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THE DETERMINATION OF OPTIMAL ROUTE OF OPEN CAPACITATED VEHICLE ROUTING PROBLEM (OCVRP) ON GARBAGE TRANSPORTATION MODEL IN KECAMATAN SEBERANG ULU II KOTA PALEMBANG

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ABSTRACT

The system of garbage transportation in Palembang usually been done in many phases. Garbage collected from homes usually been collected to the nearest temporary garbage collection place (TPS). This garbage will be delivered to final garbage collection place (TPA). This system divides into working areas (WK). The garbage truck, in this case, does not go back to the depot (TPA) after accomplishing its job. The driver usually takes the truck to his house. This problem is open capacitated vehicle routing problem (OCVRP). In this paper, the issue is OCVRP on vehicle routes of garbage transportation in Kecamatan Seberang Ulu II Kota Palembang. All OCVRPs obtained, its initial solution is integer so we cannot apply branch and cut algorithm. But for symmetric CVRP (SCVRP) its optimal solution is non integer, so we have to apply branch and cut algorithm. In OCVRP model, its solution is not valid, so we have to add new constraints to obtain valid optimal solution. The optimal solution yields optimal route of garbage transport vehicle at 7 WKs in Kecamatan Seberang Ulu II. The routes are as follows:

WK-1 : Home driver(1)-Tangga Takat(3)-TPA(0)-YaktapenaII(4)-Telaga Swidak(1)-TPA(0)
WK-2 : Home driver(1)-Jl.Jaya(2)-TPA(0)-Pasar Plaju(3)-TPA(0)
WK-3 : Home driver(1)- Bagus Kuning(2)-TPA(0)-Simpang3KA(3)-TPA(0)
WK-4 : Home driver(1)-Sentosa(2)-TPA(0)-Pasar PlajuII(3)-TPA(0)
WK-5 : TPA(0)-SD 254(2)-Patra Jaya(1)-TPA(0)-SD 254(2)-Pulau LayangI(3)-TPA(0)
WK-6 : DKP(1)-Tegal Binangun(2)-TPA(0)-Pulau Layang II(2)-TPA(0)
WK-7 : Home driver(1)-Yaktapena(2)-TPA(0)-Pertahanan(3)-TPA(0)-Bagus Kuning(4)-TPA(0)

To find a simpler OCVRP model and more efficient in its solution process, we use Preprocessing technique. The results show that the optimal route of garbage transport vehicle on OCVRP models before using preprocessing and after using preprocessing technique is the same. The results obtained show that the number of constraints, the number of variables, the number of iterations are reduced; the value z does not change; and z in the optimum model has fewer variables.

Keywords: OCVRP, Branch and Cut Algorithm, Vehicle Route of Garbage Transportation, Preprocessing Technique.

I. INTRODUCTION

Garbage transportation system in Palembang [3] usually been done in many phases. Garbage collected from homes usually been collected to the nearest temporary garbage collection place (TPS). This garbage will be delivered to final garbage collection place (TPA). This system divides into working area (WK).

In conventional CVRP problem as discussed in [3] and [6], transport of commodities required to return to the depot after completing the work. But, for some vehicle routing problems (VRP) such as garbage transportation, the conditions mentioned above can not be done.

Transport vehicle usually does not return to the depot after performing their duties, but returned to another place, such as driver home. This condition occurs, since the route must pass by becoming more efficient if the vehicle had been taken earlier by a driver who transports garbage.

Another important incidents are usually occur when schedule visit demand is not once, that is, the traveling vehicle must be divided into several sections to regulate demand. Furthermore, if period time in the visit demand is limited, it will lead to limitation of delivery time. The problems which cause new problems to be solved because the trajectories are formed not closed, but the path is open, and the demand which is divided, so the problem becomes OCVRP-st (Open CVRP-split, time deadlines) [4]. OCVRP-st problem is important to be developed because it deals with the transport of commodities such as garbage.

Garbage transportation system in Palembang city done gradually. Garbage from households is transported by a man to collect on TPS which is provided by DKP nearby. Furthermore, the existing garbage in TPS are transported by DKP' officers by using garbage transport vehicle (dump truck, amroll) to one of two TPAs, namely TPA Karya Jaya and TPA Sukawinatan. Garbage transportation been done by the DKP officers are divided according to the Working Area (WK) for each driver.

Caccetta [1] emphasized that for CVRP problem consisting of 100 vertices, Branch and cut as deemed appropriate CVRP completion method as its solution method. So, based on last research on the transportation of garbage and also on the exact solution method of branch and cut are necessary to formulate an appropriate model for OCVRP-st and also its solution techniques by branch and cut method. If the branch and cut method is applied, then preprocessing and probing techniques used to detect the tight of a model which is formed should also be applied.

Based on the background described earlier, it needs to create models and solve them with application software on garbage transportation problems in Palembang city. It also needs the algorithm of Branch and Cut method for OCVRP on garbage transportation which focuses on Kecamatan Seberang Ulu II Palembang. To find a simpler OCVRP model and more efficient in its solution process, we use Preprocessing technique.

II. RESEARCH METHOD

Design and methodology of research can be arranged in the working steps as follows:

1. The study is begun by preparing materials from various sources, including books, journals, and information on the internet relating to CVRP and their variations such as the open path conditions, delivery of commodities which are divided, the limited time of delivery between each customer, branch and cut algorithm, probing and preprocessing techniques.
2. Garbage transportation data surveys in Palembang related to vehicle and TPS capacities, time collection, and collection time sharing.
3. A general description of the work stages was done, as follows:

   a. From transportation data, set assumptions to develop CVRP to OCVRP-st, i.e.:
   - identical vehicle capacity unknown
   - demand is known with certainty and must be divided
   - single depot
   - travel expenses of all customer locations in both directions (symmetric) and one-way (anti symmetric).

   b. Develop OCVRP-st models by
   - defining the problem and the notation, by assuming the problem can be represented in a graph
   - to establish some basic reduction basic rules
   - trying to eliminate the arc which causes unfeasibility of OCVRP-st
   - trying to eliminate the arc current that is not correct solution.

   c. Developing the model, consisting of
   - two-index vehicle flow formulation
   - OCVRP-st formulation which became the basis of settlement with Branch and cut algorithm

   d. Forming valid inequalities that are used to support the branch and cut algorithm in solving OCVRP-st, such as:
   - Develop a modified adjusted comb inequalities for the problem OCVRP-st.
   - Using the sub tour elimination

   e. Analyzing the results obtained in the form of optimal routes and draw conclusions from the model.

Stage of completion model performed with 4 stages, as follows:

1. Simplifying OCVRP model, by using probing and preprocessing techniques through the strengthening of border measures on the variable of constraints, the elimination of excessive constraints and variables repairs.
2. Settlement Models with Probing Technique

a) Improvement of Variable; with these steps:

1. Identification of constraints on new formulation.

2. Arrange the final formulation based on the strengthening the boundary on the variable of constraints, the elimination of excessive constraints and variables repairs.

b) Increased Coefficient

• If $Z_k \le b_i$, then $a^i\mathbf{x} \le b_i$ become redundant. So, by assuming $x_k = 0$, the set of feasible solution will not change if b_i and a^i_k are converted into $\delta = b_i - Z_k$.

• If $Z_k \le b_i$, then $a^i\mathbf{x} \le b_i$ become redundant in the formulation. So, by assuming $x_k = 1$, the set of feasible solution will not change if b_i and a^i_k are converted into $\delta = b_i - Z_k$.

c) Identifying Logical Implications.

- If $x_j = 0$, set $x_k = 0$ so that $x_k \le x_j$
  Set $x_k = 1$ so that $1 - x_k \le x_j$
- If $x_j =1$, set $x_k = 0$ so that $x_k \le 1- x_j$
  Set $x_k = 1$ so that $1 - x_k \le 1 - x_j$

d) The procedure for setting the variable; that is done like step 2a).

3. Settlement Models with Preprocessing Technique; consists of:

a) Strengthening the Boundary on Variable of Constraint; with these steps:

1. Constructing the value of objective function based on variable restriction in the early formulation of ILP problems.
2. Strengthening the border of each variable in each constraint based on the value of variables that have been prepared.
3. Evaluating.

b) Removal of Redundant Constraints; with these steps:

1. Comparing the new formulations are obtained based on the process of strengthening the boundary on the variables of constraints with initial formulation.
2. Evaluate: Arrange the new formulation based on strengthening the boundary technique on variables of constraints and the removal of redundant constraints.

c) Improvement of the variable; that is done like step 2a).

4. Comparing the Models that is result before and after using Preprocessing Techniques

III. RESULTS AND DISCUSSION

In this section, it was discussed about additional OCVRP-st models if a solution formed is invalid, i. e. resulting in a integer solution but the fact, there are TPS which did not visit yet or TPS are visited more than once. It means breaking CVRP condition that the customer should be visited only once. In this chapter, it was also described the model and the optimal route.

OCVRP model can be obtained by modifying the standard formulation of CVRP. Mathematically, OCVRP can be stated as follows [4]:

Minimize $Z = \sum_{i,j \in V_c} c_{ij} x_{ij} + \sum_{i \in V_c} c_{0i} y_{0i}$

Subject to:

$$x(\bar{\delta}(i)) + y_{0i} + y_{i0} = 2 \quad (i = 1, 2, ..., n) \tag{1}$$

$$x(\bar{\delta}(S)) + y^-(S) + y^+(S) \ge 2k(S) \quad (S \subseteq V_c, |S| \ge 2) \tag{2}$$

$$y^-(V_c) = K, \text{ with } y^-(V_c) = \sum_{i \in V_c} y_{0i} \tag{3}$$

$$y^+(V_c) = K, \text{ with } y^+(V_c) = \sum_{i \in V_c} y_{i0} \tag{4}$$

$$x_{ij} \in \{0,1\}, \text{ for } 1 \le i < j \le n \tag{5}$$

$$y_{0i}, y_{i0} \in \{0,1\} \quad (i = 1, 2, \ldots, n) \tag{6}$$

where Z is objective function, c_{ij} is distance from location i to j, x_{ij} is route from i to j, c_{0i} is distance from the depot to the customer location i, y_{0i} is travel route from the depot to customer i, y_{i0} is route from customer i to the depot,

$x(\bar{\delta}(S)) = \sum_{i,j \in S} x_{ij}$, for $\bar{\delta}(S) = \{\{i, j\} : i \in S,\ j \in \bar{S}\}$ and $\bar{S} = V_c \setminus S$,

$x(\bar{\delta}(i)) = \sum_{i \in S} x_{i0}$,for $\bar{\delta}(i) = \{\{i,0\} : i \in S,\ 0 \in V\}$,

K is number of vehicles, V_c is a set of customers, where $V_c = V \backslash \{0\}$, $k(S)$ = lower bound of the minimum number of vehicles required to visit the customer S, $k(S) = \frac{q(S)}{Q}$, $q(S)$ is customer demand and Q is vehicle capacity.

To prevent the solution is not valid, it is necessary to add a constraint which is called the balancing inequality constraint:

$$x(\bar{\delta}(S)) + y^{+}(S) \geq y^{-}(S) \quad (S \subseteq V_c, |S| \geq 2) \tag{7}$$

The absence of such constraints on CVRP standard formula, shows that OCVRP is more complex problem than CVRP. [5] describes the additional constraints used in the completion CVRP if the optimal solution is proved invalid. Suppose the given number of vehicles is K, the vehicle capacity is C, a distance matrix is symmetric and the average volume of each TPS is d_i , where i = 1, 2, … , n,, then SCVRP model is formulated as follows:

Minimumize: $$Z = \sum_{0 \leq i \leq j \leq n} c_{ij} x_{ij} \tag{8}$$

Subject to:

$$\sum_{i \leq j \leq n} x_{0k} = 2K \tag{9}$$

The trip starts from the depot to TPS and immediately returned to the depot again. K is the number of garbage vehicles, in a case where the vehicle is equal to 1 (K = 1) per working area (WK).

$$\sum_{e \in \delta(i)} x_e = 2$$

Because of symmetric, a trip from i to j equal to the trip from j to i, while the trip i to j is calculated as 1, then the trip from i to j and the trip from j to i counted 2.

$$\sum_{0 \leq j \leq n} x_{ij} = 2 \quad \text{for all} \quad 1 \leq i \leq n \tag{10}$$

Travel is not started from the depot

$$\sum_{j \in S} x_{oj} + \sum_{(i,j) \in S} x_{ij} = 2 \leq 2b(S) \quad \text{for all}$$

$S \subset V \backslash \{0\}$; $|S| \geq 2$ (11)

$b(S)$ = lower bound (LB) of the number of vehicles required to visit customers S is obtained from $b(S) = \frac{\sum_{i \in S} d(i)}{C}$ where $d(i)$) is the total volume of TPS, C is the capacity of vehicles (trucks), and S is identical set of TPS visited.

$x_{ij} \in \{0,1,2\}$ for all $e \in \delta(0)$ which is the value of travel routes or nonnegative binary constraints.

For WK cases if the equation on the optimal solution is not formed, the minimum route and there are more than or the same as TPS that is not visited, then *b(S)* was changed to $b(S) = \left\lceil \frac{\sum_{i\in S} d(i)}{C} \right\rceil$ where: $\left\lceil \frac{\sum_{i\in S} d(i)}{C} \right\rceil$ is the smallest integer that is greater than or equal $\frac{d(i)}{C}$.

Thus Equation (11)) is also broken into its multiple permutations *b(S)* as follows:

$$\sum_{j\in S} x_{ij} = 1 \text{ for all } 1 \le i \le n \tag{12}$$

$$\sum_{i\in S} x_{ij} = 1 \text{ for all } 1 \le j \le n \tag{13}$$

$$\sum_{j\in S} x_{ij} = 2 \text{ for all } 1 \le i \le n \tag{14}$$

$$\sum_{i\in S} x_{i0} = 2 \tag{15}$$

Additional OCVRP-st Models

For the OCVRP-st model in the form (1) until (6) often occurs in the form of solutions that form the route is invalid, then it was made its permutation *b(S)* to OCVRP-st as follows:

$$y_{oi} + y_{io} = 1, (\mathrm{i} = 1,\ldots,n) \tag{16}$$

$$y_{oi} + \sum_{j\in S} x_{ij} = 1, (i = 1,\ldots,n) \tag{17}$$

$$y_{oi} = 1, (i = 1,\ldots,n) \tag{18}$$

$$y_{i0} = 1, (i = 1,\ldots,n) \tag{19}$$

$$\sum_{i\in S} x_{ij} = 1 \text{ for all } 1 \le j \le n \tag{20}$$

Furthermore, based on model (1) - (6) and (16) - (20), data were found, then they are modeled and found their solution by LINDO software application tools to get the optimal solution; i. e. the route of vehicle in every WK.

The results of research that have conducted data of garbage transportation in every district (kecamatan) in Palembang. Data processing was done by modeling the distance data into the OCVRP model (if the vehicle starts at driver home) and CVRP (if the vehicle departs from DKP office because the trajectory formed is closed) .

Listed below is the results that have been obtained with the initial routes are acquired but not yet valid and further improved by the addition of balancing constraints and additional constraints for Kecamatan Seberang Ulu II. Optimum route is obtained and depicted began moving vehicle until the expiry of the transport vehicle in TPA Sukawinatan.

[6] developed SCVRP algorithms to be solved by branch and cut method. Furthermore, for OCVRP formulations (1) until (6), solutions with branch and cut method can also be applied similar to SCVRP that discussed in [6] and [7]. B (S) is the lower bound of the number of vehicles required to visit all the locations S in the optimal solution. Some vehicles that serve the customer set S, and some of the activities of these vehicles can be described as below:

- Vehicles leaving the depot, serving customers *S* and back again to the depot.
- Vehicles leaving the depot, serve customers *S* and serve a subset of customers *S*.
- Vehicles serving the customer subset *S* before and after serving customers *S*.

With a note $b(S)$ was transferred by the lower bound (LB) should be capacity restrictions. In practice, the determination is done by calculating LB each different restrictions then form $b(S)$ as the maximum value.

In Branch and Cut scheme, $b(S)$ was calculated as the maximum value of LB resulting from restrictions capacity or distance limitation.
Below are intended algorithm for solving the OCVRP problem obtained from the above explanation.

Branch and Cut Algorithm in Solving OCVRP

Step 1. Begin by considering LP (Linear Programming) relaxation. Where the number of vehicles is variable, K based on the restricted $b(S)$. Enter the distance between customers and depot, then customer demand and the capacity of the vehicle.
Step 2. Solve LP. If the objective function value is at least or equal to the upper bound (UB) then it stops. If not proceed to Step 3.
Step 3. Remove some restrictions of the LP, which has a basic slack variables.
Step 4. If LP solution meet sub tour elimination constraints and integer, then the solution is the UB from the real problem, then stop. If not proceed to Step 5.
Step 5. If tour S is not connected with the depot, the violation will be instantly added. When it's connected to the depot, if the tour S does not violate capacity and distance limitation then the search should be attempted to find a violation of the restrictions on the capacity of the subset S. Restricted of merging violations sets are also added to the LP. Add some violation restrictions to restriction LP set and proceed to Step 2, if the violation restriction was not found then the objective function value is lower bound; stop.

Kecamatan Seberang Ulu II Model

In Kecamatan Seberang Ulu II, both two WKs have one garbage truck that is amroll (a truck with open container), so the route that is formed can not be analyzed because each vehicle transporting only one container.

a. WK 1

Use the Equation (1) - (6) thus becomes
Minimize $20.7y_{01}+15.7y_{02}+13.5y_{03}+12.9y_{04}+20.7x_{10}+5x_{12}+7.2x_{13}+7.8x_{14}+15.7x_{20}$
$+5x_{21}+2.2x_{23}+2.8x_{24}+13.5x_{30}+7.2x_{31}+2.2x_{32}+0.6x_{34}+12.9x_{40}+7.8x_{41}+2.8x_{42}+0.6x_{43}$
subject to
$y_{01}+x_{12}+x_{13}+x_{14}+y_{10}=2$
$y_{02}+x_{21}+x_{23}+x_{24}+y_{20}=2$
$y_{03}+x_{31}+x_{32}+x_{34}+y_{30}=2$
$y_{04}+x_{41}+x_{42}+x_{43}+y_{40}=2$
$y_{01}+y_{02}+y_{03}+y_{04}+y_{10}+y_{20}+y_{30}+y_{40}+x_{12}+x_{13}+x_{14}+x_{21}+x_{23}+x_{24}+x_{31}+x_{32}+x_{34}+x_{41}+x_{42}+x_{43}>=4.2$
$y_{10}+y_{20}+y_{30}+y_{40}-y_{01}-y_{02}-y_{03}-y_{04}+x_{12}+x_{13}+x_{14}+x_{21}+x_{23}+x_{24}+x_{31}+x_{32}+x_{34}+x_{41}+x_{42}+x_{43}>=0$
$y_{01}+y_{02}+y_{03}+y_{04}=1$
$y_{10}+y_{20}+y_{30}+y_{40}=1$
$y_{01}, y_{02}, y_{03}, y_{04}, y_{10}, y_{20}, y_{30}, y_{40}, x_{10}, x_{20}, x_{30}, x_{40}, x_{12}, x_{13}, x_{14}, x_{21}, x_{23}, x_{24}, x_{31}, x_{32}, x_{34}, x_{41}, x_{42}, x_{43}>=0$

<u>LINDO' Result</u>

LP OPTIMUM FOUND AT STEP 6
OBJECTIVE FUNCTION VALUE
1) 24.10000

VARIABLE	VALUE	REDUCED COST
Y01	0.000000	3.400001
Y02	0.000000	1.200000
Y03	0.000000	0.600000
Y04	1.000000	0.000000
X10	0.000000	20.700001
X12	1.000000	0.000000
X13	0.000000	2.200000
X14	0.000000	2.800000
X20	0.000000	15.700000
X21	0.000000	2.800000
X23	2.000000	0.000000

X24	0.000000	0.600000
X30	0.000000	13.500000
X31	0.000000	6.600000
X32	0.000000	1.600000
X34	2.000000	0.000000
X40	0.000000	12.900000
X41	0.000000	7.200000
X42	0.000000	2.200000
X43	1.000000	0.000000
Y10	1.000000	0.000000
Y20	0.000000	2.800000
Y30	0.000000	4.400000
Y40	0.000000	4.400000

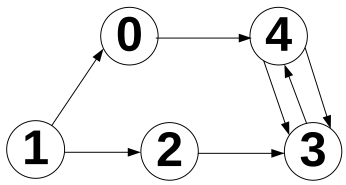


Figure 1. Initial route in on WK 1 in Kecamatan Seberang Ulu II

From the graph, there are no polling stations visited, so the constraint would be broken up and added balancing constraints to prevent invalid solutions. After the addition of balancing constraints, and been split on the constraints, then the model becomes:

minimize

$20.7y_{01}+15.7y_{02}+13.5y_{03}+12.9y_{04}+20.7x_{10}+5x_{12}+7.2x_{13}+7.8x_{14}+15.7x_{20}+5x_{21}+2.2x_{23}+2.8x_{24}+13.5x_{30}+7.2x_{31}+2.2x_{32}$
$0.6x_{34}+12.9x_{40}+7.8x_{41}+2.8x_{42}+0.6x_{43}$

subject to

$x_{13}+x_{14}=1$

$y_{02}+y_{20}=1$

$x_{42}=1$

$y_{30}=1$

$y_{04}+y_{40}=1$

$y_{01}+y_{02}+y_{03}+y_{04}+y_{10}+y_{20}+y_{30}+y_{40}+x_{12}+x_{13}+x_{14}+x_{21}+x_{23}+x_{24}+x_{31}+x_{32}+x_{34}+x_{41}+x_{42}+x_{43}>=4.2$

$y_{10}+y_{20}+y_{30}+y_{40}-y_{01}-y_{02}-y_{03}-y_{04}+x_{12}+x_{13}+x_{14}+x_{21}+x_{23}+x_{24}+x_{31}+x_{32}+x_{34}+x_{41}+x_{42}+x_{43}>=0$

$y_{10}+y_{20}+y_{30}+y_{40}=1$

$y_{01}, y_{02}, y_{03}, y_{04}, y_{10}, y_{20}, y_{30}, y_{40}, x_{10}, x_{20}, x_{30}, x_{40}, x_{12}, x_{13}, x_{14}, x_{21}, x_{23}, x_{24}, x_{31}, x_{32}, x_{34}, x_{41}, x_{42}, x_{43}>=0$

LINDO' Result

LP OPTIMUM FOUND AT STEP 4

OBJECTIVE FUNCTION VALUE

1) 38.96000

VARIABLE	VALUE	REDUCED COST
Y01	0.000000	20.100000
Y02	1.000000	0.000000
Y03	0.000000	12.900000
Y04	1.000000	0.000000
X10	0.000000	20.700001
X12	0.000000	4.400000
X13	1.000000	0.000000
X14	0.000000	0.600000
X20	0.000000	15.700000
X21	0.000000	4.400000
X23	0.000000	1.600000
X24	0.000000	2.200000
X30	0.000000	13.500000
X31	0.000000	6.600000
X32	0.000000	1.600000
X34	0.000000	0.000000
X40	0.000000	12.900000
X41	0.000000	7.200000
X42	1.000000	0.000000

X43	0.600000	0.000000
Y20	0.000000	0.000000
Y30	1.000000	0.000000
Y40	0.000000	2.800000
Y10	0.000000	15.100000
X04	0.000000	0.000000

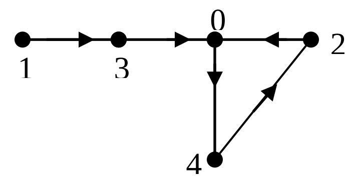


Figure 2. The final route on WK 1 Kecamatan Seberang Ulu II

From the graph, there are no polling stations visited. After the addition of balancing constraints, and been split on the constraints, then the model becomes:

minimize

$20.7y_{01}+15.7y_{02}+13.5y_{03}+12.9y_{04}+20.7x_{10}+5x_{12}+7.2x_{13}+7.8x_{14}+15.7x_{20}+5x_{21}+2.2x_{23}+2.8x_{24}+13.5x_{30}+7.2x_{31}+2.2x_{32}$

$0.6x_{34}+12.9x_{40}+7.8x_{41}+2.8x_{42}+0.6x_{43}$

subject to

$x_{13}+x_{14}=1$

$y_{02}+y_{20}=1$

$x_{42}=1$

$y_{30}=1$

$y_{04}+y_{40}=1$

$y_{01}+y_{02}+y_{03}+y_{04}+y_{10}+y_{20}+y_{30}+y_{40}+x_{12}+x_{13}+x_{14}+x_{21}+x_{23}+x_{24}+x_{31}+x_{32}+x_{34}+x_{41}+x_{42}+x_{43}>=4.2$

$y_{10}+y_{20}+y_{30}+y_{40}-y_{01}-y_{02}-y_{03}-y_{04}+x_{12}+x_{13}+x_{14}+x_{21}+x_{23}+x_{24}+x_{31}+x_{32}+x_{34}+x_{41}+x_{42}+x_{43}>=0$

$y_{10}+y_{20}+y_{30}+y_{40}=1$

$y_{01}, y_{02}, y_{03}, y_{04}, y_{10}, y_{20}, y_{30}, y_{40}, x_{10}, x_{20}, x_{30}, x_{40}, x_{12}, x_{13}, x_{14}, x_{21}, x_{23}, x_{24}, x_{31}, x_{32}, x_{34}, x_{41}, x_{42}, x_{43}>=0$

HASIL LINDO

LP OPTIMUM FOUND AT STEP 4

OBJECTIVE FUNCTION VALUE

1) 38.96000

VARIABLE	VALUE	REDUCED COST
Y01	0.000000	20.100000
Y02	1.000000	0.000000
Y03	0.000000	12.900000
Y04	1.000000	0.000000
X10	0.000000	20.700001
X12	0.000000	4.400000
X13	1.000000	0.000000
X14	0.000000	0.600000
X20	0.000000	15.700000
X21	0.000000	4.400000
X23	0.000000	1.600000
X24	0.000000	2.200000
X30	0.000000	13.500000
X31	0.000000	6.600000
X32	0.000000	1.600000
X34	0.000000	0.000000
X40	0.000000	12.900000
X41	0.000000	7.200000
X42	1.000000	0.000000
X43	0.600000	0.000000
Y20	0.000000	0.000000
Y30	1.000000	0.000000
Y40	0.000000	2.800000
Y10	0.000000	15.100000
X04	0.000000	0.000000

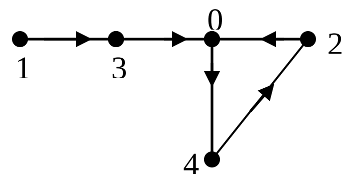


Figure 3. The valid end route on WK 1 Kecamatan Seberang Ulu II

b. WK-7

In WK-7, the type of vehicle is amroll that lifts containers. So the route is not affected by the TPS location, since each container must be removed one by one into the TPA Sukawinatan.

The optimal route through which the vehicle is as follows:

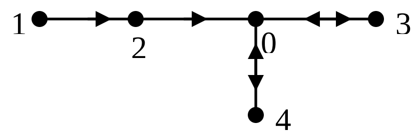
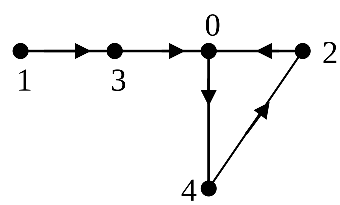
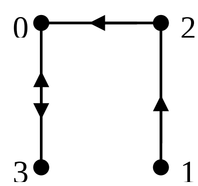
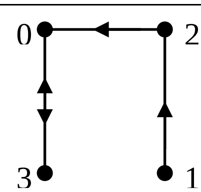
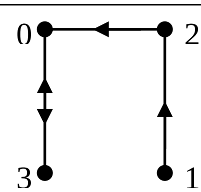
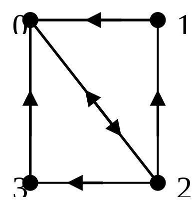
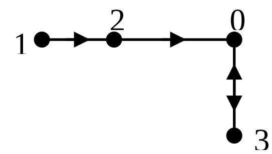
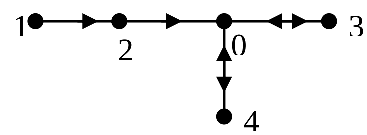


Figure 4. The final route on WK 7 Kecamatan Seberang Ulu II

All routes were completed as above, so that it can summarized in the table below which shows the end route of the vehicle after add the addition constraints.

Here, we list the routes of garbage transportation in Kecamatan Seberang Ulu II Kota Palembang.

Working Area And Model	Optimal Route
WK 1 (SCVRP Model)	
WK 2 (OCVRP Model)	
WK 3 (OCVRP Model)	
WK 4 (OCVRP Model)	

WK 5 (OCVRP Model)	
WK 6 (SCVRP Model)	
WK 7 (OCVRP Model)	

Description:

WK-1 : Home driver(1)-Tangga Takat(3)-TPA(0)-YaktapenaII(4)-Telaga Swidak(1)-TPA(0)
WK-2 : Home driver(1)-Jl.Jaya(2)-TPA(0)-Pasar Plaju(3)-TPA(0)
WK-3 : Home driver(1)- Bagus Kuning(2)-TPA(0)-Simpang3KA(3)-TPA(0)
WK-4 : Home driver(1)-Sentosa(2)-TPA(0)-Pasar PlajuII(3)-TPA(0)
WK-5 : TPA(0)-SD 254(2)-Patra Jaya(1)-TPA(0)-SD 254(2)-Pulau LayangI(3)-TPA(0)
WK-6 : DKP(1)-Tegal Binangun(2)-TPA(0)-Pulau Layang II(2)-TPA(0)
WK-7 : Home driver(1)-Yaktapena(2)-TPA(0)-Pertahanan(3)-TPA(0)-Bagus Kuning(4)-TPA(0)

Model Simplification

To simplify a OCVRP model that describes the state of TPS in a working area, it's necessary the steps as follows: OCVRP model transformation, and then simplified by using probing and preprocessing techniques. But the model has been obtained in previous studies, which have been discussed in Sari, et al. [8], Nuratika, et al. [9], and Rizta, et al. [10], OCVRP model get constraints with x_k=0 or x_k=1. This occurs, because the OCVRP model obtained by balancing technique, in which the procedure in this balancing technique similar to the technique of probing. So that the probing technique is considered to have done, and OCVRP model simplification can be continued by using preprocessing technique. To prove that the resulting model is more efficient, then compare OCVRP model before preprocessing and after preprocessing with the help of software LINDO. Based on that solution, compared to how many iterations it takes to complete the both models, the number of variables is fixed, the number of missing constraints, and the *z* optimum model.

Transformation of Elementary OCVRP Model

Preprocessing techniques can be applied to OCVRP models with constraints marked . While the initial OCVRP model formed is a model with two constraints OCVRP marked $\leq\geq$"$<$","$>$","$\leq$", or "$\geq$". OCVRP model that has been formed should be transformed in a form so that the preprocessing techniques can be applied, but did not alter the meaning of the model.

From the beginning OCVRP model established, it can be seen that the constraint should be transformed into:

$$\sum_{i\leq j\leq n} x_{0k} \leq 2K \quad \text{and} \quad \sum_{0\leq j\leq n} x_{ij} \leq 2$$

Sign "=" was changed to the sign "$\leq$" because objective function of this model is minimizing. So the new constraints that are formed means that each route (TPS-TPS or TPS-TPA) must be passed at most twice.

Comparison of OCVRP Model Results Before and After Using Preprocessing Techniques

1. The number of constraints is reduced
   In WK-1, the constraint reduced from 10 to 7, at WK-2, it reduced from 9 to 6, at WK-3, it reduced from 8 to 7, and the WK-4 it reduced from 8 to 5.
2. The number of variables is reduced.
   In WK-1 the number of variables reduced from 35 to 6, at WK-2, it reduced from 35 to 5, at WK-3 it reduced from 12 to 6, and the WK-4, it reduced from 20 to 4.
3. The number of iteration is reduced.
   In WK-1, iteration number reduced from 4 to 3, at WK-2 it reduced from 8 to 5, at WK-3, it reduced from 7 to 3.
4. The z optimum model is simplified.
   Z optimum model in each of the WK has fewer variables, as has been written in the final model OCVRP after preprocessing.
5. Optimum z value unchanged.
   Optimum z value on each of working areas in optimal route of garbage transport vehicle on OCVRP models before using preprocessing and after using preprocessing technique is the same.

IV. CONCLUSION

Branch and cut algorithms can be applied for OCVRP as long as relaxation solutions obtained is non integer shape. In this discussion, all OCVRP problems generate integer initial solution, so the branch and cut method does not need to be applied in this OCVRP model. SCVRP model often produces a non integer optimal solution, so some SCVRP models need to be solved with branch and cut method, like on one working area in Kecamatan Seberang Ulu II Palembang.

OCVRP discussed in Palembang city produces an integer optimal solution early, but the initial solution is still not valid, so the need to add additional constraints (16) - (20) to obtain the valid optimal solution.

The preprocessing' results show that the optimal route of garbage transport vehicle on OCVRP models before using preprocessing and after using preprocessing technique is the same. The results obtained show that the number of constraints, the number of variables, the number of iterations are reduced; the value z does not change; and z in the optimum model has fewer variables.

REFERENCES

[1] Caccetta, L., 2000,"Banch and Cut Methods dor Mixed Integer Linear Programming Problems,". X. Yang et al (eds), *Progress In Optimization*, pp 21-24. Kluwer Academic Publisher, Norwell

[2] Indrawati, F. M. Puspita dan C. Masita, 2007, Teknik Prerocessing untuk Mengurangi Integrality Gap pada Masalah Program Linier Integer Campuran (Mixed Integer Linear Programming), *FORUM MIPA*, Indralaya

[3] Irmeilyana, F.M. Puspita, Indrawati dan E. Roflin, 2007, *Analisis Penggunaan Model SCVRP untuk Menentukan Rute Optimal Transportasi Pengangkutan Sampah di Kota Palembang*, Research grant Project in PHK A2 of Mathematics Department, MIPA Faculty, Sriwijaya University.

[4] Letchford, A.N., Lysgaard, J., and Eglese, R.W., 2006. "A branch and cut for capacitated open Vehicle Routing Problem", http://www. lancs.ac.uk/staff/letchfoa/articles/ovrp/pdf, diakses pada tanggal 21 Maret 2007.

[5] Marina, R, F. M. Puspita dan Indrawati, 2007, *Aplikasi Metode Branch And Price Dalam Menyelesaikan Masalah Transportasi Sampah Di Kecamatan Ilir Timur I Kota Palembang*, Thesis in Undergraduate Degree of Mathematics Department, MIPA Faculty, Sriwijaya University. It's not published.

[6] Puspita, F.M. 2006. Aplikasi Teknik Preprocessing pada PBILP dan Solusinya dengan Branch and bound', *JMAP*, vol. 5, No. 2, pp.127-132.

[7] Sepputra, D., Indrawati, E. Roflin, 2007, *Aplikasi Metode Branch and Cut dalam Menyelesaikan Masalah Transportasi Sampah di Kecamatan Seberang Ulu II Kota Palembang*. Thesis in Undergraduate Degree of Mathematics Department, MIPA Faculty, Sriwijaya University. It's not published.

[8] Sari,Y.M ,2009, Indrawati, Irmeilyana, 2009, *Penentuan Rute Minimum Open capacitated Vehicle Routing Problem (OCVRP) Pada Masalah Transportasi Sampah Di Kota Palembang dengan*

mengambil contoh kasus masalah pengamgkutan sampah di Kecamatan Sako Kota Palembang, Thesis in Undergraduate Degree of Mathematics Department, MIPA Faculty, Sriwijaya University. It's not published.

[9] Nuratika, 2009, Indrawati,Irmeilyana,2009, *Penentuan Rute Minimum Open capacitated Vehicle Routing Problem (OCVRP) Pada Masalah Transportasi Sampah Di Kota Palembang" dengan mengambil contoh kasus masalah pengamgkutan sampah di Kecamatan Seberang Ulu II Kota Palembang*, Thesis in Undergraduate Degree of Mathematics Department, MIPA Faculty, Sriwijaya University. It's not published

[10] Rizta,A. ,2010, Indrawati, Irmeilyana, 2009, *Pemodelan dan Solusi Optimal Masalah Open Capacitated Vehicle Routing Problem (OCVRP) pada Transportasi Pengangkutan Sampah di Kecamatan Seberang Ulu I Kota Palembang*, Thesis in Undergraduate Degree of Mathematics Department, MIPA Faculty, Sriwijaya University. It's not published