# Timetable Creation of BRT Transmusi by Using Branch and Bound Method 

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#### Abstract

Timetable problem of public transportation is a problem in scheduling the optimal departure time to minimize the density of passengers. Timetable problem is an integer programming, so it can be solved by using Branch and Bound Method. This study used travel route of BRT Transmusi Kota Palembang in July 2016 for Sako - Pasar Gubah route and vice versa. The secondary data used are the departure schedule, the number of buses operated, and also primary data of passengers and the time taken. Based on the calculation result of Branch and Bound Method, it can be concluded that the total minimum number of passengers jostled in both routes is 145 passengers with 10 buses operated in 2 hours. Based on the timetable obtained, in the first period the number of buses needed on Sako - Pasar Gubah route are 5 buses with headway for 12 minutes headway, whereas Pasar Gubah - Sako route needs 6 buses with 10 minutes. In the second period, Sako - Pasar Gubah route needs 6 buses with headway for 10 minutes, whereas Pasar Gubah - Sako route needs 5 buses with headway for 12 minutes.


Keywords: Timetable, travel route, headway, BRT Transmusi, Branch and Bound Method.

## 1. Introduction

Transmusi is a Bus Rapid Transit (BRT) in Palembang. BRT Transmusi is one of the efforts taken by the Government of South Sumatra Province in reducing the problems of congestion and serving the better transportation.

Travel routes of BRT Transmusi are through the streets with predetermined routes, which the passengers wait in a special stop (we call it as halte). People who use Transmusi BRT public transport are very enthusiastic because there are many advantages in terms of facilities and infrastructure.

BRT Transmusi has been present in Kota Palembang for almost eight years, but there is some criticism from the public regarding the optimization of BRT Transmusi. For example, some people who want to use the BRT Transmusi sometimes cancel it, because the interval time between a bus and other buses is uncertain. Stacking passengers at halte also often occur because of the length of the arrival of the bus. Similar conditions also occur in the bus, where passengers do not get a seat, had to stand so as to make the situation inside the bus became crowded.

The operation of public transportation is set by considering the interests of passengers and service providers. One was preparation of the tables of departure time (timetable) so that no passengers were jostling and not have to wait long for departure time. Timetable of public transport is intended to maximize the number of vehicles that arrive
simultaneously at a bus stop which it is assumed the number of public transport vehicles used is not limited [1]. But the amount of public transport vehicles provided are really limited due to limited resources or venture capital. Therefore, it needs a way to solve the problem, namely by minimizing the density of passengers in the vehicle.
The purpose of this study is to develop a timetable for the departure schedule of BRT Transmusi vehicles by using Branch and Bound Method.

Restrictions problem in this research are:

1. Preparation of BRT Transmusi timetable is done only on Mondays to Thursdays at $06: 15$ to $08: 15 \mathrm{pm}$ on Sako Pasar Gubah route and vice versa.
2. Conditions during the trip is considered normally, so that the time taken BRT Transmusi from halte Sako to halte Pasar Gubah is fixed for each departure.
3. The condition of buses is good and not impaired during operation.
Timetable can be used to minimize the density of passengers in the vehicle, to obtain optimal departure time of BRT, and also to enhance passenger satisfaction in BRT Transmusi vehicles.

## 2. Literature Review

### 2.1 Planning of Public Transport System

In planning a public transport system, there are four planning stages, namely: (1) route design, (2) preparation of a timetable, (3) scheduling of vehicles, and (4) scheduling driver.

The preparation of a timetable for each route during the operational period of public transport is set by calculating the average number of passengers, which is expected to overcome the problems of passenger density. Frequency and headway must be determined beforehand. Headway is the difference of the departure time between a vehicle and the next vehicle. Frequency (within vehicles per hour) is the number of departures buses which pass at a certain point. Headway (within minutes) can be obtained by dividing the length of the period (within minutes) with frequency.

Method of preparation of a timetable that has been commonly used is clock headway method. The method is based on minute memorable by the passenger, which is one of $5,6,7,10,12,15,20,30,40,45$, or 60 minutes [2]. In the method, it is not considered the number of passengers and public transport vehicles that can be used, so it should be combined with another method to produce the headway that can minimize the density of passengers in public transport vehicles.
[1] state that the minimum number of public transport vehicles needed to serve the set of terminal $T$ is formulated as:
$N=\sum_{k \in T} D(k)=\max \sum_{k \in T} d(k, t) ; t \in\left[t_{1}, t_{2}\right]$
Where:
$N$ : minimum number of vehicles needed to serve the terminal set $T$ during the operational period $\left[t_{1}, t_{2}\right]$
$D(k)$ : minimum number of vehicles required for the departure in terminal $k$
$d(k, t)$ : The total sum of the number of departures that subtracted by the number of arrivals in terminal $k$ at time $t \in\left[t_{1}, t_{2}\right]$
Suppose the binary variable is defined as a decision variable, namely:
$x^{F}($.
$=\left\{\begin{array}{l}1 \text {; if } F \text { selected in the period } j \text { from } k_{1} \text { to } k_{2} \\ 0 \text {; other }\end{array}\right.$
$F=\mathrm{L}(),. \mathrm{L}()+1,. \mathrm{~L}()+2,. \ldots, \mathrm{U}()-1,. \mathrm{U}($.
Which:
L(.) : The minimum allowable frequency
$\mathrm{U}($.$) : The maximum frequency is determined by$ calculating the coefficient of the objective function
Suppose crowding (i.e. the number of passengers are crammed) is denoted by $c^{F}($.$) that written as:$
$c^{F}()=.\operatorname{maximum}\left\{P_{m}()-.F . d_{0}(), 0.\right\}$
with:
(.) : In the period-j for the route from terminal $k_{1}$ to terminal $k_{2} ; j \in J ; J$ is period set; $J=\{1,2,3, \ldots, n\}$
$c^{F}():$. Crowding in the period-j for the route from terminal $k_{1}$ to terminal $k_{2}$ when the frequency $F$ is selected.
$P_{m}($.$) : Maximum of the average number of passengers in$ the period-j for the route from terminal $k_{1}$ to terminal $k_{2}$.
$F \quad:$ Determined frequency in the period- $j$ for the route from terminal $k_{1}$ to terminal $k_{2}$.
$d_{0}($.$) : Occupancy (ratio between the number of passengers$ and a capacity of seating available) desired in the period-j for the route from terminal $k_{1}$ to terminal $k_{2}$.
The objective function is defined as:
Minimum $\mathrm{Z}=\sum_{\forall(.)} \sum_{F=L(.)}^{U(.)} c^{F}(.) x^{F}($.
$(.) \in J$ with $J=\{1,2,3, \ldots, n\}$.
A route in each period is only determined one frequency value to formulate a timetable. If $x^{F}()=$.1 , then for a period on a route is only allowed one variable $x^{F}($.$) that its value is$ 1 , so it is obtained constraint:

$$
\begin{equation*}
\sum_{F=L(.)}^{U(.)} x^{F}=1 ; \forall(.) \in J ; J=\{1,2, \ldots, n\} \tag{4}
\end{equation*}
$$

If there is $N_{o}$ public transport vehicles that can be used to
serve the terminal set $T$ during the operational period
[ $t_{1}, t_{2}$ ], then by Equation (1) is obtained value $d(k, t)$ is determined by
$\left.x^{F}().\right\} \leq B A(k) ; t \in T_{k} ; k \in T$
where:
$T \quad$ : The terminal set
$T_{k} \quad$ : The set of departure time from terminal $k$ for operational period $\left[t_{1}, t_{2}\right]$, where $T_{k} \subseteq\left[t_{1}, t_{2}\right]$.
$B A(k)$ : The number of public transport needed for the departure of the terminal $k$.
Constraint of total number of vehicles needed for all terminals in the terminal set $T$ is:
$\sum_{k \in T} B A(k) \leq N_{0}$
Where:
$N_{0}$ : The number of vehicles that can be used to serve the terminal set $T$ during the operational period.
$x^{F}(.) \in\{0,1\} ; \forall F ; \forall($.
with $F \in[L(),. U()$.$] dan (.) \in J$
$B A(k) \geq 0 ; B A(k) \in \mathrm{Z} ; \forall k \in T$

## 3. Research Method

In this study, we used secondary data obtained from PT Sarana Pembangunan Jaya Palembang. Primary data were obtained based on observations in July 2016 for the trips of BRT Transmusi in Sako - Pasar Gubah and vice versa. The steps in this study are as follows:

1. collecting data obtained from the trips in Sako - Pasar Gubah and vice versa at two time periods departure, i. e. long trips on both routes, departure time, the number of passengers transported, and the number of vehicles used.
2. Formulating an optimization model of the timetable on BRT Transmusi by:
2.1 Defining data into decision variables as follows: $B A(k)$ is the number of vehicles from the terminal $k$.
$x^{F}($.$) is a variable that states the value of the frequency F$ on period $j$ for the route from terminal $k_{1}$ to terminal $k_{2}$.
2.2 Formulating the objective function to minimize passenger density in the BRT by Equation (3).
2.3 Forming constraints by Equation (4) to Equation (8).
3. Creating timetable using methods Branch and Bound by:
3.1 Determining the temporary optimal value for the objective function, i.e. $z^{*}=\infty$.
3.2 Resolving the linear programming relaxation of the initial problem by using the simplex method.
3.3 If linear programming relaxation in Step (3.2) does not have a feasible solution, then this method will stop and we concluded that the initial problem has no feasible solution. If not, define $Z=$ the value of objective function for the linear programming relaxation.
3.4 If the optimal solutions of linear programming relaxation obtained are integer, then this method will stop. If not, calculate the bound of optimal value in initial objective function, namely bound $=\lceil z\rceil$, where $\lceil z\rceil$ is the smallest integer greater than or equal to $Z$.
3.5 Branching, bounding, and fathoming iteratively to obtain the optimal solution of the initial problem.
3.6 Optimality test.
3.7 If no sub problems require further testing, then this method stopped and the optimal solution found. So, we will obtain the frequency for each period on every route, the number of vehicles, and headway on each terminal.
If iterations are so many enough, the completion of the Branch and Bound method can be assisted by Software Lindo.

## 4. Results and Discussion

Based on data obtained from PT Sarana Pembangunan Jaya Palembang and result of observation in July, 2016, we obtain data shown in Table 1.

Table 1. Data Needed in Preparation Timetable of BRT Transmusi

| Rou- <br> te | Travel <br> time <br> (in <br> min. | Peri- <br> od | Departu <br> re Time | Average <br> Max. of <br> Passenger | Occu- <br> pancy | Min. <br> Freq. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sako <br> - | 60 | 1 | $06.15-$ <br> P. G |  | 27.15 | 165 |
|  | 2 | $07.15-$ <br> 08.15 | 245 | 28 | 3 |  |
| P. G | - | 45 | 1 | $06.15-$ <br> 07.15 | 131 | 23 |
| Sako |  | 2 | $07.15-$ <br> 08.15 | 158 | 28 | 3 |
|  |  |  |  | 2 |  |  |

Description: P.G. = Pasar Gubah. Total seating capacity of bus is 33 passengers. Occupancy in the $1^{\text {st }}$ period is $70 \%$ in order to obtain 23 passengers, while occupancy in the 2nd period is $85 \%$ in order to obtain 28 passengers.

Suppose the average maximum number of passengers is denoted as $\mathrm{P}_{m}($.$) . Variable c^{F}($.$) is the number of$ passengers jostling, $x^{F}($.$) is frequency value F$ in a period for a route. Based on Equation (2), it can be determined $c^{F}(1)$ in the $1^{\text {st }}$ period for Sako - Pasar Gubah route.
For $F=2$, so $c^{2}(1)=$ maximum $\{165-2 \times 23,0\}=119$.
For $F=3$, so $c^{3}(1)=$ maximum $\{165-3 \times .23,0\}=96$.
While for $2^{\text {nd }}$ period, its minimum frequency is 3 , so $c^{2}(2)$ is nothing.
In the same ways, we obtain Tabel 2. The value of $c^{F}($.$) is$ defined as a variable $x_{j}$.

Tabel 2. Decision Variable $x^{F}$ and Objective Function Coefficient $c^{F}$

|  | Sako - Pasar Gubah |  |  | Pasar Gubah - Sako |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Period 1 |  | Per. 2 |  | Per. 1 |  | Per. 2 |  |
| $\mathbf{F}$ | $\boldsymbol{c}^{\boldsymbol{F}}()$. | $\boldsymbol{x}^{\boldsymbol{F}}$ | $\boldsymbol{c}^{\boldsymbol{F}}()$. | $\boldsymbol{x}^{\boldsymbol{F}}()$. | $\boldsymbol{c}^{\boldsymbol{F}}$ | $\boldsymbol{x}^{\boldsymbol{F}}$ | $\boldsymbol{c}^{\boldsymbol{F}}$ | $\boldsymbol{x}^{\boldsymbol{F}}$ |
| 2 | 119 | $x_{1}$ | - | - | 85 | $x_{15}$ | - | - |
| 3 | 96 | $x_{2}$ | 161 | $x_{8}$ | 62 | $x_{16}$ | 74 | $x_{20}$ |
| 4 | 73 | $x_{3}$ | 133 | $x_{9}$ | 39 | $x_{17}$ | 46 | $x_{21}$ |
| 5 | 50 | $x_{4}$ | 105 | $x_{10}$ | 16 | $x_{18}$ | 18 | $x_{22}$ |
| 6 | 27 | $x_{5}$ | 77 | $x_{11}$ | 0 | $x_{19}$ | 0 | $x_{23}$ |
| 7 | 4 | $x_{6}$ | 49 | $x_{12}$ |  |  |  |  |
| 8 | 0 | $x_{7}$ | 21 | $x_{13}$ |  |  |  |  |
| 9 |  |  | 0 | $x_{14}$ |  |  |  |  |

## Description:

The average maximum number of passengers on Sako - Pasar Gubah route in period 1 with frequency 2 is 119 passengers. It can be written by $x_{1}=x^{2}$ (1, Sako, Pasar Gubah).
The average maximum number of passengers on Sako - Pasar Gubah route in period 2 with frequency 3 is 161 passengers. It can be written by $x_{8}=x^{3}$ (2, Sako, Pasar Gubah).

Based on data obtained from Table 2, the decision variables can be defined as follows:
a. Variable $x_{24}=B A$ (Sako) is the number of public transport vehicles are required from the terminal Sako to Pasar Gubah.
b. Variable $x_{25}=B A$ (Pasar Gubah) is the number of public transport vehicles are required from terminal Pasar Gubah to Sako.
So, they have coefficient 0 in the objective function.

### 4.1. Objective Function in Timetable Problem of BRT Transmusi

The objective function to minimize passenger density is based on the number of passenger density in Table 2. Based on Equation (3) and defining the decision variables, the objective function is:
Minimum $Z=119 x_{1}+96 x_{2}+73 x_{3}+50 x_{4}+27 x_{5}+4 x_{6}+0 x_{7}$ $+161 x_{8}+133 x_{9}+105 x_{10}+77 x_{11}+49 x_{12}+21 x_{13}+$ $0 x_{14}+85 x_{15}+62 x_{16}+39 x_{17}+16 x_{18}+0 x_{19}+74 x_{20}+$ $46 x_{21}+18 x_{22}+0 x_{23}+0 x_{24}+0 x_{25}$

### 4.2. Constraints in Timetable Problems of BRT

 TransmusiDue to Sako - Pasar Gubah route and vice versa in period 1 and 2 respectively only taken one value of $F$ then we obtain constraints:

$$
\begin{align*}
& x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+x_{6}+x_{7}=1  \tag{10}\\
& x_{8}+x_{9}+x_{10}+x_{11}+x_{12}+x_{13}+x_{14}=1  \tag{11}\\
& x_{15}+x_{16}+x_{17}+x_{18}+x_{19}=1  \tag{12}\\
& x_{20}+x_{21}+x_{22}+x_{23}=1 \tag{13}
\end{align*}
$$

Table 2 shows that the maximum value of $F$ is $U()=$.9 . Furthermore, minimum headway can be determined based on the maximum frequency, i.e. $\frac{60}{9} \cong 6.6$ minutes. Then we can calculate the total number of vehicles that depart and arrive at each terminal at any time that is less than or equal to 6.6 minutes.

To simplify the calculations for two periods (or 120 minutes), the headway is determined in every 6 minutes. Calculation of vehicle for each departure is +1 , while for each arrival is -1 . Furthermore, the calculation of the vehicle departing and arriving at each terminal for two periods is started from the $6^{\text {th }}$ minute till the $120^{\text {th }}$ minute.

Calculation of vehicles departure in Sako -Pasar Gubah route during period 1 is as follows. At $F$ maximum, assuming that $x_{1}=2, x_{2}=3, x_{3}=4, x_{4}=5, x_{5}=6, x_{6}=7, x_{7}=$ 8, we obtain:
a. At 6th minute there is no vehicle that departs.
b. The possible values for $F$ in the 12th minute are $x_{5}, x_{6}$, $x_{7}$ by assumed constraint as $x_{5}+x_{6}+x_{7}$
c. The possible values for $F$ in the 18 th minute are $x_{3}, x_{4}$. Based on the addition of possible values for $F$ in the 12th minute and the 18th minute, it is assumed by the constraint $x_{3}+x_{4}+x_{5}+x_{6}+x_{7}$
d. The possible values for $F$ in the 24th minute are adalah $x_{1}, x_{2}, x_{4}, x_{5}, x_{6}, x_{7}$. Based on the addition of possible values for $F$ in the 12th minute, the 18th minute, and the $24^{\text {th }}$ minute, it is assumed by the constraint $x_{1}+x_{2}+x_{3}+2 x_{4}+2 x_{5}+2 x_{6}+2 x_{7}$
And so on, also in the same way, we obtained constraints on computation of departure vehicles in both routes at both periods. They are seen in Table 3.

Constraints in Table 3 are defined as the Inequality (14) The next constraint is the limited number of buses operating during two periods in both routes. There are 10 buses, so that
$x_{24}+x_{25} \leq 10$
Then by Equation (7) and Equation (8) are obtained constraints:
$x_{j} \in\{0,1\}, j=1,2,3, \ldots, 23$
$x_{j} \geq 0, \quad x_{j} \in \mathrm{Z}, \quad j=24,25$
Table 3. Constraints Based on Number of Buses Used

| No. | Constraints |
| :---: | :---: |
| 1 | Not Yet |
|  | Not Yet |
| 2 | $x_{5}+x_{6}+x_{7} \leq x_{24}$ |
|  | $x_{19} \leq x_{25}$ |
| 3 | $x_{3}+x_{4}+x_{5}+x_{6}+x_{7} \leq x_{24}$ |
|  | $x_{16}+x_{17}+x_{18}+x_{19} \leq x_{25}$ |
| 4 | $x_{1}+x_{2}+x_{3}+2 x_{4}+2 x_{5}+2 x_{6}+2 x_{7} \leq x_{24}$ |
|  | $x_{15}+x_{16}+x_{17}+2 x_{18}+2 x_{19} \leq x_{25}$ |
| 5 | $x_{1}+x_{2}+2 x_{3}+2 x_{4}+2 x_{5}+3 x_{6}+3 x_{7} \leq x_{24}$ |
|  | $x_{15}+x_{16}+2 x_{17}+2 x_{18}+3 x_{19} \leq x_{25}$ |
| 6 | $x_{1}+2 x_{2}+2 x_{3}+3 x_{4}+3 x_{5}+4 x_{6}+4 x_{7} \leq x_{24}$ |
|  | $x_{15}+x_{16}+2 x_{17}+3 x_{18}+3 x_{19} \leq x_{25}$ |
| 7 | $x_{1}+2 x_{2}+2 x_{3}+3 x_{4}+4 x_{5}+4 x_{6}+5 x_{7} \leq x_{24}$ |
|  | $x_{15}+2 x_{16}+2 x_{17}+3 x_{18}+4 x_{19} \leq x_{25}$ |
| 8 | $x_{1}+2 x_{2}+3 x_{3}+4 x_{4}+5 x_{5}+5 x_{6}+6 x_{7} \leq x_{24}$ |
|  | $x_{15}+2 x_{16}+3 x_{17}+4 x_{18}+4 x_{19} \leq x_{25}$ |
| 9 | $x_{1}+2 x_{2}+3 x_{3}+4 x_{4}+5 x_{5}+6 x_{6}+7 x_{7} \leq x_{24}$ |
|  | $x_{15}+2 x_{16}+3 x_{17}+4 x_{18}+5 x_{19} \leq x_{25}$ |
| 10 | $\begin{aligned} & 2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}-x_{17}- \\ & x_{18}-x_{19} \leq x_{24} \end{aligned}$ |
|  | $2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19} \leq x_{25}$ |
| 11 | $\begin{aligned} & 2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}-x_{15}- \\ & x_{16}-x_{17}-2 x_{18}-2 x_{19} \leq x_{24} \end{aligned}$ |
|  | $2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19} \leq x_{25}$ |
| 12 | $\begin{aligned} & 2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+x_{10}+ \\ & x_{11}+x_{12}+x_{13}+x_{14}-x_{15}-x_{16}-2 x_{17}-2 x_{18}-2 x_{19} \\ & \leq x_{24} \end{aligned}$ |
|  | $\begin{aligned} & 2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+x_{23}-x_{5}-x_{6}- \\ & x_{7} \leq x_{25} \end{aligned}$ |
| 13 | $\begin{aligned} & 2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+x_{8}+ \\ & x_{9}+x_{10}+2 x_{11}+2 x_{12}+2 x_{13}+2 x_{14}-x_{15}-x_{16}- \\ & 2 x_{17}-2 x_{18}-3 x_{19} \leq x_{24} \end{aligned}$ |
|  | $\begin{aligned} & 2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+x_{21}+x_{22}+x_{23} \\ & x_{2}-x_{3}-x_{4}-x_{5}-x_{6}-x_{7} \leq x_{25} \end{aligned}$ |
| 14 | $\begin{aligned} & 2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+x_{8}+ \\ & x_{9}+2 x_{10}+2 x_{11}+2 x_{12}+3 x_{13}+3 x_{14}-x_{15}-2 x_{16}- \\ & 2 x_{17}-3 x_{18}-3 x_{19} \leq x_{24} \end{aligned}$ |
|  | $\begin{aligned} & 2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+x_{20}+x_{21}+x_{22} \\ & +2 x_{23}-x_{2}-x_{3}-x_{4}-2 x_{5}-2 x_{6}-2 x_{7} \leq x_{25} \\ & \hline \end{aligned}$ |
| 15 | $\begin{aligned} & 2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+x_{8}+ \\ & 2 x_{9}+2 x_{10}+3 x_{11}+3 x_{12}+4 x_{13}+4 x_{14}-x_{15}-2 x_{16} \\ & -2 x_{17}-3 x_{18}-4 x_{19} \leq x_{24} \\ & \hline \end{aligned}$ |
|  | $\begin{aligned} & 2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+x_{20}+x_{21}+x_{22} \\ & +2 x_{23}-x_{2}-x_{3}-2 x_{4}-2 x_{5}-3 x_{6}-3 x_{7} \leq x_{25} \\ & \hline \end{aligned}$ |
| 16 | $\begin{aligned} & 2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+x_{8}+ \\ & 2 x_{9}+2 x_{10}+3 x_{11}+4 x_{12}+4 x_{13}+5 x_{14}-x_{15}-2 x_{16} \\ & -3 x_{17}-4 x_{18}-4 x_{19} \leq x_{24} \end{aligned}$ |
|  | $\begin{aligned} & 2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+x_{20}+2 x_{21}+ \\ & 2 x_{22}+3 x_{23}-x_{1}-2 x_{2}-2 x_{3}-3 x_{4}-3 x_{5}-4 x_{6}-4 x_{7} \\ & \leq x_{25} \end{aligned}$ |
| 17 | $\begin{aligned} & 2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+x_{8}+ \\ & 2 x_{9}+3 x_{10}+4 x_{11}+4 x_{12}+5 x_{13}+6 x_{14}-x_{15}-2 x_{16} \\ & -3 x_{17}-4 x_{18}-5 x_{19} \leq x_{24} \\ & \hline \end{aligned}$ |
|  | $\begin{aligned} & 2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+x_{20}+2 x_{21}+ \\ & 3 x_{22}+3 x_{23}-x_{1}-2 x_{2}-2 x_{3}-3 x_{4}-4 x_{5}-4 x_{6}-5 x_{7} \\ & \leq x_{25} \end{aligned}$ |


| 18 | $2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+x_{8}+$ |
| :--- | :--- |
|  | $2 x_{9}+3 x_{10}+4 x_{11}+5 x_{12}+6 x_{13}+7 x_{14}-2 x_{15}-$ |
|  | $3 x_{16}-4 x_{17}-5 x_{18}-6 x_{19} \leq x_{24}$ |
| 19 | $2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+x_{20}+2 x_{21}+$ |
|  | $3 x_{22}+4 x_{23}-x_{1}-2 x_{2}-3 x_{3}-4 x_{4}-4 x_{5}-5 x_{6}-$ |
|  | $6 x_{7} \leq x_{25}$ |
|  | $2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+2 x_{8}+$ |
|  | $3 x_{9}+4 x_{10}+5 x_{11}+6 x_{12}+7 x_{13}+8 x_{14}-2 x_{15}-$ |
|  | $3 x_{16}-4 x_{17}-5 x_{18}-6 x_{19}-x_{21}-x_{22}-x_{23} \leq x_{24}$ |
|  | $2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+2 x_{20}+3 x_{21}+$ |
|  | $4 x_{22}+5 x_{23}-x_{1}-2 x_{2}-3 x_{3}-4 x_{4}-5 x_{5}-6 x_{6}-$ |
|  | $7 x_{7} \leq x_{25}$ |
|  | $2 x_{1}+3 x_{2}+4 x_{3}+5 x_{4}+6 x_{5}+7 x_{6}+8 x_{7}+3 x_{8}+$ |
|  | $4 x_{9}+5 x_{10}+6 x_{11}+7 x_{12}+8 x_{13}+9 x_{14}-2 x_{15}-$ |
|  | $3 x_{16}-4 x_{17}-5 x_{18}-6 x_{19}-x_{20}-x_{21}-x_{22}-2 x_{23} \leq$ |
|  | $x_{24}$ |
|  | $2 x_{15}+3 x_{16}+4 x_{17}+5 x_{18}+6 x_{19}+3 x_{20}+4 x_{21}+$ |
|  | $5 x_{22}+6 x_{23}-2 x_{1}-3 x_{2}-4 x_{3}-5 x_{4}-6 x_{5}-7 x_{6}-$ |
|  | $8 x_{7} \leq x_{25}$ |

Description: Number 1 is the 6th minute, Number 2 is the $12^{\text {th }}$ minute, and so on until Number 20 is the $120^{\text {th }}$ minute.

### 4.3. Timetable Model of BRT Transmusi

Based on Equation (9) to Equation (17), we obtain a complete formulation of optimization timetable model. The model consists of the objective function in Equation (9) and 45 constraints. Furthermore, the model is denoted as Equation (18).

### 4.4. Timetable Problem Solution Based on Branch and Bound Method

Linear programming model of timetable in Equation (18) is converted to a standard form. In this case, the right-hand side on the constraints should be as a constant whose value is based on Table 2.

The linear programming relaxation is solved by Software Lindo, so that we obtain upper bound $Z=132.5$. The optimum solution is not an integer, so it needs to be done branching. Branching is done continuously to find integer and optimal solutions. This calculation is performed until the 49th iteration. Furthermore, we solve the problem by using Software Lindo. It can be seen in Figure 1.



Figure 1. Calculation Results of Branch and Bound Method by Using Software Lindo
Based on Figure 1, the output of software can be described as follows:

1. $Z=145$ states that the total minimum number of passengers jostled is 145 passengers for 2 periods.
2. The value of the variable $x_{1}, x_{2}, x_{3}, x_{5}, x_{6}, x_{7}, x_{8}, x_{9}, x_{10}, x_{12}$, $x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{20}, x_{21}, x_{23}$ is 0 , so there is no bus dispatched on the route and the period for this variable index.
3. The value of the variable $x_{4}=1$. It means in Sako - Pasar Gubah route for period 1, the frequency of the bus is 5 buses, because of the frequency of the trip circuit is determined by the value of $x_{1}, x_{2}, \ldots, x_{7}$.
4. The value of the variable $x_{11}=1$. It means in Sako - Pasar Gubah route for period 2 , the frequency of the bus is 6 buses, because of the frequency of the trip circuit is determined by the value of $x_{8}, x_{9}, \ldots, x_{14}$.
5. The value of the variable $x_{19}=1$. It means in Pasar Gubah - Sako route for period 1, the frequency of the bus is 6 buses, because it is determined by the value of $x_{15}, x_{16}, \ldots$, $x_{19}$.
6. The value of the variable $x_{22}=1$. It means in Pasar Gubah - Sako route for period 2, the frequency of the bus is 5 buses, because it is determined by the value of $x_{20}, x_{21}, x_{22}$, $x_{23}$.
7. The value of the variable $x_{24}=4$. It means the number of buses required for the departure of halte Sako to halte Pasar Gubah is as much as 4 buses.
8. The value of the variable $x_{25}=6$. It means the number of buses required for the departure of halte Pasar Gubah to halte Sako is as much as 6 buses.
Headway can be determined by dividing 60 minutes with the frequency of each route in each period. It is shown in Table 4.

Table 4. Headway for Each Route on Each Period

| Route | Period | Freq. | Headway <br> (in minutes) |
| :---: | :---: | :---: | :---: |
| Sako - <br> P.G. | 1 | 5 | 12 |
| P.G. - | 1 | 6 | 10 |
| Sako | 2 | 6 | 10 |

Based on Table 4, it can be seen that in Sako - Pasar Gubah route for period 1 during 60 minutes, the number of
buses to be deployed is 5 buses with the difference of the departure time between a bus and the next bus is 12 minutes. As for Pasar Gubah - Sako in this first period, the number of buses to be deployed is 6 buses with the difference of the departure time is 10 minutes.

In Sako - Pasar Gubah route for period 2 during 60 minutes, the number of buses to be deployed is 6 buses with the difference of the departure time between a bus and the next bus is 10 minutes. As for Pasar Gubah - Sako in this second period, the number of buses to be deployed is 5 buses with the difference of the departure time is 12 minutes.

Based on headway calculations shown in Table 4, it can be done the scheduling of departure time on BRT Transmusi. Timetable obtained is shown in Table 5.
Table 5. Timetable of BRT Transmusi

| Departure | Route |  |
| :---: | :---: | :---: |
|  | Sako - P.G. | P.G. - Sako |
|  | Time (WIB) | Time (WIB) |
| 1 | $\mathbf{0 6 : 2 7 : 0 0}$ | $\mathbf{0 6 : 2 5 : 0 0}$ |
| 2 | $\mathbf{0 6 : 3 9 : 0 0}$ | $\mathbf{0 6 : 3 5 : 0 0}$ |
| 3 | $\mathbf{0 6 : 5 1 : 0 0}$ | $\mathbf{0 6 : 4 5 : 0 0}$ |
| 4 | $\mathbf{0 7 : 0 3 : 0 0}$ | $\mathbf{0 6 : 5 5 : 0 0}$ |
| 5 | $\mathbf{0 7 : 1 5 : 0 0}$ | $\mathbf{0 7 : 0 5 : 0 0}$ |
| 6 | $07: 25: 00$ | $\mathbf{0 7 : 1 5 : 0 0}$ |
| 7 | $07: 35: 00$ | $07: 27: 00$ |
| 8 | $07: 45: 00$ | $07: 39: 00$ |
| 9 | $07: 55: 00$ | $07: 51: 00$ |
| 10 | $08: 05: 00$ | $08: 03: 00$ |
| 11 | $08: 15: 00$ | $08: 15: 00$ |

Description: Numbers in bold at the departure time state time in period 1 .
Based on Table 5, it was obtained timetable of BRT Transmusi with 10 buses that operate for 2 hours. So, for Sako - Pasar Gubah route gained headway for 12 minutes with 5 departures starting at 6:27 to $7: 15 \mathrm{pm}$, while at 7:25 to $8: 15 \mathrm{pm}$ its headway is 10 minutes with 6 departures. For route Pasar Gubah - Sako gained headway for 10 minutes with 6 departures starting at 6:25 to 7:15 pm, while at 07:27 to $08: 15 \mathrm{pm}$ its headway is 12 minutes with 5 departures.

## 5. Conclusion

Optimum solutions on timetable problems of BRT Transmusi Palembang obtained by using Branch and Bound method generate total minimum number of passengers jostled is 145 passengers for 2 periods in both Sako - Pasar Gubah and Pasar Gubah - Sako routes. Based on the timetable obtained, buses required for the departure of Sako to Pasar Gubah in period 1 are 5 buses with headway for 12 minutes. As for the departure of Pasar Gubah to Sako are 6 buses with headway for 10 minutes. In period 2, the buses required for the departure of Sako to Pasar Gubah are 6 buses with headway for 10 minutes, while for the departure of Pasar Gubah to Sako are 5 buses with headway for 12 minutes.

## References

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