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Mathematical Modelling in Problem Solving

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Abstract. This paper is a literature review on problem solving and mathematical modelling. The steps of mathematical modelling are compared with the stages of problem solving. As a result, it was concluded that modelling is part of problem solving.

Problem solving is the heart of mathematics
Alan H. Schoenfeld, Foreword [1]

1. Introduction

Problems are faced by people in their daily. Therefore, the ability to solve problems becomes very important for everyone to have. Problems arise when people cannot immediately see how to finish it. Problems are relative; that is, a problem for someone is not necessarily a problem for other people, depending on the experience of the person facing it. This paper only addresses on mathematical problem on school levels only.

The discussion started with the definition of the problem, followed by the definition of the problem solving. The stages of problem solving were discussed in detail with examples. Next, mathematical modelling at the steps were briefly discussed. Finally, it was shown that mathematical modelling became part of the problem solving process.

2. Nature of the Problem

For a student, mathematical problems may be a problem or may not be a problem. Mathematical problems are not problems if students only need to look suitable "tools" for solving it. The "tools" themselves referred to a formula or procedure that can be used to solve the problem. If there are one or more procedures that can be directly used for solving a problem no matter how complicated the procedures are, then the problem is not a problem. This kind of problem is also known as routine problem or exercise.

Example 1. A 5 meter long ladder rests on a wall. If the foot of the stairs is 3 meters from the bottom of the wall with the assumption that the wall is perpendicular to the ground, please determine the height of the stairs on the wall.

By using perpendicular assumptions, the Pythagorean formula can be directly used to solve this problem; $\sqrt{5^2 - 3^2} = 4$ meters. Because it does not require further analysis to solve it, this problem is not a problem for students who have studied or are studying the Pythagorean Theorem.

Mathematical problems will become problems if there are no ready-made methods, techniques, or procedures that can be directly used to solve them, but it requires additional effort from the person



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who wants to solve them. Sometimes people have to change their perspective before seeing how to solve a problem. For example:

Example 2. *A 5 meters long ladder rests on a wall. If there is a water channel with a depth of 1 meter and a width of 1 meter at the foot of the wall with the assumption that the wall is perpendicular to the ground, please determine the maximum height of the ladder's back stuck on the wall.*

The Pythagorean formula cannot be directly used to solve this problem even though someone has thought of using it, because there is no information about how far the bottom of the ladder from the bottom of the wall. Therefore, this problem requires further analysis to solve it. It was noted that “maximum” is the keyword in this problem, and to reach the maximum height of the foot, the ladder must be moved as close as possible to the wall because the closer the foot of the ladder to the wall, the higher the footrest of the ladder on the wall. So we can simply determine the shortest possible distance from the foot of the ladder to the bottom of the ladder. It is certain that the shortest distance is reached when the foot of the ladder is placed on the edge of the water channel. Then, by assuming that is perpendicular, we only need to use the Pythagorean formula to calculate the height, which is $\sqrt{5^2 - 1^2} = 2\sqrt{6} = 4.9$ meters.

7 3. Problem Solving

Problem solving is the process of getting a solution. A solution is the answer along with the method used to get it. George Polya (1887-1985) was known as the father of mathematical problem solving in mathematics education. Polya defines that [2] there are 4 stages in the problem solving process such as: (1) understanding the problem, (2) drawing up a plan, (3) implementing the prepared plan, and (4) rechecking, see also [3].

9 3.1. Understanding the Problem

The first step in solving a problem is understanding the problem. At this stage, the problem is broken down into its main parts. In problems to find, the main parts are data, conditions, and what to find. In problems to prove, the main parts are hypotheses and conclusions. Making pictures, diagrams, or tables can be an important strategy at this stage.

8 3.2. Drawing up a Plan

Once the problem is clearly understood, making a plan is the next step. What concept and what method to be used are determined at this stage. The experience in solving similar problems can be helpful in choosing the right strategy to solve the problem. Some problem solving strategies can be found in [4].

3.3. Implementing the Prepared Plan

The prepared plans in the previous stage are implemented at this stage. And, all calculations are done at this stage.

3.4. Rechecking

This stage is the most important stage in the problem solving process since the learning process takes place in this stage. At this stage, the students do not only re-check whether the answers obtained is correct and match the conditions in the problem, but also allowed to look for other ways that might be better. In addition, the students can make generalizations and formulate other problems related to the problem being solved at this stage.

As an illustration, pay attention to the following example taken from [5].

Example 3. *A 5 meters-long ladder rests on a wall. If there is a cube with the side length of 1 meter at the bottom of the wall with the assumption that the wall is perpendicular to the ground, please determine the maximum height of the wall that can be reached by the ladder.*

As in Example 2, we can move the bottom of the ladder until the apparently-middle part of the ladder touches the side of the box as in Figure 1, but we still unable to determine the distance of the bottom of

the ladder to the bottom of the wall (length of AB in Figure 1) so that the Pythagorean formula cannot longer be used here.

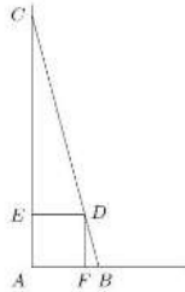


Figure 1. Stairs from the side.

The figure above really helps us in getting ideas to solve the problems. Based on Figure 1, we should calculate the length of AC, with $BC = 5$ and $ED = AF = AE = FD = 1$. In Figure 1, we get that $\triangle EDC$ is congruent with $\triangle FBD$. To simplify the notation, for example $t = AC$ and $a = AB$, the similarity between $\triangle EDC$ and $\triangle FBD$ provides a comparison $(t - 1) : 1 = 1 : (a - 1)$ with the assumption that $\triangle ABC$ is elbow, the Pythagorean formula gives equation $a^2 + t^2 = 25$. So the problem solving is equivalent to the following equation system:

$$\begin{cases} (a - 1)(t - 1) = 1, \\ a^2 + t^2 = 25. \end{cases} \quad (1)$$

Solving the system of equations (1) produces $t = 4,82$. So the maximum height of a ladder's backrest on the wall is 4.82 meters because the foot of the ladder cannot be moved closer to the bottom of the wall because it is blocked by a box.

Another way to solve [10](#) problem in Example 3 is to use a Cartesian coordinate system. According to the problem, suppose the coordinates of point A are (0,0), point B (a, 0), point C (0, t), and point D (1, 1), as in Figure 2.

In Figure 2, the equation of the line through points B and C is $ay + tx = at$, but this line passes through point D = (1; 1) so that the equation $a + t = at$ is obtained. Since the distance of points B and C is 5, we get the equation $a^2 + t^2 = 25$. Therefore, solving this problem is equivalent to solving the system of equations:

$$\begin{cases} a + t = at, \\ a^2 + t^2 = 25, \end{cases} \quad (2)$$

which is identical to the equations (1)

Then, the problem in Example 3 can be generalized to:

Example 4. A 5 meter long ladder rests on a wall. If there is a cube with a side length of s meters at the bottom of the wall with assumption that the wall is perpendicular to the ground, please determine the maximum height of the wall that can be reached by the ladder.

So that Example 3 is a specific form of the problem in Example 4. Furthermore, a problem similar to the problem in Example 3 can also be created, for example:

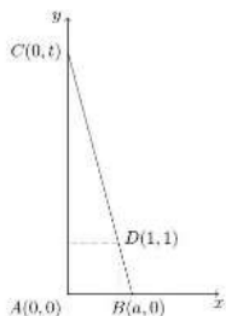


Figure 2. Cartesian coordinate system.

Example 5. A 5 meters-height wall has a cube with 1 meter side length at the down-side of the wall. Please determine the minimum length of a ladder that is laid on the wall and its' end reaching the top of the wall.

4. Mathematical Model

Real world problems cannot be solved with mathematics before the problem is translated into mathematical language in the form of mathematical models. Therefore, the process of turning real world problems into mathematical problems is called mathematical modelling. This is a process of mathematizing real-world problems and hence is also called a mathematical process [6, 7]. The steps of the mathematical modelling process for solving real-world problems are as follows (see [8, 7]):

- Understanding the problem. In this step, the problem is analyzed to distinguish the information given (givens) and the questions that must be answered (goals).
- Making Assumptions. For complex problems, assumptions are needed to simplify the model. The solution obtained only applies according to the assumptions made.
- Defining variables. All variables needed to present the problem in mathematical form are defined in this step.
- Correlating the Variables. Equations are formed in order to connect the defined variables by paying attention to the results of the problem analysis.
- Building a Model. Functional relationship between variables and assumptions (if available) forms a mathematical model. A mathematical model can be a system of equations, a number of rules, or an algorithm of variable values.
- Completing the Model. By using existed techniques and methods in mathematics, the model is solved in order to get a mathematical solution of the existing problem. If necessary, technologies like calculators and computers can be used to do all mathematical calculations.
- Interpreting the Solutions. The solution obtained is interpreted and translated back into the context of the problem given.
- Verifying the Model. The modelling process does not stop at interpreting solutions into the context of the problem. Some questions like “does the solution obtained make sense?” or “is a new assumption model needed?” still need to be answered through a review of the produced model that is validated with data and corrected if necessary in order to produce a valid model and be able to be used to study and understand problems through simulation or making predictions.

This process is illustrated schematically in Figure 3.

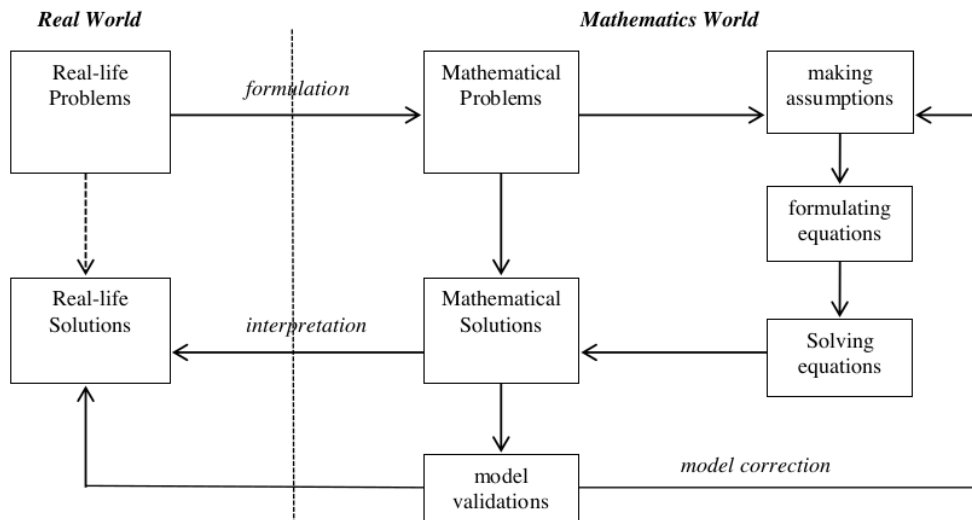


Figure 3. The mathematical modelling process [6, 9, 10].

As an illustration, consider the following example:

Example 6. Mr Udin has a number of chickens and cages. When each cage is filled by two chickens, there are two chickens that don't get the cage. However, if each cage is filled by three chickens, there are three empty cages. Please determine how many chickens and cages owned by Mr Udin.

After understanding the problem asked, suppose that many chickens are represented by a and many cages with k with both are positive integers. The next step is to look for a relationship between a and k . From the statement in the problem, it is obtained the relationship $\frac{a}{2} - 2 = k$ and $k - 3 = 3a$, so we get the following equation system:

$$\begin{cases} \frac{a}{2} - 2 = k \\ k - 3 = 3a \end{cases} \quad (3)$$

Even though $a = -2$ and $k = -3$ are the correct solution of the equation system (3), this solution is not the correct solution for the problem in Example 6. This error occurs because of the wrong model application, not the wrong calculation. If it is examined more closely, the first statement is more suitable to be represented with $a = 2k + 2$ while the second statement with $k = \frac{a}{3} + 3$, so that the equations system is obtained as follows:

$$\begin{cases} a = 2k + 2, \\ k = \frac{a}{3} + 3. \end{cases} \quad (4)$$

Equation system (4) has obtained solution of $a = 24$ and $k = 11$. This means that Mr. Udin has 24 chickens and 11 cages. Next, does this solution fit the statement in the problem? Note that if each cage is filled by two chickens, then 11 cages are only enough for 22 chickens with two chickens have no cage. If each cage is filled by three chickens, only eight cages are filled leaving three empty cages.

5. Problem Solving vs. Modelling

The success of mathematical modelling depends on the ability of the learner to choose right variables and build relationships between variables through a good understanding of the problem to be solved. Because the model is a mathematical representation of the problem, an error in setting the model will produce the wrong solution for the problem. Errors in formulating equations occur due to misunderstanding of the problem. Therefore, understanding the problem is an important step in mathematical modelling. Thus, the model selection can be equalized by setting up the plan for solving the problem. Producing a model is certainly equivalent to carrying out a plan while the interpretation and validation of the model is a looking back activity. Since the equality between the modelling process and problem solving process exists, it can be concluded that modelling is part of the problem solving process.

In contrast, according to Lesh and Zawojewski [11], mathematical modelling provides a new perspective for problem solving, that is a process of interpreting a situation mathematically which often involves an iterative cycle of expressing, testing, and revising mathematical interpretations and activities of choosing, sorting, integrating, revising, and refining the grouping of mathematical concepts for topics inside and outside mathematics. According to the experts, this definition provides wider space to understand what is given (givens) and targets (goals) in a problem.

6. Conclusions

Mathematical modelling is an important part in the problem solving process. The produced model can be used as a "tool" to understand the problem through the manipulation of variables and simulations for making the predictions.

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