Preprocessing and probing techniques in Simplifying Open Capacitated Vehicle Routing Problem (OCVRP) model

(Case study: Rubbish transportation in Kecamatan Sukarami Palembang City)

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Abstract. Rubbish transportation activity in Palembang city is one of the applications of Vehicle Routing Problem (VRP). Rubbish transportation system in Palembang usually been done in many phases. Rubbish collected from homes usually been collected to the nearest temporary rubbish collection place (TPS). This rubbish will be delivered to the final rubbish collection place (TPA). This system divides into the working areas (WK). The issue in this study is Open Capacitated Vehicle Routing Problem (OCVRP) on vehicle routes of rubbish transportation in Kecamatan Sukarami Palembang city by using Preprocessing and Probing techniques to find a simpler OCVRP. The results show that the optimal route of rubbish transportation vehicle on OCVRP models before using preprocessing and after using preprocessing technique is the same. In addition, the results obtained show that the number of constraints is reduced; the number of variables is reduced; the number of iteration is reduced; it does not change the value z; and z in the optimum model has fewer variables. To find the solution of OCVRP models was used LINDO software, so the models found were simpler and more efficient in its solution process.
Keywords: OCVRP, preprocessing technique, rubbish transportation, vehicle routes.

1. INTRODUCTION

The system of rubbish collections in Palembang is conducted into steps. Rubbish from homes is collected to be put in nearest Temporary Rubbish Disposal (TPS). Next, that rubbish is collected to Final Rubbish Disposal (TPA) by using dump truck or amroll vehicles to TPA Sukawinatan and TPA Karya Jaya. Rubbish collections were groups into Working Region (WK).

There exist various rubbish bins such as form of container, made of fibres, or concrete. In one TPS, there are many rubbish bins. Each TPS has different capacity.

In conventional Capacitated Vehicle Routing Problem (CVRP), drivers' of vehicles must return to depot (TPA) after finishing their jobs. But for vehicle route problems like transporting rubbish vehicles, that condition cannot be done.

The vehicles usually do not return to depot after finishing its job, but it returns to other places like drivers' home. It becomes new problem to be solved since the path formed is not close or open path so CVRP turns into Open Capacitated Vehicle Routing Problem (OCVRP) [4]. OCVRP is critical to develop since it concerns with commodity such as rubbish.

One way to examine the OCVRP model is by using probing and preprocessing techniques. Zulfia, et. al [3] previously discussed rubbish transportation for SCVRP model. Those techniques are discussed in this paper for OCVRP model. Those techniques involve constraint bound strengthening phase, eleminating the redundant constraints and fixing variables. Based on the application of [7], we develop probing and preprocessing techniques by identifying the nonfeasible constraints and redundant constraints, improving the bound and coefficients, and also setting up variable values. OCVRP model have been previously discussed in [1], [2], [5], and [6]. Then, we will examine validity of the model.

2. RESEARCH METHODOLOGY

2.1 Simplification of OCVRP Model

We simplify the model through probing and preprocessing techniques by strengthening bounds of constraint variables, eleminating the redundant variables and fixing variables.

2.2 Solution by Probing Technique

The steps are as follows.

a) By fixing variables

The steps are following

- Identify the constraint in new formulation formed
- Arrange the final formulation according to strengthening bound phase, eliminating redundant variable phase and fixing variables phase.

b) By improving the coefficient

- If $Z_k \leq b_i$, then $a^i\mathbf{x} \leq b_i$ will be redundant. So, by assuming $x_k = 0$, the set of feasible solutions will not change if b_i and a^i_k are changed into $\delta = b_i - Z_k$.
- If $Z_k \leq b_i$, then $a^i\mathbf{x} \leq b_i$ will be redundant on that formulation. So, by assuming $x_k = 1$, the set of feasible solution will not change if b_i and a^i_k are changed into $\delta = b_i - Z_k$.

c) By identifying the Logical Implications

- If $x_j = 0$, set $x_k = 0$ so $x_k \leq x_j$, set $x_k = 1$ so $1 - x_k \leq x_j$
- If $x_j = 1$, set $x_k = 0$ so $x_k \leq 1 - x_j$. Set $x_k = 1$ so $1 - x_k \leq 1 - x_j$

2.3 Model Solution by Preprocessing Technique

The steps consist of :

a) Strengthening the bounds of constraint variables

The steps are as follows.

1. Setting up the variable value of objective function according to variable constraint of ILP initial formulation
2. Strengthening the bound of each variable in each constraint according to variable value predetermined.
3. Conducting the evaluations

b) Eliminating the redundant constraint

The steps are as follows.

1. Comparing the new formulation obtained from strengthening bounds of constraint variable with initial formulation.
2. Conducting the evaluation by setting up new formulation based on technique of strengthening bounds on constraint variable and eliminating redundant variables.

c) Fixing variables

The steps are as follows.

1. Identifying constraints on new formulation obtained
2. Setting up final formulation based on steps of strengthening bound of constraint variables, eliminating redundant variables and fixing variables.

2.4 Comparing the model before and after conducting the preprocessing technique

3. RESULT AND DISCUSSION

3.1. Model Simplification

The steps to simplify the OCVRP model are as follows. Transform the OCVRP model and simplify using probing and preprocessing techniques. In previous researches conducted by [1], [2], [5], and [6] we obtain x_k=0 or x_k=1 in OCVRP model. This is due to application of balancing technique which has similar propreties of probing technique. So, we can continue to simplify the model using preprocessing technique. We also use LINDO application software to solve the model. We can compare the number iteration needed to solve model, number of fixing variables, number of eliminating constraints and optimum *z* value before and after conducting those techniques.

3.2. Model Simplification of Kecamatan Sukarami

(i) WK 1

Phase 1: Setting up the basic OCVRP Model

Basic OCVRP model is

Min $z= 24y_{01}+11.5y_{02}+12.5y_{03}+2y_{04}+2,7y_{05}+ 24x_{10}+10,5x_{12}+11,5x_{13}+22x_{14}+ 21,3x_{15} + 11,5x_{20} + 10,5x_{21} + x_{23} + 10,5x_{24}+9,2x_{25}+12,5x_{30}+11,5x_{31}+x_{32} + 10,5x_{34} + 9,2x_{35} + 2x_{40} + 22x_{41} + 10,5x_{42} + 10,5x_{43} + 0,7x_{45} + 2,7x_{50} + 21,3x_{51} + 9,2x_{52} + 9,2x_{53} + 0,7x_{54}$ (1)

subject to

$x_{14}+x_{15} = 1$ (2)

$y_{02} + y_{20} = 1$ (3)
$x_{24} = 1$ (4)
$y_{03} + y_{30} = 1$ (5)
$y_{04} + y_{40} = 1$ (6)
$y_{05} + y_{50} + x_{52} = 1$ (7)
$x_{53} = 1$ (8)
$y_{01}+y_{02}+y_{03}+y_{04}+y_{05}+y_{10}+y_{20}+y_{30}+y_{40}+y_{50}+x_{12}+x_{13}+x_{14}+x_{15}+x_{21}+x_{23}+$
$x_{24}+x_{25}+x_{31}+x_{32}+x_{34}+x_{35}+x_{41}+x_{42}+x_{43}+x_{45}+x_{51}+x_{52}+x_{53}+x_{54} \geq 7.2$ (9)
$y_{10}+y_{20}+y_{30}+y_{40}+y_{50}+ y_{01}+y_{02}+y_{03}+y_{04}+y_{05}+x_{12}+x_{13}+x_{14}+x_{15}+x_{21}$
$+x_{23}+x_{24}+x_{25}+x_{31}+x_{32}+x_{34}+x_{35}+x_{41}+x_{42}+x_{43}+x_{45}+x_{51}+x_{52}+x_{53}+x_{54} \geq 0$ (10)
$y_{10}+y_{20}+y_{30}+y_{40} + y_{50} =1$ (11)

with non negative constraints

$y_{01}, y_{02}, y_{03}, y_{04}, y_{05}, y_{10}, y_{20}, y_{30}, y_{40}, y_{50}, x_{10}, x_{20}, x_{30}, x_{40}, x_{50}, x_{12}, x_{13}, x_{14}, x_{15}, x_{21}, x_{23}, x_{24}, x_{25}, x_{31}, x_{32}, x_{34}, x_{35}, x_{41}, x_{42}, x_{43}, x_{45}, x_{51}, x_{52}, x_{53}, x_{54} \geq 0$

Phase 2 : Transformation of basic OCVRP model

$x_{14} + x_{15} \leq 1$ (2*)
$y_{02} + y_{20} \leq 1$ (3*)
$x_{24} \leq 1$ (4*)
$y_{03} + y_{30} \leq 1$ (5*)
$y_{04} + y_{40} \leq 1$ (6*)
$y_{05} + y_{50} + x_{52} \leq 1$ (7*)
$x_{53} \leq 1$ (8*)
$y_{01}+y_{02}+y_{03}+y_{04}+y_{05}+y_{10}+y_{20}+y_{30}+y_{40}+y_{50}+x_{12}+x_{13}+x_{14}+x_{15}+x_{21}+$
$x_{23}+x_{24}+x_{25}+x_{31}+x_{32}+x_{34}+x_{35}+x_{41}+x_{42}\ x_{43}+x_{45}+x_{51}+x_{52}+x_{53}+x_{54} \geq 5$ (9*)

Phase 3 : Preprocessing Technique

1. Strengthening Bounds of constraints variables

Since $y_{01}, y_{02}, y_{03}, y_{04}, y_{05}, y_{10}, y_{20}, y_{30}, y_{40}, y_{50}, x_{10}, x_{20}, x_{30}, x_{40}, x_{50}, x_{12}, x_{13}, x_{14}, x_{15}, x_{21}, x_{23}, x_{24}, x_{25}, x_{31}, x_{32}, x_{34}, x_{35}, x_{41}, x_{42}, x_{43}, x_{45}, x_{51}, x_{52}, x_{53}, x_{54} \in \{0, 1, 2\}$ and every variable has nonnegative constraints $\geq$ 0, we assume that x_{10}=0; x_{20}=0 x_{30}=0; x_{40}=0; x_5 =0; y_{01}=0; y_{02}=1; y_{03}=0; y_{04}=1; y_{05}=1; y_{10}=0; y_{20}=0; y_{30}=1; y_{40}=0; y_{50}=0; x_{12} =0; x_{13}=0; x_{14}=0; x_{15}=1 ; x_{21}=0; x_{23}=0; x_{24}=1; x_{25}=0; x_{31}=0; x_{32}=0; x_{34}=0; x_{35}=0; x_{41}=0; x_{42}=0; x_{43}=0; x_{45}=0; x_{51}=0; x_{52}=0; x_{53}=1; x_{54}=0 so substituting the value of x_{15}=1 into (2*) so we the bound of variable x_{14} can be strengthened into x_{14}=0 since $x_{14}\geq 0$. Variable x_{15} can be strengthened into $x_{15}\geq 0$. Strengthening bound of y_{02} variable in (3*) constraint is $y_{02}\geq 0$ and y_{20} variable is $y_{20}\geq 0$. Strengthening bound in (4*) is $x_{24}\geq 0$. Strengthening bound of variable y_{03} in (5*) is $y_{03} \geq 0$. Strengthening bound of variable y_{30} in (5*) is $y_{30}\geq 0$.

Strengthening bound of variable y_{04} in (6*) is $y_{04}\geq 0$ and for y_{40} is $y_{40}\geq 0$. Strengthening bound of variable y_{05} in (7*) is $y_{05}\geq 0$, for y_{50} is $y_{50}\geq 0$ and for x_{52} is $x_{52}\geq 0$. Strengthening bound of variable x_{53} in (8*) is $x_{53}\geq 0$. Next, for variable y_{10} in (11) is $y_{10}\geq 0$, for y_{20} is $y_{20}\geq 0$, for y_{30} is $y_{20}\geq 0$, for y_{40} is $y_{40}\geq 0$ and for y_{50} is

$y_{50}\geq 0$.

Constraint (9*) can be eliminated since none of the variables can be stregthened or satisfy the integer constraints. Nonnegative constraint formed are $x_{14}=0$, $x_{24}=1$, $x_{15}=1$, $y_{20}=0$, $y_{02}=1$, $y_{30}=1$, $y_{03}=0$, $y_{40}=0$, $y_{04}=1$, $y_{50}=0$, $y_{05}=1$, and $x_{53}=1$. Constraint (10) can be eliminated since none of the variables can be stregthened or satisfy the integer constraints

2. Eliminating the redundant constraints

By using the upper bound of nonnegative constraint variable from strengthening bound phase, we can say that constraint (11) satisfies the upper bound of nonnegative constraint. So, constraint (9*) and (10) is redundant and can be eliminated. Constraint (11) satisfies the upper bound and lower bound of nonnegative constraint variable, so we can also eliminate that constraint because it is redundant.

3. Fixing variables

Since number of their largest coefficients exceed RHS then constraint (2*), (3*), (5*), (6*) and (7*) cannot be eliminated. Variable values $x_{14}=0$, $y_{20}=0$, $y_{03}= 0$, $y_{04}=0$, $y_{50}=0$ dan $x_{51}=0$ can be substituted into model so we have new OCVRP model after conducting preprocessing as follows.

$$\text{Min } z = 11{,}5\, y_{02} + 2\, y_{04} + 2{,}7\, y_{05} + 21{,}3\, x_{15} + 10{,}5\, x_{24} + 9{,}2\, x_{53} \quad (10)$$

Subject to

$$x_{15} = 1 \quad (11)$$
$$y_{02} = 1 \quad (12)$$
$$x_{24} = 1 \quad (13)$$
$$y_{30} = 1 \quad (14)$$
$$y_{04} = 1 \quad (15)$$
$$y_{05} = 1 \quad (16)$$
$$x_{53} = 1 \quad (17)$$

with $0\leq x_{15}\leq 1$, $0\leq x_{24}\leq 1$, $0\leq y_{02}\leq 1$, $0\leq y_{30}\leq 1$, $0\leq y_{04}\leq 1$, $0\leq y_{05}\leq 1$.

We obtain optimal solution of 3 iterations with optimal vehicle routes are sketched in Figure 1. The routes are driver's home – Perum Sukajaya – Perum PDK – TPA and TPA-Kebun Bunga – Kuburan Cina – TPA.

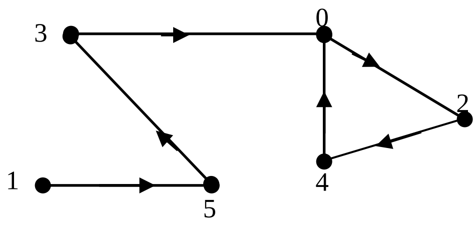

Fig. 1. Final Route of Rubbish Transportation in WK 1

(ii). WK 2

We obtain optimal solution of 5 iterations with optimal vehicle routes are sketched in Figure 2. The optimal routes are driver's home- Km.5 – Sukarami Indah – TPA and TPA-Jl.Perindusrian – Simpang Sukarela – TPA.

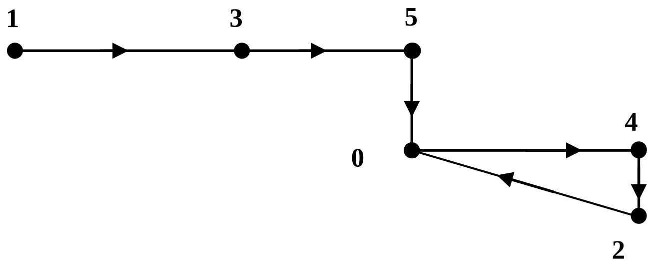

Fig. 2. Final Route of Rubbish Transportation in WK 2

(iii). WK 3

We obtain optimal solution of 3 iterations with optimal vehicle routes are sketched in Figure 3. The optimal routes are TPA – Jl.Simp.Soak – Bandara Kanan – TPA and TPA – Bandara Kanan – Bandara Kiri – TPA.

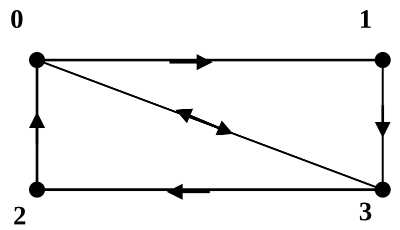

Fig. 3. Final Route of Rubbish Transportation in WK 3

(iv). WK 4

We obtain optimal solution of 3 iterations with optimal vehicle routes are sketched in Figure 4. The optimal routes are driver's home–Depo Kebun Bunga–TPA and TPA – Sukarela – Bandara –TPA .

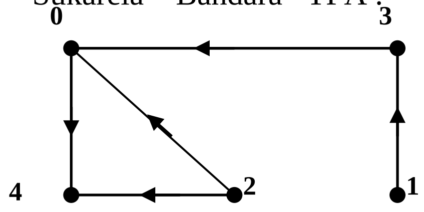

Fig. 4. Final Route of Rubbish Transportation in WK 4

(v). WK 5 and WK 6

In these WK, the vehicle is *amroll* where the routes are not affected by TPS location, since every container must be taken one by one by the vehicle to TPA Sukawinatan. The optimal routes for WK 5 and WK 6 are in Figure 5 and 6, respectively.

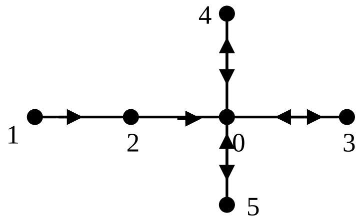

Fig. 5. Final Route of Rubbish Transportation in WK 5

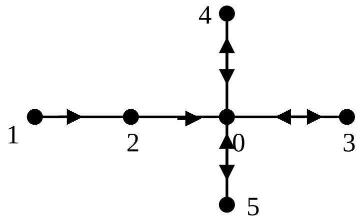

Fig. 6. Final Route of Rubbish Transportation in WK 6

3.3. Comparison between two Models before and after conducting the Preprocessing technique

After conducting preprocessing technique for OCVRP model in WK 1-WK4, so

1. the constraint numbers decrease
   In WK 1, we have 10 constraints and decrease into 7; in WK 2, from 9 into 6; in WK 3, from 8 into 7 and in WK 4 from 8 into 5.
2. the variable numbers decrease
   In WK 1, we have 35 variables and decrease into 6, in WK 2 from 35 into 5, in WK 3 from 12 into 6, and in WK 4 from 20 into 4.
3. the iteration numbers decrease
   In WK 1, we have 4 iteration numbers and decrease into 3, in WK 2 from 8 into 5, in WK 3 from 7 into 3.
4. simpler objective function of z are obtained
   In each WK we obtain simpler objective function value.
5. the optimal value of z for each WK is not changed
   We obtain same optimal objective function value for each WK

4. CONCLUDING REMARK

From above result and discussion we can conclude that the optimal routes that can be travelled by rubbish transportation vehicles in Kecamatan Sukarami are same routes obtained without the preprocessing techniques with the exception that we obtain decreasing number of variables, iterations, simpler model but no change in objective function value.

For future work, we suggest that it could be possible to also include other factors that affect the routes such as the conditions of road travelled, conditions of vehicles, etc.

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