quality and obtain the profit from available resources. The knowledge to develop the new pricing plan which fulfills the consumer and provider requirements is available, but few involving QoS network [1], [2] dan [3].

Sain and Herpers [4] had investigated the pricing scheme for internet by considering the price, total network capacity and level of QoS for each offered service The model then solve as an optimization model and solved by using optimization tool to obtain the maximum profit for ISP. The extended investigation proposed by [5] is by generating the improved internet pricing model based on [3, 4, 6] by adding the new parameter, the decision variables, the constraints, and by considering the base price and quality premium to yield better maximum revenue than previous model.

The research on the improved model of single link internet pricing scheme under multi service network and multi class QoS networks are due to [1-5, 7-15] under the original model proposed by [5] and [9] by fixing and varying both base price and quality premium and setting out the QoS level to obtain better maximum revenue for ISP from previous model discussed. That model applies 3 services for multi service network and 2 users and classes in single link multiclass QoS network. In reality, in enhancing the quality, ISP provides many services and many classes to the consumers.

This paper basically attempt to show the generalized optimal solution of the internet pricing scheme model with numerous services based on model presented [3, 5] for the case when the base price and quality premium are constants, the case where the base price is constant whereas the quality premium as a variable, the case when the base price and quality premium are as variable and the case where the base price is as variable and quality premium is as a constant. The obtained solution can assist ISP to choose the best pricing scheme.

II. RESEARCH METHOD

In this paper, the internet pricing scheme model is solved by using LINGO 11.0 to obtain the optimal solution. We apply set-endset and data-enddata to have structured coding to enable us to apply the optimization model with many numbers of users. We fix 9 services to be served in the plan. The solutions will help us to clarify the current issue on internet pricing, network share, network capacity and level of QoS and also the number of services offered is compatible with the real situation in the internet network.

III. MODELS

We adopt models from [5] by considering for cases when the best price (α) and quality premium (β) as constant, α constant and β as variable, α and β as variables and α as variable and β as a constant. The QoS level for each case is modified into three conditions

$$Ii = Ii\text{-}1 \text{ or } Ii > Ii\text{-}1 \text{ or } Ii < Ii\text{-}1. \quad (1)$$

For the case when β is variable then the ISP will be able to promote the certain service, so

$$\beta i = \beta i\text{-}1 \text{ or } \beta i > \beta i\text{-}1 \text{ or } \beta i < \beta i\text{-}1. \quad (2)$$

For the case when α then ISP is able to conduct market competition, so

$$\alpha i = \alpha i\text{-}1 \text{ or } \alpha i > \alpha i\text{-}1 \text{ or } \alpha i < \alpha i\text{-}1 \quad (3)$$

IV. RESULT AND ANALYSIS

We use the same model proposed by [5] with the parameter value of α = 0.5 and β=0.01. Table I below presents the other parameter values in the model.

TABEL I
PARAMETER VALUES IN MULTI SERVICE NETWORK

i	Parameter								
	C	d_i	p_i	m_i	n_i	l_i	b_i	c_i	g_i
1	102400	97.5	3	0.01	20	0.01	0.5	0	1
2	102400	13312.3	45	0.01	20	0.01	0.5	0	1
3	102400	367,9	15	0.01	20	0.01	0.5	0	1
4	102400	825,8	35	0.01	20	0.01	0.5	0	1
5	102400	593,5	32	0.01	20	0.01	0.5	0	1
6	102400	489,3	25	0.01	20	0.01	0.5	0	1
7	102400	98,9	5	0.01	20	0.01	0.5	0	1
8	102400	1407,2	38	0.01	20	0.01	0.5	0	1
9	102400	393,5	20	0.01	20	0.01	0.5	0	1

Case 1: α and β as constants.

$$Max\ R \ = \sum_{i=1}^{9}(\alpha + \beta \cdot I_i) \cdot p_i \cdot x_i = (0{,}5 + 0{,}01I_1) \cdot 3x_1 + (0{,}5 + 0{,}01I_2) \cdot 45 \quad (0{,}5 + 0{,}01I_3) \cdot 15x_3 + \cdots + (0{,}5 + 0{,}01I_9) \cdot 20x_9 \tag{4}$$

Subject to

$$95{,}7\ I_1x_1 \le 102.400a_1$$

$$13.312{,}3x_2 \le 102.400a_2$$
$$367{,}9I_3x_3 \le 102.400a_3$$
.
.
.

$$393{,}5I_{10}x_{10} \le 102.400a_{10} \tag{5}$$

$$97{,}5I_1 * x_1 + 13312{,}3I_2 * x_2 + 367{,}9I_3 * x_3 + \cdots +$$

$$393{,}5I_9 * x_9 \le\ 102.400 \tag{6}$$

$$a_1 +\ a_2 +\ a_3 + \cdots +\ a_9 = 1 \tag{7}$$

$$0\ \le\ \ a_i \le\ 1 \tag{8}$$

$$0{,}01\ \le\ I_i \le 1 \tag{9}$$

$$0 \le x_i \le 20 \quad ; \forall\ i = 1{,}2, \ldots ,9 \tag{10}$$

$$\{x_1, x_2, x_3, x_4, x_5, \ldots , x_9\} \text{ integer} \tag{11}$$

By modifying the QoS level and index quality we add the following constraints.

If $I_i = I_{i-1}$ then
$$I_i - I_{i-1} = 0 \tag{12}$$

If $I_i > I_{i-1}$ then

$$I_i - I_{i-1} > 0 \tag{13}$$

If $I_i < I_{i-1}$then
$$I_i - I_{i-1} < 0 \tag{14}$$

Case 2: for $\boldsymbol{\alpha}$ as constant and $\boldsymbol{\beta}$ as variable

$$Max\ R \ = \sum_{i=1}^{9}(\alpha + \beta_i \cdot I_i) \cdot p_i \cdot x_i = (0{,}5 + \beta_1 I_1) \cdot 3x_1 + (0{,}5 + \beta_2 I_2) \cdot 45x_2 + (0{,}5 + \beta_3 I_3) \cdot 15x_3 + \cdots + (0{,}5 + \beta_5 I_9) \cdot 20x_9 \tag{15}$$

subject to (4)-(14) and additional constraints

$$\beta_i \cdot I_i \ge \beta_{i-1} \cdot I_{i-1}\ ; \forall\ i = 2{,}3, \ldots ,9 \tag{16}$$

$$0{,}01 \le \beta_i \le 0{,}5 \ \ ; \forall\ i = 1{,}2, \ldots ,9 \tag{17}$$

With modifying the quality premium (β) as a variable then we add these constraints.

If β as $\beta_i = \beta_{i-1}$, then
$$\beta_i - \beta_{i-1} = 0 \tag{18}$$

If β as $\beta_i > \beta_{i-1}$, then
$$\beta_i - \beta_{i-1} > 0 \tag{19}$$

If β as $\beta_i < \beta_{i-1}$, then
$$\beta_i - \beta_{i-1} < 0 \tag{20}$$

Case 3: α and β as variable

$$Max\ R \ = \sum_{i=1}^{9}(\alpha_i + \beta_i \cdot I_i) \cdot p_i \cdot x_i = (\alpha_1 + \beta_1 I_1) \cdot 3x_1 + (\alpha_2 + \beta_2 I_2) \cdot 45x_2 +\ (\alpha_3 + \beta_3 I_3) \cdot 15x_3 + \cdots + (\alpha_9 + \beta_5 I_9) \cdot 20x_9 \tag{21}$$

subject to (4)-(14) and (16)-(20) and additional constraints

$$\alpha_i + \beta_i \cdot I_i \ge \alpha_{i-1} + \beta_{i-1} \cdot I_{i-1}\ ; \forall\ i = 1{,}2, \ldots ,9 \tag{22}$$

$$0 \le \alpha_i \le 1\ ; \forall\ i = 1{,}2{,}3, \ldots ,9 \tag{23}$$

And

If α as $\alpha_i = \alpha_{i-1}$, then
$$\alpha_i - \alpha_{i-1} = 0 \tag{24}$$

If α as $\alpha_i > \alpha_{i-1}$, then
$$\alpha_i - \alpha_{i-1} > 0 \tag{25}$$

If α as $\alpha_i < \alpha_{i-1}$, then
$$\alpha_i - \alpha_{i-1} < 0 \tag{26}$$

Case 4: α as variable and β as constant

$$Max\ R = \sum_{i=1}^{9}(\alpha_i + \beta \cdot I_i) \cdot p_i \cdot x_i = (\alpha_1 + 0{,}01I_1) \cdot 3x_1 + (\alpha_2 + 0{,}01I_2) \cdot 45x_2 + (\alpha_3 + 0{,}01I_3) \cdot 15x_3 + \cdots + (\alpha_9 + \beta_5 I_9) \cdot 20x_9 \tag{27}$$

subject to (4)-(14) and (23)-(26) and additional constraints

$$\alpha_i + I_i \geq \alpha_{i-1} + I_{i-1}\ ;\ \forall\ i = 2,3,\dots,9 \quad (28)$$

We will solve the model by using LINGO 11.0 then

1) Case 1: α and β as constant by modifying the QoS level so we divide Case 1 into three sub cases.
2) Case 2: α as constant and β as a variable by modifying the quality premium and QoS level so we divide Case 2 into 9 sub cases.
3) Case 3: α and β as variables by modifying the base price, quality premium and QoS level so we divide Case 3 into 27 sub cases.
4) Case 4: α as variable and β as constant so we divide Case 4 into 9 cases.

We have total of 48 sub cases. According to the results of LINGO 11.0 we have one solution of sub case from each case as follows.

1) In Case 1: α and β as constant for Ii=Ii-1
2) In case 2 : α as constant and β as βi = βi-1 for Ii=Ii-1
3) In case 3: α as αi = αi-1 and β as βi = βi-1 for Ii=Ii-1
4) In case 4: α as αi = αi-1 and β as constant for Ii=Ii-1

Table II to Tabel V below present the optimal solution of our four cases. Tabel II shows that in Case 1: α and β as constant for Ii=Ii-1, we obtain the optimal solution 192.7. The value of quality premium is 0.5 for each service with the number of users is 20, which means that the service provider offer all services to the users. Total capacity used is 103,399.99 kbps or 99.99% of total capacity available. The highest profit is obtained in Service 2 of 452.6 with capacity used of 77,523.4 kbps atau 75.7% of total capacity used.

Table III explains that in Case 2: α as constant dan β as βi=βi-1 for Ii=Ii-1, we obtain the optimal solution of 2814.76. The quality premium is 0.5 for each service with QoS level is 0.291 or 29.1%. The users utilize the service is 20 users, which means that the service provider offer all services to the users. Total capacity used is 103,399.99 kbps or 99.99% of total capacity available. The highest profit obtained from service 2 is 581.03 with the capacity used of 77,523.4 kbps or 75.7% of total capacity used and this value is the highest capacity usage from every service.

TABLE II

CASE 1 SOLUTION WITH α AND β AS CONSTANTS FOR $I_i=I_{i-1}$

Service (*i*)	**QoS level (I_i)**	**# of User (x_i)**	**Capacity Used ($I_i \cdot d_i \cdot x_i$)**	**Profit ($(\alpha+\beta_i \cdot I_i) \cdot p_i \cdot x_i$)**
1	0.291	20	557.3	30.17
2	0.291	20	77523.4	452.6
3	0.291	20	2142.4	150.9
4	0.291	20	4809	352.04
5	0.291	20	3456.2	321.86
6	0.291	20	2849.2	251.46
7	0.291	20	575.9	50.29
8	0.291	20	8194.8	382.2
9	0.291	20	2291.5	201.16
Total Capacity			102399.99	-
Total Profit				2192.7

TABLE III

CASE 2 SOLUTION WITH β AS $\beta_i = \beta_{j-1}$ FOR $I_i=I_{i-1}$

Service (*i*)	**QoS level (I_i)**	**# of User (x_i)**	**Capacity Used ($I_i \cdot d_i \cdot x_i$)**	**Profit ($(\alpha+\beta_i \cdot I_i) \cdot p_i \cdot x_i$)**
1	0.291	20	557.3	38.74
2	0.291	20	77523.4	581.03
3	0.291	20	2142.4	193.68
4	0.291	20	4809	451.9
5	0.291	20	3456.2	413.18
6	0.291	20	2849.2	322.79
7	0.291	20	575.9	64.56
8	0.291	20	8194.8	490.65
9	0.291	20	2291.5	258.23
Total Capacity			102399,99	-
Total Profit				2814.76

Table IV shows that in Case 3: α as $\alpha_i = \alpha_{i-1}$ and β as $\beta_i=\beta_{i-1}$ for $I_i=I_{i-1}$ we obtain the optimal solution of 4994.76. The base price and quality premium are 1 and 0.5 for each service with the QoS level of 0.291 for each service or 29.1%. The number of users apply the service is 20 users, which means that the service provider offer all services to the user. The total capacity used is 103,399.99 kbps or 99.99% of total capacity used. The highest profit of 1031.03 is in service 2 with total capacity used is 77,523.4 kbps or 75.7% of total capacity used. This capacity is the highest capacity used from other services.

TABLE IV

CASE 3 SOLUTION WITH α AS $\alpha_i = \alpha_{i-1}$ AND β AS $\beta_i = \beta_{j-1}$ FOR $I_i=I_{i-1}$

Service (*i*)	**QoS level (I_i)**	**# of User (x_i)**	**Capacity Used ($I_i \cdot d_i \cdot x_i$)**	**Profit ($(\alpha+\beta_i \cdot I_i) \cdot p_i \cdot x_i$)**
1	0.291	20	557.3	68.74
2	0.291	20	77523.4	1031.03
3	0.291	20	2142.4	343.68
4	0.291	20	4809	801.91
5	0.291	20	3456.2	733.18
6	0.291	20	2849.2	572.79
7	0.291	20	575.9	114.56
8	0.291	20	8194.8	870.65
9	0.291	20	2291.5	458.23
Total Capacity			102399,99	-
Total Profit				4994.76

TABLE V

CASE 4 SOLUTION WITH α AS $\alpha_i = \alpha_{i-1}$ AND β AS A CONSTANT FOR $I_i=I_{i-1}$

Service (*i*)	**QoS level (I_i)**	**# of User (x_i)**	**Capacity Used ($I_i \cdot d_i \cdot x_i$)**	**Profit ($(\alpha+\beta_i \cdot I_i) \cdot p_i \cdot x_i$)**
1	0.291	20	557.3	60.17
2	0.291	20	77523.4	902.62
3	0.291	20	2142.4	300.87
4	0.291	20	4809	702.04
5	0.291	20	3456.2	641.86
6	0.291	20	2849.2	501.46
7	0.291	20	575.9	100.29
8	0.291	20	8194.8	762.21
9	0.291	20	2291.5	401.16
Total Capacity			102399,99	-
Total Profit				4372.7

Table V depicts that in Case 4: α as $\alpha_i = \alpha_{i-1}$ and β as a constant for $I_i=I_{i-1}$, we obtain the optimal solution of 4372.7. The base price value is 1 for each service and QoS level for each service is 29.1%. The number of users apply the service is 20 user, which means that the provider offers all services. Total capacity used is 103,399.99 kbps or 99.99% of total capacity available. The highest profit obtained is 902.62 in service 2. Total capacity used for service 2 is 77,523.4 kbps or 75.7% of total capacity used.

TABEL VI
RECAPITULATION OF FOUR CASE SOLUTIONS

	Case			
	1	2	3	4
Total capacity used	102,399.99	102,399.99	102,399.99	102,399.99
Percentage of total capacity used	99.99%	99.99%	99.99%	99.99%
Profit per service	452.6	581.03	1031.03	902.62
Total Profit	2192.7	2814.76	4994.76	4372.7

The summary of the results is presented in Table VI menunjukkan that the maximum total profit is obtained in case 3: α as $\alpha_i=\alpha_{i-1}$ and β as $\beta_i=\beta_{i-1}$ for $I_i=I_{i-1}$ which is 4994.76. So, ISP adopts the internet pricing scheme by setting up the base price and quality premium as a variable with the condition of the base price, quality premium and the QoS level to be the same value for each service. The solution will enable ISPs to compete in the market and promote the certain service to the users. The number of service offered and the number of users apply the service will yield higher total profit for ISPs.

V. CONCLUSION

The generalized model of internet pricing scheme based on the base price, quality premium to be fixed or varied and modified quality index, quality premium and QoS level enable ISP to achieve the maximum profit according the ISP's goals. The solutions show that the connection among index quality, capacity needed and number of users applied the service is important in determining the total capacity used. In all cases, the highest profit and capacity used is in service 2 due to highest service sensitivity price from the services offered. All cases show that the total capacity used is 99.99% of total capacity available with the QoS level of 29.1%. However, the maximum total profit is in case 3 by fixing the base price and varying the quality premium. Toward these generalized models, ISPs can obtain better and higher maximum profit with service offered is close to real internet traffic.

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