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On the Vibrations of a Linear and a Weakly 1-D Wave Equations with Non-classical Boundary Damping

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Abstract: In this study initial-boundary value problems for a linear and a weakly nonlinear string (or wave) equation are considered. One end of the string is assumed to be fixed and the other end of the string is attached to a spring-mass-dashpot system, where the damping generated by the dashpot is assumed to be small. This problem can be regarded as a simple model describing oscillations of flexible structures such as overhead power transmission lines. For a linear problem a semigroup approach will be used to show the well-posedness of the problem as well as the asymptotic validity of formal approximations of the solution on long time-scales. It is also shown how a multiple time-scales perturbation method can be used effectively to construct asymptotic approximations of the solution on long timescales. The main problem of this paper is to study how efficiently these boundary dampers work.

Key words: Boundary damping, asymptotic approximation, multiple timescales, perturbation method, semigroup approach

INTRODUCTION

There are examples of flexible structures such as suspension bridges, overhead transmission lines, dynamically loaded helical springs that are subjected to oscillations due to different causes. Simple models which describe these oscillations can be expressed in initial-boundary value problems for wave equations (Keller and Kogelman, 1970; Van Horssen, 1988; Van Horssen and Van der Burgh, 1988) or for beam equations (Castro and Zuazua, 1998; Boertjens and Van Horssen, 1998). To suppress the oscillations various types of boundary damping can be applied (Castro and Zuazua, 1998).

In most cases simple, classical boundary conditions are applied (Boertjens and Van Horssen, 1998; Keller and Kogelman, 1970; Van Horssen, 1998) to construct approximations of the oscillations. For more complicated, non-classical boundary conditions (Castro and Zuazua 1998), it is usually not possible to construct explicit approximations of the oscillations. In this study such an initial-boundary value problem with a non-classical boundary condition will be studied and explicit asymptotic approximations of the solution, which are valid on a long time-scale will be constructed. The main problem of this paper is to study how efficiently these boundary dampers work. The method which can be used to investigate these problems are multiple timescales methods (Kevorkian and Cole, 1981; Van Horssen, 1988), Galerkin truncation methods and combinations of these methods. From the asymptotic point of view it is also interesting to study the convergence properties of the applied perturbation methods for these types of initial-boundary value problems. A string which is fixed at $x = 0$ and attached to a spring-mass-dashpot system at $x = l$ will be considered.

To derive a model for flexible structures such as suspension bridges or overhead transmission lines it refers to Boertjens and Van Horssen (1998). It is assumed that l (the length of the string), p (the mass-density of the string), T (the tension in the string), rii (the mass in the spring-mass-dashpot system), γ (the stiffness of the spring) and , a , ϕ (the damping coefficients of the dashpot) are all positive constants. Furthermore, the only vertical displacement $u(x,t)$ of the string is considered, where x is the place along the string and t is time.

After applying a simple rescaling in time and in displacement $\bar{t} = \phi t$, $\bar{u}(x,\bar{t}) = u(x,t)$ putting m=p.m,y=y.T, and a=..frpa simple model for the oscillations of the string the following initial-boundary value problem:

$$u_{tt} - u_{xx} + P^2 u = \varepsilon f(x,u,u_t), 0<x<l, t>0, \quad (1)$$

$$u(0,t)=0, t>0, \quad (2)$$

$$u_x(l,t)=-g(t), t>0, \quad (3)$$

u(x,O)=ȹ(x),O < x-c (4)
1

(5)

u,(x,O)=\Jf(X),O < X<
1

is obtained, where E is a small parameter with 0 < E « 1 and where the function f is an external force (for instance a wind force) and where g (t) is the boundary control force defined by:

g(t)=nu. 0,t)+)'U 0,t)+on,0,t)

The functions qi and tlJ represent the initial displacement of the string and the initial velocity of the string, respectively.

MATERIALS AND METHODS

Different cases are considered for f, m, y, a and ϕ· In this paper, it will be considered the following three cases, namely:

p^2 = 0, f(x,μ, μ,)=O,
m, y =O (1),a=O(E)

2 p^2 tcQ f(x, uu) =I\ $\frac{1}{3}$I\ ,r
m, Y, a =O(E)

For the first case a semigroup approach (Goldstein 1985), is used to show the well-posedness of the problem for suitable initial conditions as well as to prove the asymptotic validity of the formal approximations of the solution on long time-scales. Although the problem is linear the construction of these approximations is far from being elementary because of the complicated, non• classical boundary condition. Using some kind of

balancing procedure we solve the linear wave equation and construct approximations. In fact, the procedure is an extension of the classical way to solve a linear wave equation using the method of separation of variables. For the second case, it will be analyzed the behavior of the solutions of the problem where a justification is given whether truncation of the infinite series for the formal

approximation of the solution is valid or not. We will

be used. The idea of such an approach is to reformulate the problem into an abstract Cauchy problem. To use this approach we introduce the following auxiliary functions defined as follows; a (t) = u(•, t) , b(t) = u,(•, t) and ri(t) = mu, (1, t). For simplicity, we denote a, b, TJ, for a (t), b (t) TJ

(t), respectively. The following function spaces are defined as follows:

v: ={aEH'[0,1],a(0) = 0} (6)

'.)(= {y(t) = (a, b, 11)E vx L' [0,1)x9t}

(7) Now the space '.}{ is equipped with the inner

product
'Hx '.}{ -+ffl defined by:

(y,y}: g (a,+ a,+ bb)dx+ya(1)a(1)±_l (8)
rifi
m

where, y = (a, b, TJ) and y=(abij are in '.}{. Observe that this inner product is based upon the energy of the string. For that reason we call the space the energy space 'H. The energy space '.}{ together with the inner product\.,.) is a Hilbert space. Next, it needs to define the unbounded operator A: D (A) c '.}{ϕ'.}{
by:

$$Ay(t):=\begin{pmatrix} a_t \\ b_t \\ \eta_t \end{pmatrix}, y\in D(A),$$

where,
D (A):= {y(t) = (a, b, TJ) E (H2[0, 1] n v xv x ffl; TJ = mb (1)}

It then follows that the form of the abstract Cauchy problem of the initial-boundary problem (1)-(5) is of the form:

y=
Ay

y (0)
= ф

where, $y = \frac{dy(t)}{dt}$ and …

show that mode interactions occur only between modes with non-zero initial energy (up to O(E)). For a sufficiently large value of the damping parameter α it will be shown that all solutions tend to zero.

Case 1: To prove the well-posedness of the initial-boundary value problem (1)-(5) a semigroup approach will

It is shown (Darmawijoyo and Van Horssen 2007), using the Lumer-Philips theorem, that (9)-(10) is well-posedness for tz 0 and that the problems (1)-(5) and (9)-(10) are equivalent (in classical sense) if qi (x) E H^3 (0, 1), qi (0) = qi" (0) = 0 and tlJ (x) EH^2 (0, 1) n V, tlJ (0) = 0.

To construct an approximation of the solution of the initial-boundary value problem the two-time scales perturbation method will be used. Using such a method the function u (x, t) is supposed to be a function of x, t and r = Et. For that reason, it is put

$$u(x, t) = w(x, t, r; E) \quad (11)$$

It is assumed that w (x, t, t; E) can be approximated by the formal expansion:

$$u_0(x,t,t)+E u_1(x,t,r)+E^2 u_2(x,t,t)+E^3 \ldots \quad (12)$$

From (3), it is reasonable to expand the initial displacement φ (x; E) and the initial velocity ψ(x; E) of the string in Fourier series, that is:

$$\varphi(x)=\varphi_0(x)+E\varphi_1(x)+\ldots, \quad (13)$$

$$\psi(X)=\psi_0(x)+E\psi_1(x)+\ldots, \quad (14)$$

Substituting (12-14) into (10-5) and after equating the coefficients of like powers in E, it follows that the solution of $u_0(x, t, t)$ is given by:

$$u_0(x,t,T)=\sum_{n=1} A_n(\tau)\sin(\sqrt{\lambda_n}t)+B_n(\tau)\cos(\sqrt{\lambda_n}t)\sin(\sqrt{\lambda_n}x) \quad (15)$$

where, λ_n is the n-th non-negative zero of:

$$\cot(\sqrt{\lambda_n})\frac{m\lambda n-\gamma}{\sqrt{\lambda n}} \quad (16)$$

and where two different eigenfunctions are orthogonal with respect to the inner product defined by:

$$(X,Y)=\int_0^1[1+m\delta(x-1)]\, XYdx$$

(δ is the delta function).

The O(E) - problem for is given by:

$$u_{1tt}-u_{1xx}=-2u_{0t\tau},\ 0<x<1, t>0$$

$$u_1(0,t,\tau)=0, t>0, \tau>0,$$

$$u_1(x,0,0)=\varphi_1(x),\ 0<x<1 \quad (20)$$

$$u_{1t}(x,0,0)=\psi_1(x)-u_{0\tau}(x,0,0),\ 0<x<1 \quad (21)$$

To solve (17-21) the eigenfunction expansion approach will be used. Using such an approach we have to pay special attention to the non-classical boundary condition at x = 1.

Making boundary conditions homogeneous is the usual way to solve initial-boundary value problems when the inhomogeneous boundary conditions are of classical type (that is, are of Dirichlet, Neumann, or of Robin type). For the non-classical boundary condition at x = 1 this approach turns out to be not applicable. When we apply the eigenfunction expansion to solve the initial- boundary value problem (17-21) the left-hand side of (17) at x = 1 and that of (19) are of the same form. So, to solve the problem correctly the right-hand side of (17) at x = 1 and that of (19) should match, that is, should be proportional. To obtain this matching we introduce the following transformation:

$$u_1(x,t,\tau)=xg(t,\tau)+v(x,t,\tau) \quad (22)$$

Taking $v(x,t,T)=\sum v_n(t,T)\sin(\sqrt{\lambda_n}x)$ we find:

$$g(t,\tau)=\frac{1}{\gamma+1}u_0(1,t,\tau) \quad (23)$$

Using transformation (23) the boundary condition at x = 1 now becomes:

$$m v_{tt}(1,t,T)+\gamma v(1,t,\tau)+v_x(1,t,T)= -2mu_{0t\tau}(1,t,\tau)+m\frac{1}{\gamma+1}u_{0tt}(1,t,\tau),\ t>0 \quad (24)$$

It should be observed that if m is equal to zero then the boundary condition at x = 1 becomes a classical boundary condition. From (24) it can readily be seen that in that case the boundary condition (19) at x = 1 becomes an homogeneous one after the transformation (23). By using the eigenfunction expansion for v (x, t, r) we find that $v_n(t, r)$ has to satisfy:

$$(1,t,t)+\gamma u_1(1,t,t)+u_{1x}(1,t,t) \quad (17)$$

$$(18)$$

(19)

Yntt
+J..n
vn=
(2 ,
A .
A'n
+

__S,_
Xi

sin(
A ,)
AJ
sin(
A ,t
)–
(2
A ,
B : +
y+
c
y,
f,
r;;-:
Xn
sm(
VJ..n
)
BJ
cos
((VJ
..nt)
+
x
y+1
y+1
L
sin(
N
(AP
sin(
.jf;
t) –
BP
cos(
.jf;
t))

(25)

=–2mu∞ tc (l.t, t)–uϕ (l.t.tj.t »
0,t>0

p–1
p=n

where:

$$c_n = \frac{2(\gamma+1)\sin(\sqrt{\lambda_n})}{\lambda_n + (m\lambda_n + \gamma)\sin^2(\sqrt{\lambda_n})}$$

Observe that v (x, t, r) now automatically satisfies the boundary conditions v (0, t, r) = 0 and (24). In order to remove secular terms, it now easily follows from (25) that A_n and B_n have to satisfy:

$$A'_n + \frac{c_n}{2(\gamma+1)} A_n \sin(\sqrt{\lambda_n}) B_n = 0, \quad (26)$$

$$B'_n + \frac{c_n}{2(\gamma+1)} A_n \sin(\sqrt{\lambda_n}) B_n = 0 \quad (27)$$

Using (16) and defining:

$$a_n = \frac{c_n}{2(\gamma+1)} A_n \sin(\sqrt{\lambda_n}) = \frac{A_n \sin^2(\sqrt{\lambda_n})}{\lambda_n + (m\lambda_n + \gamma)\sin^2(\sqrt{\lambda_n})} > 0 \quad (28)$$

the solution of (26)-(27) is given by:

$$A_n(\tau) = A_n(0)\exp(-\alpha_n \tau) \quad (29)$$

$$B_n(\tau) = B_n(0)\exp(-\alpha_n \tau) \quad (30)$$

It is easy to see that the infinite series representation (15) for u_0 is twice continuously differentiable with respect to x and t and infinitely many times with respect to r. From (16) it follows that $\sqrt{\lambda_n} \approx (n-1)\pi$ as $n \to \infty$. So, c_n tends to 0 as n tends to ∞. From (29) and (30) it then follows that u_0 is stable but not uniform. After removing secular terms $v_n(t, r)$ can now be determined completely, yielding:

$$v_n(t,\tau) = D_n(\tau)\cos(\sqrt{\lambda_n} t) + E_n(\tau)\sin(\sqrt{\lambda_n} t) + \sum_{\substack{p=1 \\ p\neq n}}^{\infty} \frac{1}{\gamma+1} \frac{\lambda_p^{3/2}}{\lambda_p - \lambda_n} \sin(\sqrt{\lambda_p})(-A_p(\tau)\sin(\sqrt{\lambda_p} t) + B_p(\tau)\cos(\sqrt{\lambda_n} T))$$

where, $D_n(\tau)$ and $E_n(\tau)$ are still arbitrary functions which can be used to avoid secular terms in $u_1(x, t, r)$. At this that $|A_n(0)|$, $|B_n(0)| \le$ constant/n', $|C_n(0)|$, $|D_n(0)| \le$ constant/n. So far we have constructed a formal approximation $\bar{u}(x, t) = u_0(x, t, r) + \epsilon u_1(x, t, r)$ of u (x, t), where $u_0(x, t, r)$ and $u_1(x, t, r)$ are twice continuously differentiable with respect to x and t and infinitely many times with respect to r. It can be shown that: u (x, t) - ($u_0(x, t, r) + \epsilon u_1(x, t, r)$) = O (E) and $u_t(x,t) - (u_0(x, t, t)) = O$ (E) on $0 \le t \le L$, ϵ^{-1} and $0 \le x \le 1$. And so we obtained asymptotic approximations.

Case 2: In this case we will analyse the asymptotic behaviour of the solution for small E and large values of t. For classical boundary conditions this problem has been studied (Keller and Kogelman, 1970; Van Horssen, 1988). It was shown that the solution of the initial value problem with classical boundary conditions tends to a combination of a finite number of periodic solutions (Keller and Kogelman, 1970). We will show that for a sufficiently large value of the damping coefficient these periodic solutions tend to a stable-zero solution. The problem we consider is:

$$u_{tt} - u_{xx} + p^2 u = \epsilon(u_t - \frac{1}{3}u_t^3), \quad (31)$$

$$0 < x < \pi;\ t > 0,\ u(0,t) = 0,\ t > 0, \quad (32)$$

$$u_x(x,t) = -E\ (mu_{tt} + \gamma u + \alpha u_t),\ x = \pi,\ t > 0 \quad (33)$$

$$u(x,0) = \varphi(x),\ 0 < x < \pi \quad (34)$$

$$u_t(x,0) = \psi(x),\ 0 < x < \pi \quad (35)$$

As we did the function u (x, t) is supposed to be a function of x, t and r where r = Et. For that reason we put u (x, t) = v (x, t, r). After expanding v (x, t, r) into a formal power series in E as in (12) and after substituting this into the equations (31)-(35) and after equating coefficients of like power in E, it follows that the solution for v_0 is given by:

$$v_0(x,t,T) = \sum_{n=0}^{\infty} (A_n(T)\cos(\sqrt{\lambda_n} t)) + B_n(T)\sin(\sqrt{\lambda_n} t)\sin(\tfrac{1}{2}+n)x \quad (36)$$

moment, however, we are not interested in the higher order approximations. For that reason we will take $D_n(\mathbf{r}) = D_n(0)$ and $E_n(\mathbf{r}) = E_n(0)$. It is shown (VanHorssen, 2003) where, $\lambda_n = p^2+(n+1/2)^2$ is an eigenvalue. $A_n(\mathbf{r})$ and $B_n(\mathbf{r})$ will be determined to avoid secular terms in v_1. The function v_1 should satisfy:

$$v|_{e} - v|_{\phi} + p2v = vd: -2vm - \frac{1}{3}v! \quad (37)$$

0xx-cn.tc-0

$$v_{,}(0,t l:)=0, t>0 \; v_{|,}(\pi,t;t)=- \quad (38)$$

$$(mvett + yv_{o} + av'') \quad (39)$$

x=π,t>0

$$v_{l}(x,0,0)=0, 0<x<lt \quad (40)$$

$$v_{l|}(x,0,0)=v''(x,0,0), 0<x<lt \quad (41)$$

In order to solve problem (37-41) we make the boundary conditions (39) homogeneous. For that purpose we define the following transformation:

$$v(x,t,l:)=v_{l}(x,t,l:)+x(mv_{,.}+yv_{o}+av''') \quad (42)$$

Substituting (42) into (37-41) and putting:

$$v(x,t:t)=\sum_{n=0} vn(t,l:)\sin(\frac{1}{2}+n)x \quad (43)$$

we obtain the following equation for vnCt, l:):

$$\begin{aligned} &vn, +llnvn=n = li \; H)' (Ak -p')(Ck cai(./A:t) \\ &+D_k\sin(\sqrt{\lambda_k}t)+\sqrt{\lambda_n}[2A'_n(\tau)-A_n(\tau)]\sin(\sqrt{\lambda_k}t) \\ &+\sqrt{\lambda_n}[B_n(\tau)-2B_n(\tau)]\cos(\sqrt{\lambda_k}t) \\ &-\frac{1}{4}\left(\sum_{k+l-m=n}^{} - \sum_{k-l-m-l=n}^{\infty} \sum_{k+l+m+l=n}^{\infty}\right)H_kH_lH_m \end{aligned} \quad (44)$$

where:

$$H_n=\sqrt{\lambda_n}\left(-A_n(\tau)\sin(\sqrt{\lambda_n}t)+B_n(\tau)\cos(\sqrt{\lambda_n}t)\right)$$

where:

$$C_n=m\lambda_nA_n-\gamma A_n-\alpha\sqrt{\lambda_n}B_n \text{ and } D_n=m\lambda_nB_n-\gamma B_n+\alpha\sqrt{\lambda_n}A_n$$

In order to avoid secular terms we have to take the coefficients of sin ..pj and cos ..pj in the right - hand side of (44) to be equal to zero. This will give us equations for AJ-i;) and BJl:). It can also be shown that in order to determine the approximation u, of the solution completely.

k+l+nr=n.or k-lm-l=n,
ork+l+m+l=nand
±A,=A-A+Jf::,or
±A,=A-A-Jf::,or
A=..P::+Jr:-S: (45)

To solve these equations we use a similar technique to the one used in Van Horssen (1988). By substituting n = k + l - morn= k -1 - morn= k + l + m + l into (45) and then squaring the equations with the square roots twice and after rearranging terms and using some algebraic manipulations we find that secular terms in the last term of (44) can only occur (fork, m, l and n in Z' and $p^2 > 0$) if

- ,[i;.=A+Ji:,-A, and k+l-m. In this case the solution of the equation is give by l = m and n = l
- ./(=A-A-A, and k-r l= me n In this case the soltion of the equation is give by 1 = m and n=k
- J'f:.=-A+A+A, and k+Lvm=n: In this case the soltion of the equation is give by k = m and n = l

Byputting: ./(An(T)=Rn(T)cos(_φn(T))and ./(Bn(T)Rn(T) sin (φn(T)) secular terms in v_l can be avoided if Rn (l:) and Φn (l:) satisfy:

$$R'_n(\tau)=\frac{R_n(\tau)}{2}\left(1-\frac{2}{\pi}\alpha+\frac{1}{16}R_n^2-\frac{1}{4}\sum_{m=0}^{\infty}R_m^2\right) \quad (46)$$

and

$$\varphi'_n(T)=\_l\_[\phi \; ff''_n - A] \quad (47)$$

rt H"V

for n = 0, 1, 2.,...,

From equation (46) it follows that if we start with zero initial energy in the th mode (that is, An (0) = B, (0) = Rn (0) = 0) c then there will be no energy present up to O (E) for 0<l:<O(1). In this case we say the coupling between the modes is of O (E). This allows us to truncate to those modes which have non-zero initial energy. As example we will consider Eq. 46 for two modes only by assuming R. (0) = 0 for nz Z. The equations for R_0 and R_l are given by:

We have to determine the secular terms in equation by solving the Diophantine-like equation:

$$R'_0(T) = \frac{R_0(T)}{2}\left(1 - \frac{3}{\pi}a - \frac{\text{[illegible]}}{16}R_0^2 - \frac{1}{4}R_1^2\right) \qquad (48)$$

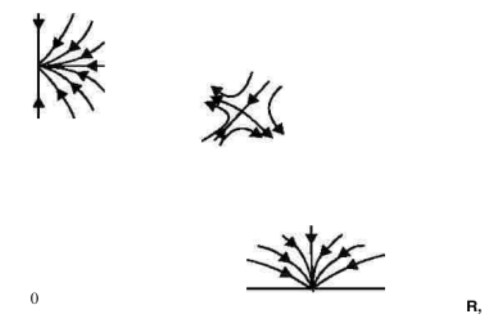

Fig. 1: Phase plane for $0 < \alpha < \pi/2$

Table 1: The Behaviour of the critical points

	Critical point	Behaviour
$0<a<\frac{\pi}{2}$	cp_1	Unstable node
	cp_2	Stable node
	cp_3	Stable node
	cp_4	Saddle point
$a<\frac{\pi}{2}$	cp_1	Stable node

$$R'_1(\tau)=\frac{R_1(\tau)}{2}\left(1-\frac{2}{\pi}\alpha-\frac{1}{4}R_0^2-\frac{3}{16}R_1^2\right) \qquad (49)$$

The critical points of the equations (48) and (49) are $CP_1 = (0, 0)$, $cp_2 = \{\blacklozenge\blacklozenge;(\Pi-2a),0\}$ $cp_3 = (0, \blacklozenge\blacklozenge;(\pi - 2a))$ and $cp_4 = (*\blacklozenge;(\Pi-2a), *\blacklozenge;(\Pi-2a))$ for $0<\alpha<\pi/2$ and for $\alpha \geq \pi/2$ the only critical point is (0, 0). By linearizing the equations (48) and (49) around the critical points for $0<\alpha<\pi/2$ we obtain two stable nodes, one unstable node and one saddle and for $\alpha>\pi/2$ the critical point is a stable node (Table 1).

From the table we can see that if α (the damping coefficient) is increased then all critical points will move to the stable node. The behaviour of the solutions of the equation (48 and 49) locally can be shown in Fig. 1 and 2. To see the qualitative behaviour of the solution we implement the numerical continuation package DSTOOL on the R_0-R_1 plane and by taking $\alpha = 0.5$ the result can be seen in Fig. 3.

Also for more general initial values we can show that u tends to zero for $\alpha > \pi/2$ So far we have shown that it is possible to construct secular free approximations $v_0 + \epsilon v_1$

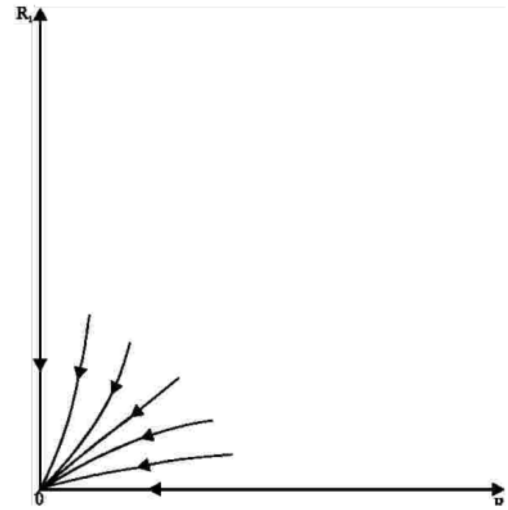

Fig. 2: Phase plane for $\alpha > \pi/2$

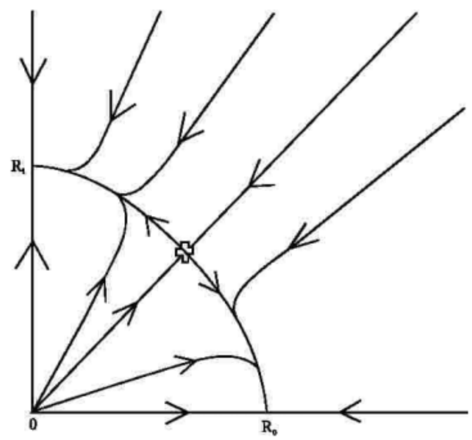

Fig. 3: Qualitative behaviour of the solution of system (48) and (49) on the R_0-R_1 plane for $\alpha = 0.5$

and v_ϵ of the exact solution u of the initial-boundary value problem (31-35).

CONCLUSIONS

In the first part of this study an initial - boundary value problem for a weakly damped string has been considered. It can be shown that (using a semigroup approach) the initial-boundary value problem (1-5) is well-posed for $0 \le x \le 1$ and $t \ge 0$. Although the problem in this part is linear, the construction of the approximation is far from being elementary. For instance it is not possible to solve (17-21) in the classical way by making the boundary condition at $x = 1$ homogeneous. This is due to the non-classical boundary condition at $x = 1$. It can only be done by balancing or matching the right-hand side of (17) and that of (19) by transforming u in an appropriate way. It also should be noted that the way to solve the wave equation with a non-classical boundary condition (using the eigenfunction expansion is an extension of the classical way to solve such problem. In the second part of this study we considered an initial-boundary value problem for a weakly nonlinear wave equation with a non-classical boundary condition. We have constructed formal approximations of order ε. It has been showed that for all values of $p^2>0$ mode interactions of O (1) occur only between modes with non-zero initial energy. In this case we say the coupling between the modes is of O (ε) and truncation is allowed, restricted to those modes that have non-zero initial energy. It has been showed that for large values of t the system will oscillate in only one mode up to O (ε) It has also been showed that for a sufficiently large value of the damping parameter α all solutions tend to zero. If the term u, in the boundary condition at $x = \pi$ is proposed, it can be shown that this term gives rise to a singularly perturbed problem. Six scalings (four time scales and two space scales) are necessary to describe the behaviour of the solution correctly for large values of t. It can also be shown that the problem is well-posed for all $t > 0$. It refers to Darmawijoyo (2010).

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