YILDIZ TECHNICAL UNIVERSITY

INTERNATIONAL CONFERENCE ON APPLIED ANALYSIS AND ALGEBRA

20 - 24 June 2012 Istanbul - Turkey

Certificate of Participation

This is to Certify that

Irmeilyana Saidi Ahmad

has presented a talk entitled
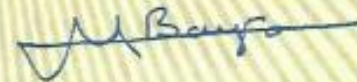
Clique Identification In Flight Route Problem

during the International Conference on
Applied Analysis and Algebra,
held on 20 - 24 June 2012 at the
YILDIZ TECHNICAL UNIVERSITY, ISTANBUL - TURKEY

Prof. Dr. Mustafa BAYRAM
Conference Chair

Clique Identification in Flight Route Problem

Irmeilyana[1]*, Indrawati[1], and Serly Angraini[1]

[1]*Department of Mathematics, Faculty of Mathematics and Natural Sciences, Sriwijaya University, Inderalaya, South Sumatera Indonesia*

*Email Address: imel_unsri@yahoo.co.id

Abstract

Clique is a subset of a directed graph which has at least three vertices. For each pair of vertices P_i and P_j in these subsets, both $P_i \rightarrow P_j$ and $P_j \rightarrow P_i$ is true. The subsets are made largest possible; in the sense that it is impossible to add other vertices into these subsets. By identifying *clique* we can know the relationship among the airports so that known which cities can be traversed to go from one city to another. It also can know which airports are busy or large airport of on every island in Indonesia. *Clique* identification is done on the flight route Sriwijaya Air and Garuda Indonesia. Each clique consists of only three cities, and includes Jakarta. The cities that form the *clique* are Banda Aceh, Medan, Batam, Pekanbaru, Palembang, Semarang, Surabaya, Banjarmasin, Balikpapan, Ujung Pandang, Gorontalo, Manado, Ambon, Kendari, Palu, Ternate, Denpasar, Kupang, Timika, Biak, and Jayapura. The number of airports that make up *clique* on Sriwijaya Air flight is less than the Garuda Indonesia flight. The number of *clique* that can be formed on Sriwijaya Air flight more than on Garuda Indonesia flight.

Keywords: Clique, Flight Route, Relationship among airports in Indonesia.

I.   INTRODUCTION

Graf is a Mathematical model that can be used to determine a solution of the problems, such as routing and transport issues, and Markov chains problem. A graph representation depends on the data of a particular case, one example is clique in the flight route.

In an aviation, there are connecting routes among cities, so one flight phase can be also through several stages of flight routes. In this case it can be seen which route should be passed to save costs and time.

This route can be identified as a clique which is a directed sub graph consisting of at least three vertices. Clique must consist of a subset of vertices of the directed graph. For each pair of vertices P_i and P_j in these subsets, there must be an arc from P_i to P_j and vice versa. The subsets are made largest possible; in the sense that it is impossible to add other vertices into these subsets.

By identifying a clique in airline routes, it can be seen where the towns are interconnected and can be determined which airports are 'busy' or large airports.

The purpose of this study was to identify a clique on the flight route Sriwijaya Air and Garuda Indonesia. Problems in the study was limited to the flight routes in Indonesia based on data from January to March 2011.

II.   METHODS

The steps in this study are as follows:

1. Retrieving flight routes data of Sriwijaya Air and Garuda Indonesia.
2. Sketch flight routes in the form of graphs. Airport is represented by vertices and route is denoted by an arc directed.
3. Compiled vertex matrix M.
4. Determining the stage of relationship among the airports.
5. Interpretation of Step 4.
6. Identify the clique by:

6.1 Formulate the matrix $S = [s_{ij}]$

$$s_{ij} = \begin{cases} 1; & for & P_i \leftrightarrow P_j \\ 0; & otherwise \end{cases} \tag{1}$$

6.2 Determining the matrix S^3

6.3 Checking $s_{ii}^{(3)} \neq 0$ (the diagonal entries of S)

7. Interpretation of Step 6.

III.   RESULTS AND DISCUSSION

Sriwijaya Air airline is one of the private airlines that has 31 flights in Indonesia. Meanwhile, Garuda Indonesia is one of the state-owned airlines that has more flight routes, especially in Indonesia. Garuda Indonesia airline has 32 flights.

3.1 Sketch the routes of flights in Graf

Once the data is obtained then we sketch flight routes in graph. P_i is defined as an airport in the city i and then P_i is written as a city where the airport i was.

(i). Sriwijaya Air Flight

Suppose that:

P_1 : Banda Aceh P_{12} : Tanjung Pandan P_{22} : Denpasar

P_2 : Medan P_{13} : Jakarta P_{23} : Kupang

P_3 : Jambi P_{14} : Bandung P_{24} : Ujung Pandang

P_4 : Padang P_{15} : Pontianak P_{25} : Kendari

P_5 : Palembang P_{16} : Palangkaraya P_{26} : Palu

P_6 : Bengkulu P_{17} : Malang P_{27} : Tarakan

P_7 : Tanjung Karang P_{18} : Solo P_{28} : Balikpapan

P_8 : Pekanbaru P_{19} : Semarang P_{29} : Gorontalo

P_9 : Batam P_{20} : Banjarmasin P_{30} : Ambon

P_{10} : Pangkal Pinang P_{21} : Surabaya P_{31} : Manado

P_{11} : Tanjung Pinang
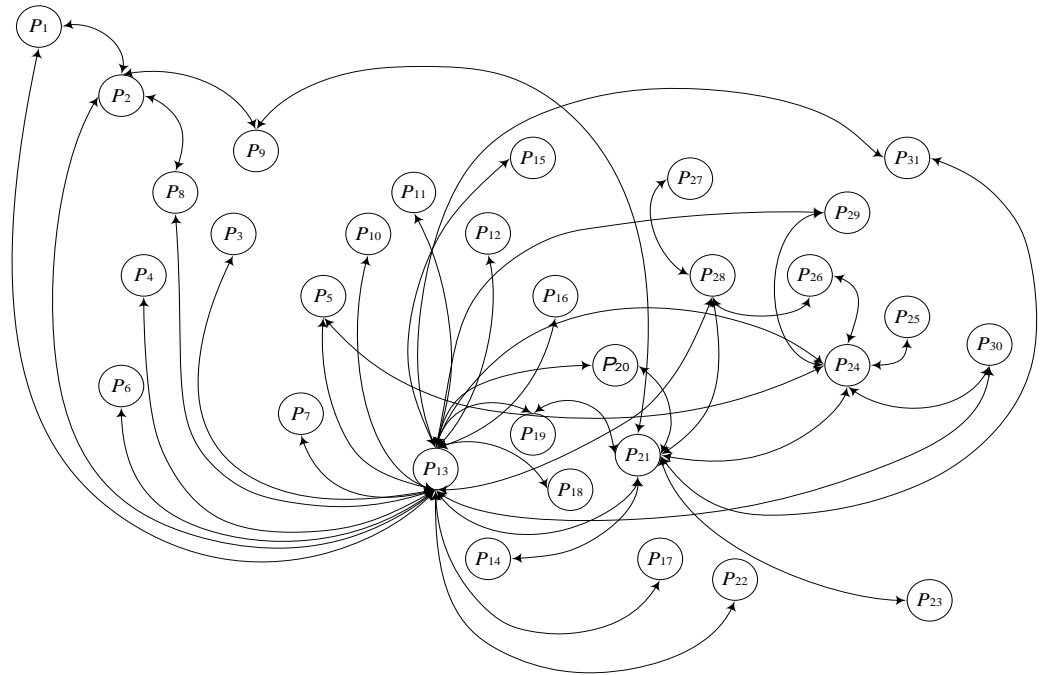
Figure 1. Graf of Sriwijaya Air Flight Routes

(ii). Garuda Indonesia Flight

Suppose that:

Q_1 : Banda Aceh

Q_2 : Medan

Q_3 : Jambi

Q_4 : Padang

Q_5 : Palembang

Q_6 : Tanjung Karang

Q_7 : Pekanbaru

Q_8 : Batam

Q_9 : Pangkal Pinang

Q_{10} : Jakarta

Q_{11} : Yogyakarta

Q_{12} : Malang

Q_{13} : Solo

Q_{14} : Semarang

Q_{15} : Pontianak

Q_{16} : Palangkaraya

Q_{17} : Surabaya

Q_{18} : Mataram

Q_{19} : Denpasar

Q_{20} : Kupang

Q_{21} : Banjarmasin

Q_{22} : Balikpapan

Q_{23} : Palu

Q_{24} : Ujung Pandang

Q_{25} : Kendari

Q_{26} : Ambon

Q_{27} : Gorontalo

Q_{28} : Manado

Q_{29} : Ternate

Q_{30} : Biak

Q_{31} : Jayapura

Q_{32} : Timika

Figure 2. Graf of Garuda Indonesia Flight Routes

Based on Figure 1 and Figure 2, both Sriwijaya Air and Garuda Indonesia airlines has flight route to Jakarta from the airport in each city. In this case there is a direct flight routes from each airport to Jakarta.

3.2 Preparation of vertex matrix

Based on the flight routes in Figure 1 and Figure 2, the vertex matrix M of every airline can be constructed.

(i). Vertex matrix for Sriwijaya Air Flight

There are 31 flight routes in 31 cities (represented by P_1, P_2, …, P_{31}). It can be arranged by vertex matrix M_{31x31}, as follows:

$$M = \begin{bmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&1&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&1&1&1&1&1&1&1&1&0&0&1&1&1&1&1&1&1&1&1&0&1&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&1&1&0&0&0&0&1&1&0&0&0&1&1&0&0&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&1&0&0&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0
\end{bmatrix}$$

Based on the matrix M can be seen that $m_{12} = 1$. It means there is one way (route) from P_1 to P_2 with one stage, i.e. $P_1 \rightarrow P_2$. For $m_{13} = 0$, there is no way from P_1 to P_3.

(ii). Vertex Matrix for Garuda Indonesia Flight

Routes of Garuda Indonesia flight are in 32 cities (represented by Q_1, Q_2, …, Q_{32}), then they can be constructed by vertex matrix $M_{32\times32}$, as follows:

$$M = \begin{bmatrix}
0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&1&1&1&1&1&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0&1&0&1&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&1&0
\end{bmatrix}$$

Based on the matrix M can be seen that $m_{21} = 1$. It means there is one way from Q_2 to Q_1 by 1 stage. The way is $Q_2 \rightarrow Q_1$.

3.3 Determination of Relationship among The Airports

To determine the relationship among the airports can be seen from the elements of matrix M. If its element is not zero, then it gives one stage relationship. If $m_{ij} = 1$, then there is 1 stage relationship from P_i to P_j. Whereas if $m_{ij} = 0$, then there is not relationship from P_i to P_j (no arc connecting P_i to P_j).

Matrix M^2 express the two stages relation of the airports, and M^3 express three stages relation of the airports. Elements of M^2, i.e. $m_{ij}^{(2)}$ provides number of ways on 2 stages relationship from vertex P_i to P_j. While $m_{ij}^{(3)}$ provides number of ways on three stages relationship from vertex P_i to P_j. As a note, $m_{ij}^{(2)}$ and $m_{ij}^{(3)}$ are assumed by $i \neq j$.

3.4 Interpretation of the matrix M, M^2, dan M^3

Based on the element of vertex matrix M, M^2, M^3 can be known that to go from one city to another can be through one stage, two stages and three stages. For example, in flight route of Sriwijaya Air, $m_{18}^{(2)} = 2$, then there are two ways for the two stages flight route from P_1 to P_8. The routes are $P_1 \rightarrow P_2 \rightarrow P_8$ or can also $P_1 \rightarrow P_{13} \rightarrow P_8$. If $m_{2,13}^{(2)} = 3$, then there are three ways for the two stages flight route from P_1 to P_{13}. The routes are $P_2 \rightarrow P_1 \rightarrow P_{13}$ or $P_2 \rightarrow P_8 \rightarrow P_{13}$ or can also $P_2 \rightarrow P_9 \rightarrow P_{13}$.

Further in the same way, it can also be interpreted for the other $m_{ij}^{(r)}$, with $i \neq j$. In fact, only one or more of the routes are used by customers (passengers).

An example is someone who will travel from Banda Aceh (P_1) to Pekanbaru (P_8) using Sriwijaya Air flight, the route chosen by him is through the 2 stages relationship, namely Banda Aceh (P_1)→Medan (P_2)→Pekanbaru (P_8). The person is unlikely (even impossible) to take flight route Banda Aceh (P_1) → Jakarta (P_{13}) → Pekanbaru (P_8) or three stages relationship route; for example: Aceh (P_1) → Jakarta (P_{13}) → Medan (P_2) → Pekanbaru (P_8). These routes lead to cost and price are not efficient.

3.5 Identification of the clique
3.5.1 Formulate a matrix S

Based on the vertex matrix M, then the S matrix can be constructed as Equation (1). In this case $s_{ij} = 1$ if $m_{ij} = m_{ji} = 1$ and $s_{ij} = 0$ if $m_{ij} = m_{ji} = 0$. The s_{ij} is the value of the entries in the i-th row and j-th column of the matrix S. Consequently S is a symmetric matrix with all their diagonal elements are 0 (because $m_{ii} = 0$ or no one stage relationship from P_i to P_j.

Element S can also be determined based on the graph. If there is a two-way route from P_i to P_j (represented by arc with two arrowheads), then $s_{ij} = 1$. Conversely, if there is no two-way route, then $s_{ij} = 0$.

3.5.2 Determining matrix S^3

Having obtained the matrix S can be determined matrix S^3.
(i). Matrix S^3 in Sriwijaya Air Flight Routes

$$S^3 = \begin{bmatrix}
2 & 7 & 1 & 1 & 2 & 1 & 1 & 2 & 3 & 1 & 1 & 1 & 28 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 8 & 1 & 1 & 5 & 1 & 2 & 1 & 2 & 2 & 2 & 2 \\
7 & 6 & 3 & 3 & 4 & 3 & 3 & 7 & 9 & 3 & 3 & 3 & 29 & 2 & 3 & 3 & 3 & 3 & 5 & 5 & 9 & 3 & 2 & 8 & 1 & 2 & 1 & 5 & 4 & 4 & 5 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
2 & 4 & 1 & 1 & 2 & 1 & 1 & 2 & 4 & 1 & 1 & 1 & 29 & 2 & 1 & 1 & 1 & 1 & 3 & 3 & 7 & 1 & 2 & 11 & 1 & 2 & 1 & 4 & 2 & 2 & 3 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
2 & 7 & 1 & 1 & 2 & 1 & 1 & 2 & 3 & 1 & 1 & 1 & 28 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 8 & 1 & 1 & 5 & 1 & 2 & 1 & 2 & 2 & 2 & 2 \\
3 & 9 & 2 & 2 & 4 & 2 & 2 & 3 & 4 & 2 & 2 & 2 & 34 & 1 & 2 & 2 & 2 & 2 & 3 & 3 & 17 & 2 & 1 & 6 & 2 & 4 & 2 & 3 & 4 & 4 & 3 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
28 & 29 & 25 & 25 & 29 & 25 & 25 & 28 & 34 & 25 & 25 & 25 & 24 & 6 & 25 & 25 & 25 & 25 & 31 & 31 & 37 & 25 & 6 & 37 & 4 & 5 & 1 & 34 & 29 & 29 & 31 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 6 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 9 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 2 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
2 & 5 & 1 & 1 & 3 & 1 & 1 & 2 & 3 & 1 & 1 & 1 & 31 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 15 & 1 & 1 & 5 & 2 & 4 & 2 & 2 & 3 & 3 & 2 \\
2 & 5 & 1 & 1 & 3 & 1 & 1 & 2 & 3 & 1 & 1 & 1 & 31 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 15 & 1 & 1 & 5 & 2 & 4 & 2 & 2 & 3 & 3 & 15 \\
8 & 9 & 6 & 6 & 7 & 6 & 6 & 8 & 17 & 6 & 6 & 6 & 37 & 9 & 6 & 6 & 6 & 6 & 15 & 15 & 12 & 6 & 9 & 24 & 1 & 2 & 1 & 18 & 7 & 7 & 8 \\
1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 25 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 6 & 0 & 1 & 4 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 6 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 9 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 2 & 1 \\
5 & 8 & 4 & 4 & 11 & 4 & 4 & 5 & 6 & 4 & 4 & 4 & 37 & 1 & 4 & 4 & 4 & 4 & 5 & 5 & 24 & 4 & 1 & 8 & 7 & 10 & 3 & 5 & 11 & 11 & 5 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 7 & 0 & 0 & 0 & 3 & 1 & 1 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 2 & 2 & 5 & 2 & 2 & 2 & 2 & 2 & 4 & 4 & 2 & 2 & 2 & 10 & 0 & 0 & 0 & 7 & 2 & 2 & 4 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 3 & 0 & 0 & 0 & 4 & 1 & 1 & 2 \\
2 & 5 & 1 & 1 & 4 & 1 & 1 & 2 & 3 & 1 & 1 & 1 & 34 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 18 & 1 & 1 & 5 & 3 & 7 & 4 & 2 & 4 & 4 & 2 \\
2 & 4 & 1 & 1 & 2 & 1 & 1 & 2 & 4 & 1 & 1 & 1 & 29 & 2 & 1 & 1 & 1 & 1 & 3 & 3 & 7 & 1 & 2 & 11 & 1 & 2 & 1 & 4 & 2 & 2 & 3 \\
2 & 4 & 1 & 1 & 2 & 1 & 1 & 2 & 4 & 1 & 1 & 1 & 29 & 2 & 1 & 1 & 1 & 1 & 3 & 3 & 7 & 1 & 2 & 11 & 1 & 2 & 1 & 4 & 2 & 2 & 3 \\
2 & 5 & 1 & 1 & 3 & 1 & 1 & 2 & 3 & 1 & 1 & 1 & 31 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 15 & 1 & 1 & 15 & 2 & 4 & 2 & 2 & 3 & 3 & 2
\end{bmatrix}$$

Some diagonal entries of matrix S^3 are non-zero value, one example is $s_{55} = 2$. It means there are two ways of P_5 to itself with 3 stages. The two ways are $P_5 \rightarrow P_{24} \rightarrow P_{13} \rightarrow P_5$ and $P_5 \rightarrow P_{13} \rightarrow P_{24} \rightarrow P_5$.

(ii). Matrix S^3 on Garuda Indonesia Flight Route

Some diagonal entries of matrix S^3 are non-zero value, one example is $s_{22} = 2$. It means there are two ways of Q_2 to itself with 3 stages. The two ways are $Q_2 \rightarrow Q_1 \rightarrow Q_{10} \rightarrow Q_2$ and $Q_2 \rightarrow Q_{10} \rightarrow Q_1 \rightarrow Q_2$.

$$S^3 = \begin{bmatrix}
2 & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 31 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 4 & 2 & 1 & 1 & 2 & 6 & 2 & 2 & 1 & 2 & 2 & 2 & 3 & 3 \\
3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 31 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 4 & 2 & 1 & 1 & 2 & 6 & 2 & 2 & 1 & 2 & 2 & 2 & 3 & 3 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
31 & 31 & 30 & 30 & 30 & 30 & 30 & 30 & 30 & 22 & 30 & 30 & 30 & 30 & 30 & 30 & 30 & 35 & 30 & 38 & 33 & 30 & 30 & 35 & 38 & 35 & 35 & 5 & 31 & 31 & 32 & 33 & 35 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 35 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 4 & 3 & 1 & 1 & 2 & 12 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 4 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
4 & 4 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 38 & 3 & 3 & 3 & 3 & 3 & 3 & 4 & 3 & 6 & 7 & 3 & 3 & 4 & 16 & 4 & 4 & 4 & 4 & 4 & 5 & 5 & 9 \\
2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 33 & 1 & 1 & 1 & 1 & 1 & 1 & 3 & 1 & 7 & 2 & 1 & 1 & 3 & 6 & 3 & 3 & 2 & 2 & 2 & 2 & 4 & 3 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\
2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 35 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 4 & 3 & 1 & 1 & 2 & 12 & 2 & 2 & 1 & 2 & 2 & 2 & 3 & 4 \\
6 & 6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 38 & 5 & 5 & 5 & 5 & 5 & 5 & 12 & 5 & 16 & 6 & 5 & 5 & 12 & 10 & 12 & 12 & 7 & 6 & 6 & 6 & 8 & 7 \\
2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 35 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 4 & 3 & 1 & 1 & 2 & 12 & 2 & 2 & 1 & 2 & 2 & 2 & 3 & 4 \\
2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 35 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 4 & 3 & 1 & 1 & 2 & 12 & 2 & 2 & 1 & 2 & 2 & 2 & 3 & 4 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 7 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 2 \\
2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 31 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 4 & 2 & 1 & 1 & 2 & 6 & 2 & 2 & 1 & 2 & 3 & 2 & 3 & 3 \\
2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 31 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 4 & 2 & 1 & 1 & 2 & 6 & 2 & 2 & 1 & 3 & 2 & 2 & 3 & 3 \\
2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 32 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 5 & 2 & 1 & 1 & 2 & 6 & 2 & 2 & 1 & 2 & 2 & 5 & 3 \\
3 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 33 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 5 & 4 & 2 & 2 & 3 & 8 & 3 & 3 & 1 & 3 & 3 & 5 & 4 & 7 \\
3 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 35 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 9 & 3 & 2 & 2 & 4 & 7 & 4 & 4 & 2 & 3 & 3 & 3 & 7 & 4
\end{bmatrix}$$

3.5.3 Checking $s_{ii}^{(3)} \neq 0$

(a). Sriwijaya Air flight

The diagonal entries of matrix S^3 are associated with the vertices which can form a clique. On Sriwijaya flight, the non-zero diagonal entries of S^3 ($s_{ii}^{(3)} \neq 0$) are $s_{11}^{(3)}$, $s_{22}^{(3)}$, $s_{55}^{(3)}$, $s_{88}^{(3)}$, $s_{99}^{(3)}$, $s_{13,13}^{(3)}$, $s_{19,19}^{(3)}$, $s_{20,20}^{(3)}$, $s_{21,21}^{(3)}$, $s_{24,24}^{(3)}$, $s_{28,28}^{(3)}$, $s_{29,29}^{(3)}$, $s_{30,30}^{(3)}$, $s_{31,31}^{(3)}$. In this case the value i of P_i are 1, 2, 5, 8, 9, 13, 19, 20, 21, 24, 28, 29, 30, and 31. Figure 3 states that a graph containing the vertices that can form a clique.

(b). Garuda Indonesia Flight

Non-zero diagonal entries of S^3 on Garuda Indonesia are $s_{11}^{(3)}$, $s_{22}^{(3)}$, $s_{10,10}^{(3)}$, $s_{17,17}^{(3)}$, $s_{19,19}^{(3)}$, $s_{20,20}^{(3)}$, $s_{23,23}^{(3)}$, $s_{24,24}^{(3)}$, $s_{25,25}^{(3)}$, $s_{26,26}^{(3)}$, $s_{28,28}^{(3)}$, $s_{29,29}^{(3)}$, $s_{30,30}^{(3)}$, $s_{31,31}^{(3)}$, $s_{32,32}^{(3)}$. In this case the vertices Q_i ($i = 1, 2, 10, 17, 19, 20, 23, 24, 25, 26, 28, 29, 30, 31, 32$) are member of the cliques. They can be seen in Figure 4.
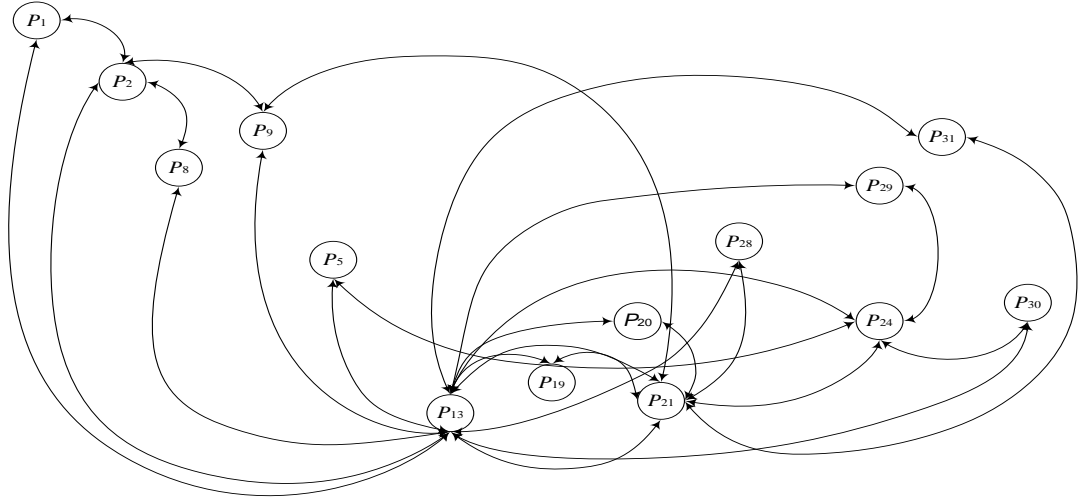
Figure 3. Graf of clique on Sriwijaya Air Flight Routes
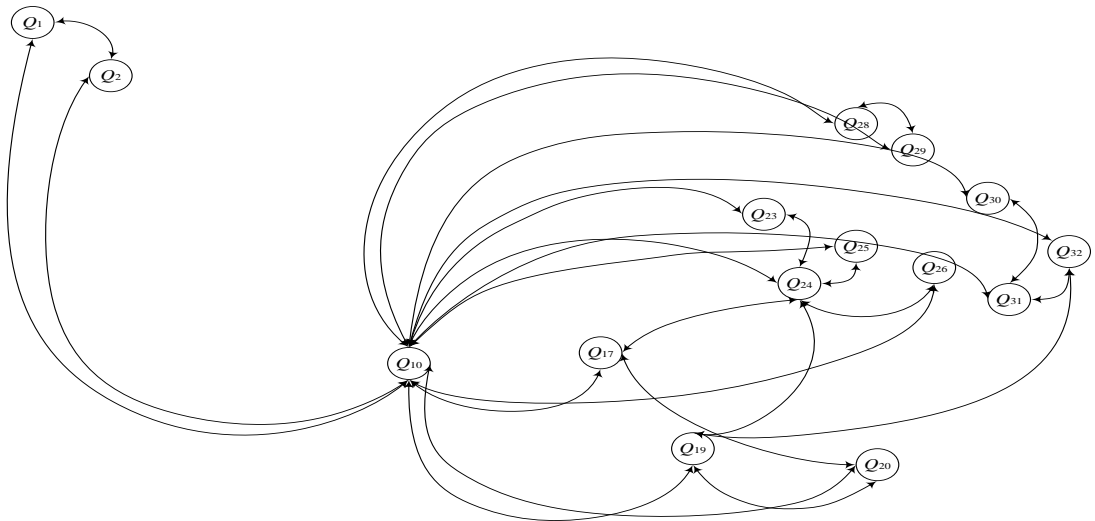
Figure 4. Graf of clique on Garuda Indonesia Flight Routes

3.6 Interpretation of Value $s_{ii}^{(3)} \neq 0$
(a). Sriwijaya Air flight

In matrix S^3 of Sriwijaya Air flight, there are 14 non-zero diagonal entries. Thus P_1, P_2, P_5. P_8, P_9, P_{13}, P_{19}, P_{20}, P_{21}. P_{24}, P_{28}, P_{29}, P_{30}, P_{31} are part of the cliques. Because of a clique must contain at least three vertices, so the Sriwijaya Air flight has 12 cliques, namely:

1. $\{P_1, P_2, P_{13}\}$ = {Banda Aceh, Medan, Jakarta}
2. $\{P_2, P_8, P_{13}\}$ = {Medan, Pekanbaru, Jakarta}
3. $\{P_2, P_9, P_{13}\}$ = {Medan, Batam Jakarta}
4. $\{P_5, P_{13}, P_{24}\}$ = {Palembang, Jakarta, Ujung Pandang}
5. $\{P_9, P_{13}, P_{21}\}$ = {Batam, Jakarta, Surabaya}
6. $\{P_{13}, P_{19}, P_{21}\}$ = {Jakarta, Semarang, Surabaya}
7. $\{P_{13}, P_{20}, P_{21}\}$ = {Jakarta, Banjarmasin, Surabaya}
8. $\{P_{13}, P_{21}, P_{24}\}$ = {Jakarta, Surabaya, Ujung Pandang}
9. $\{P_{13}, P_{21}, P_{28}\}$ = {Jakarta, Surabaya, Balikpapan}
10. $\{P_{13}, P_{21}, P_{31}\}$ = {Jakarta, Surabaya, Manado}
11. $\{P_{13}, P_{24}, P_{29}\}$ = {Jakarta, Ujung Pandang, Gorontalo}
12. $\{P_{13}, P_{24}, P_{30}\}$ = {Jakarta, Ujung Pandang, Ambon}

All clique that formed on Sriwijaya Air flight route includes the airport in Jakarta.

b. Garuda Indonesia Flight

In the Garuda Indonesia flight, there are 15 non-zero diagonal entries. Thus Q_1, Q_2, Q_{10}, Q_{17}, Q_{19}. Q_{20}, Q_{23}, Q_{24}, Q_{25}, Q_{26}, Q_{28}, Q_{29}, Q_{30}, Q_{31}, Q_{32} are part of the cliques. Because of a clique must contain at least three vertices, so that Garuda Indonesia has 11 cliques, namely:

1. $\{Q_1, Q_2, Q_{10}\}$ = {Banda Aceh, Medan, Jakarta}
2. $\{Q_{10}, Q_{17}, Q_{24}\}$ = {Jakata, Surabaya, Ujung Pandang}
3. $\{Q_{10}, Q_{19}, Q_{20}\}$ = { Jakata, Denpasar, Kupang}
4. $\{Q_{10}, Q_{19}, Q_{24}\}$ = { Jakata, Denpasar, Ujung Pandang}
5. $\{Q_{10}, Q_{19}, Q_{32}\}$ = { Jakata, Denpasar, Timika}
6. $\{Q_{10}, Q_{23}, Q_{24}\}$ = { Jakata, Palu, Ujung Pandang}
7. $\{Q_{10}, Q_{24}, Q_{25}\}$ = { Jakata, Ujung Pandang, Kendari}
8. $\{Q_{10}, Q_{24}, Q_{26}\}$ = { Jakata, Ujung Pandang, Ambon}
9. $\{Q_{10}, Q_{28}, Q_{29}\}$ = { Jakata, Manado, Ternate}
10. $\{Q_{10}, Q_{30}, Q_{31}\}$ = { Jakata, Biak, Jayapura}
11. $\{Q_{10}, Q_{31}, Q_{32}\}$ = { Jakata, Jayapura, Timika}

All clique that formed on Garuda Indonesia flight route includes the airport in Jakarta.

If cliques on Sriwijaya Air and Garuda Indonesia flight routes are compared, then there are the cities (airport) that are only member of a clique on only one flight.

The cities (airports) of clique member on Sriwijaya Air flight routes that are not a member of the clique at Garuda Indonesia are Pekanbaru, Batam, Palembang, Semarang, Banjarmasin, Balikpapan, and Gorontalo. Instead, the cities (airports) are only as member of the clique at Garuda Indonesia flight are Denpasar, Kupang, Palu, Kendari Ternate, Biak, Jayapura and Timika.

While cities that are members of the clique on both flight routes are Banda Aceh, Medan, Jakarta, Surabaya, Ujung Pandang and Manado and Ambon.

IV. CONCLUSIONS AND RECOMMENDATIONS

Based on the identification of clique on Sriwijaya Air and Garuda Indonesia flight routes can be concluded:

1. Each clique consists of only three cities.
2. At Sriwijaya Air flight:
2.1 There are 14 airports that can form cliques, and there are 12 cliques.
2.2 For Sumatra island, "big airports" are Banda Aceh, Medan, Batam, Pekanbaru, and Palembang.
2.3 For Sulawesi island, "big airports" is Gorontalo, Manado, and Ujung Pandang.
2.4 For the Maluku Islands and Papua, "big airport" is Ambon.
2.5 For Borneo island, "big airporst" are Banjarmasin and Balikpapan.
2.6 For Java island, "big airports" are Semarang and Surabaya.

3. In the Garuda Indonesia flight:
3.1 There are 15 airports to form the clique, and there are 11 cliques.
3.2 For Sumatra island, "big airporst" are Banda Aceh and Medan.
3.3 For Sulawesi island, "big airports" are Ujung Pandang, Palu, Kendari, and Manado.
3.4 For the Islands of Maluku and Papua, "big airports" are Ambon, Ternate, Biak, Timika and Jayapura.
3.5 For Java island, "big airport" is Surabaya.
3.6 For Bali and East Nusa Tenggara islands, "big airports" are Denpasar and Kupang.
4. The number of airports that make up the clique on Sriwijaya Air flight is less than the airline Garuda Indonesia. However, the clique that can be formed on Sriwijaya Air flight more than the airline Garuda Indonesia.

REFERENCES

Aldous, J and Wilson, R. 2000. *Graph and Applications*. The Open University, Great Britain.

Anton, H & Rorres, C. 1994. *Elementary Linear Algebra Applications Version*. 7th Ed. Jhon Wiley and Sons, Inc., New York.

Anton, H & Rorres, C. 2002. *Aljabar Linier Terapan Versi Aplikasi*. Edisi ke delapan Jilid 2. Terjemahan dari : *Elementary Linear Algebra Applications Version/Eight Edition*. Erlangga, Bandung.

Harary, F. 1994. *Graph Theory.* Addison-Wesley Publishing Company, London.

Thulasariman, K & Swamming, MNS. 1992. *Graph : Theory and Algorithms*. John Willey and Sons, Inc., New York.

http://www.dapatinfo.com/2011/01/jadwal-penerbangan-sriwijaya-air.html. Diakses 28 Januari 2011.