Numerical Solution of Internet Pricing Scheme Based on Perfect Substitute Utility Function

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Abstract— **In this paper we will analyze the internet pricing schemes based on Perfect Substitute utility function for homogeneous and heterogeneous consumers. The pricing schemes is useful to help internet service providers (ISP) in maximizing profits and provide better service quality for the users. The models on every type of consumer is applied to the data traffic in Palembang server in order to obtain the maximum profit to obtain optimal. The models are in the form of nonlinear optimization models and can be solved numerically using LINGO 11.0 to get the optimal solution. The results show that the case when we apply flat fee, usage-based and two part tariff scheme for homogenous we reach the same profit and heterogeneous on willingness to pay we got higher profit if we apply usage based and two part tariff schemes. Meanwhile, for the case when we apply usage based and two part tariff schemes for heterogeneous on demand, we reach better solution than other scheme.**

Keywords— **Utility functions, perfect substitute, pricing schemes, consumer homogeneous, heterogeneous consumers.**

I. INTRODUCTION

Internet has an important role in the economy and education around the world. The Internet is a multimedia library, because it has a lot of information that is complete [5]. Complete information and quickly make consumers interested in becoming a consumer internet services. Consumers who make a lot of Internet Service Providers (ISPs) compete to provide services of the highest quality (Quality of Service) and the optimal prices for consumers. In addition to maintaining the quality of service and optimal prices for consumers, Internet Service Provider (ISP) should also consider profits.

There are some assumptions for utility function to be applied in the model but the researchers usually use the bandwidth function with fixed loss and delay and follow the rules that marginal utility as bandwidth function diminishing with increasing bandwidth [1-14]. The other reason dealing with the choices of utility function is that the utility function should be differentiable and easily to be analyzed the homogeneity and heterogeneity that impacts the choice of pricing structure for the companies. Kelly [15] also contends that the utility function also can be assumed to be increasing function, strictly concave and continuously differentiable.

The studies on pricing schemes based on utility function analytically originate from [16-22]. This paper essentially seeks to provide optimal solutions numerically for three internet pricing schemes which are flat fee, usage-based, and two-part tariff for homogeneous and heterogeneous consumers based on perfect substitute using LINGO 11.0 [23]. The results can help ISPs to choose a better pricing schemes to improve their profit.

II. RESEARCH METHOD

In this paper, the internet pricing schemes will be completed by the program LINGO 11.0 to obtain the optimal solution. The solution obtained will help determine the optimal price on the flat fee, usage-based, and two-part tariff pricing schemes.

III. MODEL FORMULATION

The general form of utility function based perfect subtitute $U(X\,,Y) = \;ax + by$

For the case of homogeneous consumers

Consumer Optimization Problems

$$\max_{X,Y,Z} aX + bY - P_X X - P_Y Y - PZ \tag{1}$$

with constraints

$$X \le \bar{X}Z \tag{2}$$
$$Y \le \bar{Y}Z \tag{3}$$
$$aX + bY - P_X X - P_Y Y - PZ \ge 0 \tag{4}$$
$$Z = 0 \text{ or } 1 \tag{5}$$

For the case of heterogeneous upper class and lower class consumers, suppose that there are m consumers upper class (i= 1) and n lower class consumers (i = 2). It is assumed that each of these heterogeneous consumers have a limit on the same $\bar{X}$ and $\bar{Y}$ with each one is the level of consumption during peak hours and during off-peak hours, $a_1 > a_2$ dan $b_1 > b_2$.

For consumer optimization problems:

$$\max_{X_i,Y_i,Z_i} aX + bY - P_x X_i - P_y Y_i - PZ_i \tag{6}$$

with constraints :

$$X_i \le \bar{X}_i Z_i \tag{7}$$
$$Y_i \le \bar{Y}_i Z_i \tag{8}$$
$$aX + bY - P_x X_i - P_y Y_i - PZ_i \ge 0 \tag{9}$$
$$Z_i = 0 \text{ or } 1 \tag{10}$$

As for the case of heterogeneous consumers of a high level of usage and low usage level classes, suppose that we assume the two types of consumers, high consumer consumption level (i = 1) with a maximum consumption rate of $\bar{X}_1$ dan $\bar{Y}_1$ and low consumer usage rate (i = 2) with a maximum consumption rate of $\bar{X}_2$ dan $\bar{Y}_2$. There are m consumers of type 1 and n consumers type 2 with $a_1 = a_2 = a$ dan $b_1 = b_2 = b$.

IV. OPTIMAL SOLUTION

Table I-III below show the parameter value used in the model. The values originally from local server internet traffic.

TABLE I
PARAMETER VALUES FOR HOMOGENOUS CASE

Case	a	b	X	Y	*Px*	*Py*	*P*	Z
1	4	3	2656.2	5748.8	0	0	27871.3	1
2	4	3	2656.2	5748.8	2.2	3.8	0	1
3	4	3	2656.2	5748.8	2.5	3.6	2.9	1

TABLE II
PARAMETER VALUES FOR HETEROGENEOUS CASE FOR HIGH AND LOW CLASS CONSUMERS

Case	X_1	X_2	Y_1	Y_2	Z_1	Z_2	*Px*	*Py*	*P*
4	2656.2	2314.4	5748.8	2406.8	1	1	0	0	19814.1
5	2656.2	2314.4	5748.8	2406.8	1	1	0.1	4.8	0
6	2656.2	2314.4	5748.8	2406.8	1	1	4.8	0.1	0.1

TABLE III
PARAMETER VALUES FOR HETEROGENEOUS CASE FOR HIGH AND LOW CLASS CONSUMER CONSUMPTION

Case	X_1	X_2	Y_1	Y_2	Z_1	Z_2	*Px*	*Py*	*P*
7	2656.1	2314.4	5748.8	2406.8	1	1	0	0	15611.6
8	2656.1	2314.4	5748.8	2406.8	1	1	3.7	0.1	0
9	2656.1	2314.4	5748.8	2406.8	1	1	0.1	3.7	0.1

Then, we substitute the parameter values in Table I-III above to each model, then we have as follows.

Case 1: For flat fee Pricing schemes we set $P_X = 0, P_Y = 0$ and $P > 0$, meaning that the prices used by the service provider has no effect on the time of use.

Case 2: For Usage-based pricing scheme we set $P_X > 0, P_Y > 0$ and $P = 0$,meaning that service providers deliver differentiated prices, the price of consumption during peak hours and when the price of consumption at off-peak hours.

Case 3: For the pricing scheme with a two-part tariff scheme, we set $P_X > 0, P_Y > 0$ and $P = 0$ which means that service providers deliver differentiated price, i.e the price of consumption during peak hours and the price of consumption at off-peak hours.

Case 4: For the pricing scheme by setting a flat fee scheme, we set $P_X = 0, P_Y = 0$ and $P > 0$, meaning that the prices used by the service provider has no effect on the time of use, then consumers will choose the maximum consumption rate of $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y}$, dan $Y_2 = \bar{Y}$.

Case 5: For Usage-based pricing scheme by setting $P_X > 0, P_Y > 0$ and $P = 0$, with a maximum consumption rate $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y}$, dan $Y_2 = \bar{Y}$. Then consumers will choose the maximum consumption rate $X_1 = \bar{X},\ X_2 = \bar{X}$, $Y_1 = \bar{Y}$, dan $Y_2 = \bar{Y}$.

Case 6: For the pricing scheme with a two-part tariff scheme, we set $P_X > 0, P_Y > 0$ and $P = 0$, with a maximum consumption rate $X_1 = \bar{X},\ X_2 = \bar{X},\ Y_1 = \bar{Y}$, dan $Y_2 = \bar{Y}$. then consumers will choose the maximum consumption rate $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y}$, dan $Y_2 = \bar{Y}$.

Case 7: For the flat fee pricing schemes then we set $P_X = 0, P_Y = 0$ and $P > 0$, by choosing the level of consumption $X_1 = \bar{X}_1,\ Y_1 = \bar{Y}_1$ atau $X_2 = \bar{X}_2,\ Y_2 = \bar{Y}_2$.

Case 8: For Usage-based pricing scheme by setting $P_X > 0, P_Y > 0$ and $P = 0$ we choose the level of consumption $X_1 = \bar{X}_1,\ Y_1 = \bar{Y}_1$ atau $X_2 = \bar{X}_2,\ Y_2 = \bar{Y}_2$.

Case 9: For the pricing scheme with a two-part tariff scheme, we set $P_X > 0, P_Y > 0$ and $P = 0$, by choosing the level of consumption $X_1 = \bar{X}_1,\ Y_1 = \bar{Y}_1$ atau $X_2 = \bar{X}_2,\ Y_2 = \bar{Y}_2$.

Table IV below explains the data usage at peak and off-peak hours.

TABLE IV
DATA USAGE AT PEAK AND OFF-PEAK HOURS

	Mail **(*byte*)**	***Mail*** **(*kbps*)**
$\bar{X} - \bar{X}_1$	2719914.01	2656.17
$\bar{X}_2$	2369946.51	2314.40
$\bar{Y} - \bar{Y}_1$	5886849.92	5748.88
$\bar{Y}_2$	2464637,66	2406.87

where

1. $\bar{X}$ or $\bar{X}_1$ is the maximum possible level of consumption during peak hours both in units of kilo bytes per second.
2. $\bar{X}_2$ is the maximum possible level of consumption during off-peak hours in units of kilo bytes per second.
3. $\bar{Y}$ or $\bar{Y}_1$ is the maximum possible level of consumption both during peak hours in units of kilo bytes per second.
4. $\bar{Y}_2$ is the maximum possible level of consumption during peak hours in units of kilo bytes per second.

Table V below describes the optimal solution of using the perfect substitute utility function with the aid of LINGO 11.

TABLE V
OPTIMAL SOLUTION FOR ALL CASES

Objective	**Case**		
	1	**2**	**3**
Profit	27871.3	27871.3	27871.3
Objective	**Case**		
	4	**5**	**6**
Profit	99070.7	107105	107105
Objective	**Case**		
	7	**8**	**9**
Profit	78058	84370.5	84370.5

We can see from Table V that in homogenous case, we obtain the same maximum profit for all case of flat fee, usage based and two part tariff schemes. In other case, when we deal with heterogeneous high end and low end user consumers, the maximum profit is achieved when we apply the usage based and two part tariff. The last case when dealing with high and low demand users, again, the usage based and two part tariff yield the maximum profit.

If we compare the result in [16, 24], we have slightly difference. If using the modified Cobb-Douglass utility function, the maximum profit achieved when we apply the flat fee and two part tariff schemes for homogenous case. For heterogeneous case, maximum profit occurs when we apply the flat fee and two part tariff schemes. In our utility function, the three schemes yield the same profit in homogeneous case, while in heterogeneous case we obtain higher profit if we apply usage based and two part tariff schemes in heterogeneous case.

In using the perfect substitute utility function, the provider has more choices in applying pricing schemes that attract the customer to join the schemes.

V. Conclusions

Based on the application of the model on each data traffic, the use of perfect substitute utility functions for homogeneous and based on the flat fee, usage-based and two-part tariff pricing scheme obtained the same optimal solution, while the problem of heterogeneous consumer's consumption levels pricing schemes based on usage-based and two-part tariff obtained more optimal than the flat fee pricing schemes.

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