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Spatial Visualization Tasks To Support Students' Spatial Structuring In Learning Volume Measurement

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Abstract

Many prior researches found that most of students in grade five tended to have difficulty in fully grasping the concept of volume measurement because they have to build their competence in spatial structuring. The unit of volume "packing" measurement must be integrated and coordinated in three-dimension. On the other hand, it is revealed the errors that students made on the volume measurement tasks with three dimensional cube arrays are related to some aspects of spatial visualization, such as the skill to "read off" two-dimensional representation of solid objects. For those reasons, this research is aimed to develop classroom activities with the use of spatial visualization tasks to support students' spatial structuring in learning volume measurement. Consequently, design research was chosen as an appropriate means to achieve this research goal. In this research, a sequence of instructional activities is designed and developed based on the hypothesis of students' learning processes. This research was conducted in grade 5 of SD Pupuk Sriwijaya Palembang, Indonesia.

Keywords: volume measurement, spatial structuring, spatial visualization, design research.

Abstrak

Banyak peneliti terdahulu menemukan bahwa siswa kelas 5 sekolah dasar memiliki kesulitan dalam memahami konsep pengukuran volume, karena mereka harus memiliki kompetensi spatial structuring. Unit dalam pengukuran volume harus diintegrasikan dan dikoordinasikan dalam tiga dimensi. Di sisi lainnya, hasil penelitian-penelitian tersebut antara lain menyebutkan bahwa kesalahan yang dilakukan siswa dalam menyelesaikan tugas yang berkaitan dengan susunan kubus satuan berhubungan dengan aspek visualisasi spasial, seperti kemampuan untuk membaca gambar dua dimensi dari benda padat Oleh karena itu, serangkaian aktivitas di desain untuk membantu siswa menghubungkan kemampuan visualisasi spasial mereka dengan bagaimana mereka memahami struktur dari susunan kubus satuan. Penelitian ini bertujuan untuk mengembangkan kegiatan pembelajaran dengan menggunakan kegiatan yang berhubungan dengan kemapuan visualisasi spasial untuk mendukung kemampuan strukturisasi spasial siswa dalam belajar mengenai pengukuran volume. Dalam penelitian ini, design research dipilih sebagai jenis penelitian yang tepat untuk mencapai tujuan tersebut. Dalam penelitian ini, serangkaian instruksi pembelajaran di desain dan dikembangkan berdasarkan hipotesis proses pembelajaran siswa, dan pendekatan pemelajaran yang diguanakan adalah Pendidikan

Matematika Realistik. Penelitian ini dilaksanakan di kelas 5 SD Pusri Palembang, Indonesia.

Kata Kunci: pengukuran volume, strukturisasi spasial, visualisasi spasial, design research, pendidikan matematika realistik.

Introduction

The students in grade 5 often have difficulty in fully grasping the concept of volume. In those grades is the transition period from primary to secondary education when more abstract methods for measuring volume are introduced. It was revealed that what makes a measure of volume difficult is that it requires students to build their competence in spatial structuring, because the cubic unit in volume must be defined, coordinated and integrated in three-dimension. In particular, Ben – Haim et al. (1985) indicated the errors that students in grades 5-8 made on the volume measurement tasks with three dimensional cube arrays are related to some aspects of spatial visualization, such as the skill to "read" two-dimensional representation of solid objects. In that study, the answers students gave to solve the task tended to only count to either the number of faces, or the number of visible small cubes. The students seem not consider about the interior part of the object. It indicates that the students need to practice with concrete tasks in which they can well perceive the constructed views of the organization of a three dimensional rectangular array made of unit cubes before engaging with its pictorial representation.

Considering the important of that domain and realizing that lack of research about this domain in Indonesia, we designed a study to develop classroom activities, which RME underlies its design, with the use of spatial visualization tasks to support students' spatial structuring in learning volume measurement in grade 5 elementary school of Indonesia. This report discusses an experimental study in which we aimed to better understanding the emergence of the relationship between spatial visualization and spatial structuring in learning volume measurement during students' activities. Therefore, this study pose a question: How can spatial visualization tasks support students' spatial structuring in learning volume measurement in grade 5?

Theoretical Framework

Literature was studied to find out what former studies have shown about the development of students' understanding of volume measurement. Furthermore, this literature is also useful as a basis to design a sequence of instructional activities about volume measurement. Since it was designed under the Realistic Mathematics Education environment, the literature about realistic mathematics education is also needed to explain and to investigate how the contextual situations could be shifted to more formal mathematics.

1. Volume Measurement

Volume may be measured in two ways. In one method, the space is filled by iterating a fluid unit which takes the shape of the container. In this method, the unit structure is one-dimensional. In the second method, the space is packed with a three-dimensional array unit which is iterated in the third dimension. To differentiate these two methods, we shall call them volume (filling) and volume (packing) respectively (Curry and Outhred, 2005). Related to measurement of (packing) volume, spatial structuring competence is needed to be built because the unit must be integrated and coordinated in three-dimension.

Battista and Clements (1998) in their study found that co-ordination, integration and structuring appear to be required for students in the third, fourth and fifth grades to conceptualize and enumerate the cube units in three dimensional rectangular arrays. A developmental sequence was identified in which at the initial stage students focused on the external aspects of the array and perceived it as an uncoordinated set of faces. At later stage as they reflected on experience of counting or building cube configurations, students gradually become capable of coordinating the separate views of the arrays and they integrated them to construct one coherent and global model of the array.

2. Spatial Visualization in Volume Measurement

Sarama & Clements (2009) have emphasized students' spatial structuring ability as an essential factor in learning about volume "packing" measurement. Students' spatial structuring abilities provide the necessary input and organization for the numerical procedures that the students use to count an array of cubes. Using spatial structuring strategy allows students to determine the number of cubes in term of layers and then multiple or skip-count to obtain the total number of cubic units. In addition, Ben-Haim et.al (1985) suggests that, in order to be able to count the volume of an object made of small cubes, students need to be able to coordinate and integrate the views of an array either in real blocks arrangement or in drawing representation. On the other hand, the skill to "read" two-dimensional drawing representation of solid objects is a part of the spatial visualization ability (Ben- Haim et.al, *ibid*). The skill to "read off" two-dimensional drawing representation of solid objects is a part of the spatial visualization ability (Ben- Haim et.al, *ibid*).

In general, spatial visualization can be meant as the ability to mentally manipulate two dimensional and three dimensional figures. In addition, Titus & Horsman (2009) define spatial visualization as the ability that involves skill to mentally manipulate and rotate an image into another arrangement and to mentally imagine what is inside of a solid object.

In particular, Ben-Haim et.al (1985) reported that in a spatial visualization unit of instruction developed for training the students in middle grades about three-dimensional arrays construction, the students are asked to draw flat view of the isometric drawing of a cube building and then count how many cubes in the drawing. In the present research, spatial visualization will be used to support students' in learning about volume measurement. Therefore, the instructional activities designed in the present study will involve the spatial visualization tasks to help the students perceive their spatial structuring ability in learning volume measurement.

3. Realistic Mathematics Education

In the process of doing mathematics, Freudenthal (1991) emphasizes that students should be allowed and encouraged to invent their own idea and use their own strategies. In the other words, they have to learn mathematics in their own way. Freudenthal argued that mathematics is as 'a human activity'. Instead of giving algorithms, mathematics should be taught in the way where students can do and experience to grasp the concepts. Therefore, this study develops an instructional unit on teaching and learning volume measurement in which the students could gain more insight about how to measure the volume of an object through experiencing a sequence of meaningful activities instead of only memorizing the volume formula.

In each activity, the students are free to discuss what strategies they are going to use in solving the task or problem given. Therefore, social interaction emerging in the classroom is important part of the whole class performance. Working in groups will build a natural situation for social interaction.

One of the principals in RME is bridging from a concrete level to a more formal level by using models and symbols. In the present study, the students can develop their own model or symbol to represent the arrangement of the packages in three dimension arrays. Gravemeijer (1994) described how models-of a certain situation can become models-for more formal reasoning. Actually, the sequence of activities designed in this study is only a part of longer series of learning trajectories in learning volume measurement. We will go further with the Hypothetical Learning Trajectory (HLT) of this study in the next section.

4. Emergent Perspective

Before starting the process of learning, it is conjectured that the students have their own belief about their own roles, the others' roles, the teacher's roles and the mathematics that will be learnt. In this study, during the process of learning, the teacher will initiate and develop the social norms that sustain classroom culture characterized by explanation and justification of solution, and argumentation: attempting to make sense of explanation given by others, indicating agreement and disagreement, and questioning alternatives in solutions in which a conflict in interpretation or solution has become apparent (Gravemeijer & Cobb, 2006).

In this research, we will focus on the normative aspect of mathematics discussion specific to students' mathematical activity. To clarify this distinction, we will use the term socio-mathematical norms rather than social norms. We describe socio-mathematics norms as normative understanding of what counts as mathematically different, mathematically sophisticated, an acceptable mathematical explanation and justification. Students will develop their ways of judging, whether a solution is efficient or different, and the teacher is not the only one who decides the acceptable solutions. In this way, socio-mathematical norms are negotiated as the teacher and students participated in the discussions.

Hypothetical Learning Trajectory

Hypothetical learning trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process.

The HLT in this study had several learning goals expected to be reached by the students during the three weeks study. To reach the goals formulated, we design a sequence of instructional learning for volume measurement which is elaborated on the following table:

Table 1. Overview of the HLT

Sequence of Activities	Goals	Descriptions
Picture Packages	Students can relate the visible part and interior side of a three-dimensional objects arrangement	• We arranged some stuffs such as tea boxes, tissue packages and wafer bars in a three-dimensional array arrangement on their tables and then asked them to draw the arrangement on the paper so that the people who see their drawing can understand the situation • We conjectured that some students will try to draw the layers to explain to others about the situation while some students still have difficulties in representing the layers in their drawing.
Building Blocks	Students can construct a cube-blocks building based on different views pictures.	• The students are asked to build a construction made of cube blocks. The teacher will give them pictures of the construction from side, top and front views. • Later, after they finish with their construction, they are asked to draw their construction into a single picture in which they can see the side, front and top views in the drawing. • We predicted that some students will build the construction first from the top view which makes them possible to build the base of the building and then build the layers until match with front-view picture;

		some others might start with front view or side view and make outer parts of the construction and then try to fill in the center part of the building.
Count the Blocks	Students can count the number of cube blocks in a 3D cube blocks construction	• The students are asked to count the number of blocks in their friend's drawing (from the previous activity, building blocks) and then check it by seeing the real construction. • In the real construction, the students can touch and tag the blocks while in the drawing they have to imagine the situation of the blocks arrangement. • We predicted that the students will count the blocks both in their drawing and in the real construction by first counting one layer, either in rows or in columns, and then multiplying it with the number of layers.
Predict the Number of Blocks	Students can estimate the Number of Blocks needed to cover up an empty space (box)	• We gave them a box and four cube blocks. We asked them to predict the number of cube blocks needed to cover up the box prepared. • By observing the students doing this activity, we would like to know if the previous lesson could help the students who had difficulties in perceiving the structures of the objects arrangement in three-dimensional arrays. • We predicted that some students still saw the arrangement as unstructured objects but we predicted that the class discussion

blocks construction could promote the using of layers either in columns or in rows in counting or in estimating the number of objects in an arrangement or inside a box.

Methods

a. Participants

We work with a teacher and 32 five grade students of SD Pusri Palembang. The students are on age 10 to 11. In each lesson they worked in groups of 4 or 5. The teacher classified the students based on academic ability and gender. So, in each group there are high achievers, average students and also low achievers.

b. Materials and Procedure

As the main goal of this study, we designed the activities for the students to know how they visualize the three-dimensional object into the two-dimensional drawing, to investigate their ability in reading off the drawing of a three-dimensional array arrangement and how they solve the volume measurement task related to a three-dimensional array arrangement. In the first section of the activities, we arranged some stuffs such as tea boxes, tissue packages and wafer bars in a three-dimensional array arrangement on their tables and then asked them to draw the arrangement on the paper so that the people who see their drawing can understand the situation. In the next lesson, the students were asked to build a construction made of cube blocks. The teacher gave them pictures of the construction from side, top and front views. After they finished with their construction, they were asked to draw their construction into a single picture in which they can see all three views – side, front and top views – in the drawing. Then, the students were asked to count the number of blocks in their friend's construction. And also they were asked to estimate or to predict the number of blocks needed to cover up such an empty rectangular box.

As mentioned in my HLT, each lesson brings some essential features that are as my expectations. We would look into students' ability of reasoning. Therefore, the result will be analyzed qualitatively. The reliability of this design research is, of course, accomplished in qualitative way. The qualitative reliability is conducted in two ways, data triangulation and cross interpretation. The data triangulation in this study involves different sources: the videotaping of the activities, the students' works and field notes. The parts of the data of this research will be also cross interpreted with observers. This is conducted to reduce the subjectivity of the researcher's point of view.

Result and Analysis

Before the students work in the activities, We gave them a pre-test to know their pre-knowledge and ability. We found that the students seemed have little difficulties in determining the number of concrete objects in the pictures such as tea boxes or soap bars arrangement because they are familiar with the soap and the tea; they can imagine the situation. On the other hand, they have difficulties in determining the number of blocks in the drawing which is more abstract. They tended to count the number of squares rather than count the number of blocks. It indicates that they need more concrete task before they work with pictorial representation of the objects that are not familiar with them such as cube blocks arrangement.

"Picture Packages" Activity

There were four different arrangements on their tables: tissue packs, tea boxes, and two different kinds of wafer bars as in figure 1. The students were asked to draw the nearest food arrangement with their chairs. The packages were arranged in three-dimensional arrays and the students had to make its representation in a paper. We predicted that some students will make isometric drawings as we saw in the preliminary design and some others will had difficulties in representing the objects as a concise building.

Figure 1. Food Packages Arrangement

In our observation, we found two groups of students made drawings from separates views. When we asked them how they know the number of objects in their drawings, the first group explained their drawing as shown in the figure below:

Figure 2. Rafli's Group drawing

They explained that they saw 4 tea boxes in the bottom part and they saw two stacks in front and in the back. So, they multiply those become 8 and it was what they saw from the top because there were three box high, then they multiply it by 3 and all together is 24 (see figure 2). The other group who drew the same objects with them also drew the tea boxes arrangement from separates views as shown in figure below:

Figure 3. Dwi's group drawing

The group work was continued by group presentation. Dwi and her group drew separates views of the tea boxes arrangement as shown in figure 3. They drew the tea boxes from three views but from the fragment, in interpreting the drawing she only saw two views: top and right side. From their drawing of right side, there are 2 rows of three tea boxes. Then, to interpret her drawing, she said that we have to look at the top, eight boxes, and then multiply with the tea boxes she saw from the right side, three. So, she interpreted her drawing as 24 tea boxes.

The other group who explained their drawing was group 6. We could not hear very clear their voice in the video, so we could not transcribe it. In the video, we can only see the drawing as in figure 4 and one of them, Syahrul was explaining. However, based on our observation, they explained that they made a building of wafer arrangement. There were three rows of 12 wafers. So, all together is 24. They are the group who work with us in the preliminary design. They had experience in drawing the objects arrangement and saw a lot more isometric drawings.

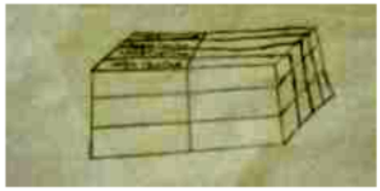

Figure 4. Syahrul's group drawing

The last group who explained their drawing was group 7. They told the other friends that they had wrong drawing because they count that there were 18 tissue packs on the table but they had 21 tissue packs in their drawing as in figure 5.

Figure 5. Amel's group drawing

We observed that Amel and her group tended to count the number of squares in their drawing. That's why they thought that they made wrong drawing. They had difficulties in perceiving the structures of the tissue packs in their drawing. However, Fadilla then tried to help this group. She first seemed influenced by how Amel count the drawing of tissue packs. She said there were five packs of tissue in the drawing in each column. However, then when the teacher asked once more, she changed her answer become nine. And then Amel said that there were two rows of nine. This discussion could help Amel realized that there was nothing wrong with their drawing. The way to count is the mistakes they did.

Then, the teacher asked one of the students, Yudha, to interpret Dwi's drawing and Syahrul's drawing. He said that he saw 28 objects in Dwi's drawing. The teacher asked Dwi if Yudha answered correctly, and the group said it was not correct because there were 24 tea boxes, not 28. He seemed to count 8 tea boxes from the front view and added 6 tea boxes from the side view and multiply it by two since there were two rows of it. He tended to be influenced by Dwi who explained that she interpreted the drawing from the top and side views. However, he did not pay much attention when Dwi explaining. However, Yudha then changed his answer become 26. Although the others told him that there were 24 tea boxes, but he did not listen. When he changed his answer become 26, he said 8, 14, 26. He seemed to add all the squares he saw in Dwi's group drawing, 26 tea boxes. Then, Yudha continued to count the object in Syahrul's group drawing. He answered 24. He explained that he could see 12 objects from their drawing and all is 24.

Throughout this lesson students could make representation of the arrangement of three-dimensional objects. It is difficult for most of the students to make a drawing of the objects arrangement as a concise building. They had their own strategies to visualize the situation they saw. Also, they had different interpretations of a drawing. One drawing is easier to interpret than the other one. Even, one group of students thought that they made wrong drawing because they could not well perceive the structures of the objects in their drawing. This activity has provided a bridge for students to develop their thinking process. Later, in the next activity they would have to make arrangement of objects from drawings given.

"Building Blocks" Activity

Firstly, the teacher remind the students about yesterday lesson in which each group made a drawing of an arrangement of tea boxes, tissue packs or wafer bars. Then, each group got a box contains some cube blocks and an instruction sheet. In the instruction, they are asked to construct a building made of the cube blocks as they saw in the drawings. The drawings are from separated views: top view, side view and front view. They have to relate those views to make a concise building of the cube blocks. Every group got different number of blocks and had to build different building. The students had to write down their steps in making the construction and then explained to others. They were also asked to draw their building on the paper. The drawings would be used in the second activity, count the blocks.

One of the groups, group 1, arranged the cube blocks really the same with the drawing in the instruction sheet. They arranged the blocks separately, from the top, side and front views as shown in the figure. They asked for more blocks, but the teacher told them that she had no more blocks to give. We observe that they looked around; they saw that other groups made a concise building from the drawings and not separate building as they thought. Finally, this group succeeded to build their construction well. They restarted their work by build the front and then the side view.

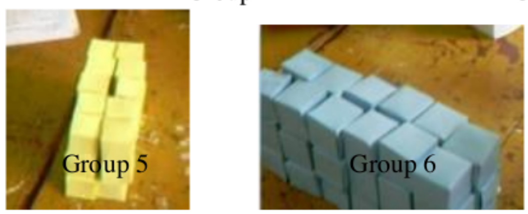

Figure 6. Students' building blocks

The other groups also explained that they build the front view and then the side view. Some students fill in the interior part of their building after they had the outer parts of the building. Some others build the layers, after they had the first row of the building (the front side), then they build the second and the other layers as many as they saw in the side view. However, one group of students, group 7, said that they needed more blocks because they only had outer part of their building, and the blocks were not enough to make the building as instructed in the drawing. After all students finished working, the teacher asked that group to explain their construction.

We observed that the group wrongly interpreted the side view of the building. They saw two columns of 4 blocks in the side view in the drawing, but indeed they build 3 columns of 4 blocks. They did not count the blocks in front as part of the side view. They perceived it as separate buildings. When the teacher asked them to fix their building into the right one as instructed in the sheet, then they realize that it must be a concise building which could be seen from different views, not separately built.

"Count the Blocks" Activity

The count the blocks activity is the continuation of the previous activity, building a construction made of cube blocks and making its drawing. The teacher asked the students to come to the nearest neighbor-group and then count their blocks in the real construction and in their drawing. The teacher asked some students to count it and there was an interesting moment as we transcribe in the following fragment.

Teacher : How many blocks in the construction of group 5?

Tasya : 40.

Teacher : How did you get 40?

Tasya : (Pointed out to the blocks in the front)

Teacher : How about you Rafif?

(Rafif count one by one and pointed out to every single block)

Rafif : 42.

Teacher : How did you get 42?

Rafif : I add all.

(The teacher again asked Tasya to count the blocks but Tasya looked frustrated with counting the blocks. Therefore, the teacher stopped asking her about the blocks but then she asked Tasya to count the blocks in the drawing)

Teacher : Now, in the drawing. How many Tasya you looked in the drawing?

Tasya : 32?

Teacher : How did you get 32?

Tasya : I add all.

In the fragment, we observed that Tasya and Rafif could not well perceive the structures of the blocks. Rafif tended to count the blocks from the front, the back, the left and the right side. That's why he found 42 blocks rather than 30 blocks, the number of blocks in the construction. Tasya also had difficulties in perceiving the structures of the blocks. In counting the real blocks, she could count the number of blocks in the front, 15. But then she did not directly multiply it by two, the number of layers she saw in the side view. She then tried to count the blocks one by one and got 40. She tended to count the number of blocks in front, in the back, and added the top part she saw, so all together was 40. But she could not explain it. Then, she was influenced by the teacher question and changed every time. She could not decide which her answer was. She looks frustrated in counting the number of blocks in the construction.

The teacher did not ask the right answer from that group. Indeed, she did not ask other students to help the group. She also looked frustrated with that group. Then, the teacher asked other students to count the number of blocks in other construction and other drawing. She then pointed to Rizki. Rizki explained that he found there were 36 blocks in the construction of group 2 and also in their drawing. He explained that he count it by multiplying 4 by 3 times 3 in the construction. He seemed to count the number of blocks from the front and then saw the number of rows. And in the drawing he did the same thing. He found that there was the same number of blocks both in the real construction and in the drawing. Based on his group explanation, they built the blocks constructed from the front wall and then built the blocks to the back. His strategy in counting the blocks indicates that he was influenced by his strategy in building the blocks. He tended to count the number of blocks in front and then count the number of layers in the construction.

The teacher then asked Tasya to share her ideas. She said that she counted 40 blocks, by added 15 blocks from the front, plus 15 from the back plus 10 from the top. When the teacher asked other students to find Tasya's mistakes, Landok helped her to figure it out. He said that he only count the number of blocks in front, and again count the number of blocks in the back, and then added it.

From that example, we observed that the students learnt about their friends' mistakes and their friends' strategies in counting the number of blocks in the real construction. Most of the students could help in determining which strategy is correct and which one is incorrect. The lesson was continued by discussing about how to count the number of blocks in the drawing.

The teacher drawn an object made of a cube blocks in the white board in front of the class. The teacher then asked some of the students to present their ideas in front of class.

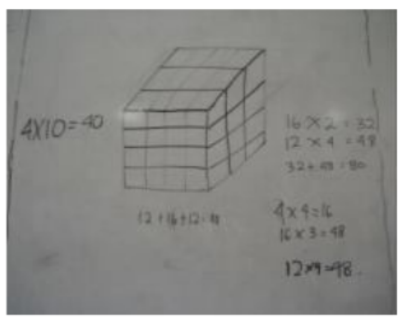

Figure 7. Students explained how they count the blocks in the drawing

Maria explained that she count the number of squares in front, 16, and then the same will be in the back, 16. It makes 32. Then, at the top 12, the same will be in the bottom, 12. And, the same was also in both sides. So, 4 times of 12 makes 48. All together were 80.

Nadia counted differently, she wrote 4 times 10. She could not explained well in front of class but during our observation, we talked with her and she explained that that she counted the number of squares in the border, 4 squares from left to right and then 3 from front to back and then 3 to the top. All makes 10 and because there are 4 layers high, she multiplied 4 times 10. She did in the same way to answer the second question.

Bagus explained that he also got 40. He explained that he added 12 from the top, 16 squares from the front and 12 squares from the side (as in figure 7). We observed that some students still had difficulties in seeing the structures of the blocks arrangement in the drawing. Even, when they are asked to determine which strategy is correct, some of them agreed with Bagus and Nadia. However, in the discussion, they can share ideas about how they count and perceive the number of blocks in the drawing. Dinda and Syahrul promoted the using of layers structures in counting the blocks in the drawing to their friends.

Throughout the discussion about how they count the number of blocks in the real construction and in the drawing, we could see that some of the students had difficulty not only in interpreting a drawing but also they had problem with their spatial ability in perceiving the structures of the blocks even in the real construction. We expected that by having this discussion could help the students to better perceiving the structures of the unit blocks arrangement in three-dimensional arrays.

Predicting The Number Of The Blocks

In the previous lessons, the students worked with boxes, packages arrangement, cube blocks and its drawings. In this lesson, we also prepared boxes and cube blocks. We asked them to predict the number of cube blocks needed to cover up the box. We predicted that some students still saw the arrangement as unstructured objects but perhaps the students' experience of the previous lesson could promote the using of layers either in columns or in rows in counting or in estimating the number of objects in an arrangement or inside a box.

The teacher started the lesson by telling the students that today she would give a box and a few blocks to each group. Then, the students were asked to estimate the number of cube blocks needed to cover up the box. We observed that during working with the box and the blocks they asked us if it is the same thing with estimating the number of dodol needed to cover up the carton and plastic boxes. They seemed to recall their strategies in that previous activity, but some others just try to solve the problem given. After five minutes or so, the teacher asked the students to share their strategies in predicting the number of cube blocks can be put inside the box. The teacher then asked Anggi to tell her strategy. During showed her strategy, Anggi did not say much words. Based on our observation, Anggi try to iterate the blocks along the base and then she tried to imagine what might happen if she could iterate it to the top as shown in figure 8. However, when the teacher asked her how many blocks needed, she did not say any number as the result. She only mentioned that on the base there were 16 but she could not get the total number of blocks needed. The teacher asked her about her estimation about the total number of blocks needed but she kept working with blocks and did not give any single answer.

Figure 8. Anggi predicting number of blocks

Then, her friends in the group, Amel and Dinda, helped her. Amel and Dinda told her that there were sixteen times three equal to 48 blocks in the box. Then, Anggi said to the teacher exactly the same words as Dinda and Amel told her.

The teacher then asked other students about their way in predicting the number of blocks needed. Most of the students seemed to cover up the base and then multiplied the number of blocks on the base and the number of layers. One of the students, Rafli, said that he imagined covering up the base first. He added four blocks plus four blocks, repeated it and got 16. Then he estimated the number of blocks can be arranged to the top, that is 3. And all together is 16 times 3, 48.

Then, the teacher asked if other students had different strategies. But, most of students seemed to do the same thing, multiplied the number of blocks on the base with the number of layers.

Conclusion

This research hypothesized that the students will not employ the layers structures in counting the three-dimensional units configuration unless they realize that it is a structured arrangement. They needed to practice with more concrete tasks of increasing structural complexity through which they can acquire personally constructed views of the organization of the three dimensional rectangular arrays. We found evidence that in visualizing the arrays into drawing, some of them could represent the arrangement from different views. They were aware that the arrangement could not be seen only from one side. However, at this initial stage students focused on the external aspects of the array and perceived it as an uncoordinated set of faces.

Therefore, we conducted activities in which students experienced of building and counting cube configurations. We found evidence that the building blocks activity has helped the students to coordinate the separates views of the arrays. And it influenced the students in counting the blocks arrangement. Some of students tended to count the number of faces on different views but some others who were able to coordinate and integrate the different views could see that it was an arrangement of layers. These students built the construction from the base to the top or from the front to the back. So, these students only count the number of blocks on the top or on the front or from the left or right side and then count the number of its layers.

We also conducted activity in which students had to predict or estimate the number of blocks needed to cover up a box. In this stage we conjectured that they would reflect on the previous activities. Our findings showed that the students gradually become capable of coordinating the separate views of the arrays and they integrated them to construct one coherent model of the array. Based on our findings, most of students were aware of using layers in predicting the number of cube blocks needed to cover up the boxes. They tended to first count the number of blocks in the row (base) or on the base and then do skip counting or multiplied it with the number of layers.

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