Mathematical Modelling on information Service Provider Based-Independent Goods Utility Function

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Abstract. This research analysis the internet pricing scheme model to maximize profits for ISPs (Internet Service Providers) by considering the utility function of Independent Goods. This study analyzes the types of flat-fee, usage-based, and two-part tariff pricing schemes for homogeneous consumers and heterogeneous consumers (high-end and low-end) and heterogeneous consumers (high-demand and low-demand). The pricing scheme model is then solved differentially by being applied to the local server data, SISFO traffic data. The results obtained are that ISPs will get maximum profit by implementing flat-fee, usage-based, and two-part tariff pricing schemes for homogeneous consumers. For high-end and low-end heterogeneous consumer types and high-demand and low-demand heterogeneous consumer types, the ISP will get maximum profit on the application of flat-fee, usage-based, and two-part tariff pricing schemes. However, ISPs will get the highest profit on homogeneous consumer types compared to the application of high-end and low-end heterogeneous consumer types and high-demand and low-demand heterogeneous consumers

INTRODUCTION

The increasing development of information technology today has made the internet a necessity whose use has become unlimited. The use of the internet that is not limited by distance, space, and time is a means for the community to meet needs in the field of information and communication technology [1]. This development was also accompanied by the ease of access received by each person in reaching the internet. To achieve ease of accessing the internet, users can use internet services that are currently widely available [2].

An internet service provider or Internet Service Provider (ISP) [3]-[4] is a company that provides internet services for the public, where the ISP will connect consumers to the internet network [5]. In this case, ISPs are challenged to provide the best Quality of Service (QoS)[6]-[10] at an appropriate price for consumers to meet customer satisfaction.

According to Li, et al. [11] , meeting the level of customer satisfaction based on the decision - making method by taking into account the factors of the provider and the consumer can be done by considering the use of the right utility function [12]. The level of customer satisfaction with service performance, in this case, can be a benchmark for ISPs in improving the quality of service for their users to achieve maximum profit. The utility function itself has been widely applied [13]-16]. In the research of Wu and Banker [17] who previously analyzed internet pricing schemes with the modified Cobb-Douglas utility function and Indrawati et al [18], to produce a utility function that is profitable for their ISPs, they used three internet pricing schemes, namely flat fee, usage based, and two part tariff [17][19]-[20].

In this study, the author will apply the utility function of Independent Goods to internet pricing schemes to produce a theory of pricing plans that are expected to maximize profits for ISPs. The research gap identified is due to the disadvantage of recent research in lack of detail to explain the phenomenon when the utility function involved is other variety of utility functions existed. Various utility functions that are very useful to develop the satisfaction of consumers based on their goal of preference. That is why this research is worth to be attempted by utilizing other forms of utility function namely independent goods utility function [21] which has a demand cross elasticity of zero. This utility function will then be applied to three pricing schemes for information services [12] that are a flat fee, usage based, and two part tariff [17][22] to find more optimal price and maximum profit for ISP. Therefore, this research novelty is due to the development of independent goods utility function in information service pricing scheme analytically using the differential form to design models to be utilized by ISP. The lemmas obtained will be a new point of view in designing the models dealing with an optimization problem. Finally, the research contribution will be new lemmas dealing with the pricing scheme set up to satisfy consumers [22][23] and gain benefit for ISP [24] [25]. This research will be carried out differentially regarding about homogeneous and heterogeneous of high-end and low-end and heterogeneous of high-demand and low-demand consumers.

RESEARCH METHOD

The steps conducted in this research are as follows:

1. Defining variable decisions and parameters based on the function of the utility Independent Goods on homogeneous and heterogeneous of high-end and low-end and heterogeneous of high-demand and low-demand consumers
2. Analyzing in analytic functions of a utility Independent Goods in three schemes of pricing the internet which is distinguished by the type of homogeneous and heterogeneous of high-end and low-end and heterogeneous of high-demand and low-demand consumers
3. Testing models of the scheme of pricing the internet in step 3 by using traffic data Sisfo which consists of inbound and outbound obtained from the local server data (January 4, 2021-January 31, 2021).
4. Comparing and concluding about the optimal internet pricing scheme model based on Step 3.

RESULT AND DISCUSSION

The study is intended to maximize profits by using a scheme of pricing flat-fee, usage-based and two-part tariffs for homogeneous consumers and heterogeneous consumers.

The general form of the Independent Goods utility function [26] is$(A, B) = A^x B^{1-x}$ with x > 0.

3.1 Homogeneous Consumer

Consumer Optimization Problems will be as follows:

$$\max_{A,B,C} A^x B^{(1-x)} - H_A A - H_B B - HC \tag{1}$$

Subject to

$$A \le \bar{A}C \tag{2}$$

$$B \le \bar{B}C \tag{3}$$

$$A^x B^{(1-x)} - H_A A - H_B B - HC \ \ge 0 \tag{4}$$

$$C = 0 \text{ or } 1 \tag{5}$$

With Eq.(1) is intended for ISP to maximize its revenue subject to Eq.(2)-(5) that state the conditions that should be fulfilled by the objective function.

Optimization Problems of the providers will be as follow:

$$\underset{H,H_A,H_B}{\text{Max}} \sum_i (H_A A^* + H_B B^* + HC^*) \tag{6}$$

With $(A^*, B^*, C^*) = \arg\max A^x\, B^{(1-X)} - H_A A - H_B B - HC$

Subject to (2) – (5),

Eq.(6) states that the consumers want to maximize their satisfaction, subject to constraints stated in Eq.(2)-Eq.(5).

Next, the setup for the model analytically is stated as follows.

Case 1. For service providers who choose to use the flat-fee pricing scheme, it will be determined that $H_A = 0, H_B = 0$, and $H > 0$, then the optimization of consumer problems in the flat-fee pricing scheme is as follows:

$$\begin{aligned}
&\underset{A,B,C}{\text{Max}}\, A^x\, B^{(1-x)} - H_A A - H_B B - HC \\
&= \underset{A,B,C}{\text{Max}}\, A^x\, B^{(1-x)} - (0)A - (0)B - H(1) \\
&= \underset{A,B,C}{\text{Max}}\, A^x\, B^{(1-x)} - H
\end{aligned}$$

By using Eq.(4), then:

$$\begin{aligned}
&A^x B^{(1-x)} - H_A A - H_B B - HC \ \geq 0 \\
&\Leftrightarrow H \leq A^x B^{(1-x)}
\end{aligned}$$

Optimization of provider became:

$$\begin{aligned}
&\underset{H,H_A,H_B}{\text{Max}} \sum_i (H_A A^* + H_B B^* + HC^*) \\
&= \underset{H,H_A,H_B}{\text{Max}} \sum_i ((0)A^* + (0)B^* + H(1)) \\
&= \underset{H,H_A,H_B}{\text{Max}} \sum_i ((0)A^* + (0)B^* + A^x B^{(1-x)}(1)) \\
&= \underset{H,H_A,H_B}{\text{Max}} \sum_i (\, A^x B^{(1-x)})
\end{aligned}$$

By providing this price, the level of consumer usage will be maximum at $A = \bar{A}$ and $B = \bar{B}$ with maximum utility, so that ISPs can provide prices $A^x B^{(1-x)}$ to consumers. So with flat-fee ISPs can provide maximum prices for consumers$A^x B^{(1-x)}$ namely with optimal profits $\sum_i [A^x B^{(1-x)}]$. Based on this case, we get Lemma 1.

Lemma 1: If the ISP chooses a flat-fee pricing scheme, then it will get a price that is $A^x B^{(1-x)}$ for consumers with optimal benefits as $\sum_i [A^x B^{(1-x)}]$.

Case 2. For providers of services who choose to use the scheme of pricing the usage-based then it will be determined $H_A > 0, H_B > 0$, and $H = 0$, where will be given differences in prices for the consumer at the time of hour is busy and hours are not busy. Optimization problems consumers on usage-based pricing scheme are:

$$\underset{A,B,C}{\text{Max}}\, A^x\, B^{(1-x)} - H_A A - H_B B \tag{7}$$

To maximize the price, differentiation A and B will be carried out in Eq. (7) as follows:

$$\frac{\partial (A^x B^{(1-x)} - H_A A - H_B B)}{\partial A} = 0,$$

$$\text{Then} \quad x A^{x-1} B^{1-x} = H_A \tag{8}$$

$$\Leftrightarrow A^{x-1} = \frac{H_A}{x B^{1-x}} \Leftrightarrow A^* = \left(\frac{H_A}{x B^{1-x}}\right)^{\frac{1}{x-1}}$$

and

$$\frac{\partial(A^x B^{(1-x)} - H_A A - H_B B)}{\partial B} = 0,$$

Then $(1-x)A^x B^{-x} = H_B$ the 9)

$$\Leftrightarrow B^{-x} = \frac{H_B}{(1-x)A^x} \Leftrightarrow B^* = \left(\frac{H_B}{(1-x)A^x}\right)^{\frac{1}{-x}}$$

Optimization Problems of the providers will be as follow:

$$\underset{H,H_A,H_B}{\text{Max}} \sum_i (H_A A^* + H_B B^*)$$

$$= \sum_i \left[H_A \left(\frac{H_A}{xB^{1-x}}\right)^{\frac{1}{x-1}} + H_B \left(\frac{H_B}{(1-x)A^x}\right)^{\frac{1}{-x}} \right]$$

$$= \sum_i \left[\frac{H_A{}^{(1+\frac{1}{x-1})}}{x^{\frac{1}{x-1}}B^{-1}} + \frac{H_B{}^{(1+\frac{1}{-x})}}{(1-x)^{\frac{1}{-x}}A^{-1}} \right]$$

$$= \sum_i \left[\frac{(xA^{x-1}B^{1-x})^{(1+\frac{1}{x-1})}}{x^{\frac{1}{x-1}}B^{-1}} + \frac{((1-x)A^x B^{-x})^{(1+\frac{1}{-x})}}{(1-x)^{\frac{1}{-x}}A^{-1}} \right]$$

$$= \sum_i \left[A^x B^{(1-x)} \right]$$

To get the maximum profit, the service provider must minimize the value of H_A and H_B. If the service provider places restrictions on H_A and H_B, then $A^* = \bar{A}$ dan $B^* = \bar{B}$. With restriction the providers service will generate H_A and H_B optimized, namely $H_A = xA^{x-1}B^{1-x}$ and $H_B = (1-x)A^x B^{-x}$ with optimal profit $\sum_i [A^x B^{(1-x)}]$. Based on this case, Lemma 2 was obtained.

Lemma 2: If the ISP chooses a usage-based pricing scheme, it will get a price that is $H_A = xA^{x-1}B^{1-x}$ and $H_B = (1-x)A^x B^{-x}$ for consumers with optimal benefits $\sum_i [A^x B^{(1-x)}]$.

Case 3. For service providers who choose to use a two-part tariff pricing scheme, it will be determined $H_A > 0, H_B > 0$, and $H > 0$.

By using equation (8)-(9) are substituted into the equation (4) then optimization problem of consumers in the scheme of pricing of a two-part tariff is as follows:

$$A^x B^{(1-x)} - H_A A - H_B B - HC \geq 0$$

$$\Leftrightarrow A^x B^{(1-x)} - xA^x B^{1-x} - (1-x)A^x B^{1-x} - H \geq 0$$

$$\Leftrightarrow H \leq A^x B^{(1-x)} - xA^x B^{1-x} - (1-x)A^x B^{1-x}$$

Optimization problems of the providers will be as follow:

$$\underset{H,H_A,H_B}{\text{Max}} \sum_i (H_A A^* + H_B B^* + HC^*)$$

$$= \underset{H,H_A,H_B}{\text{Max}} \sum_i \left[H_A \left(\frac{H_A}{xB^{1-x}}\right)^{\frac{1}{x-1}} + H_B \left(\frac{H_B}{(1-x)A^x}\right)^{\frac{1}{-x}} + A^x B^{(1-x)} - xA^x B^{1-x} - (1-x)A^x B^{1-x}) \right]$$

$$= \sum_i \left[\frac{H_A{}^{(1+\frac{1}{x-1})}}{x^{\frac{1}{x-1}}B^{-1}} + \frac{H_B{}^{(1+\frac{1}{-x})}}{(1-x)^{\frac{1}{-x}}A^{-1}} + A^x B^{(1-x)} - xA^x B^{1-x} - (1-x)A^x B^{1-x} \right]$$

$$= \sum_i \left[\frac{(xA^{x-1}B^{1-x})^{\left(1+\frac{1}{x-1}\right)}}{x^{\frac{1}{x-1}}B^{-1}} + \frac{((1-x)A^x B^{-x})^{\left(1+\frac{1}{-x}\right)}}{(1-x)^{\frac{1}{-x}}A^{-1}} + A^x B^{(1-x)} - xA^x B^{1-x} - (1-x)A^x B^{1-x} \right]$$

$= \sum_i \left[xA^x B^{(1-x)} + (1-x)A^x B^{(1-x)} + A^x B^{(1-x)} - xA^x B^{1-x} - (1-x)A^x B^{1-x}\right]$
$= \sum_i \left[A^x B^{(1-x)}\right]$

Such restrictions were done by provider service and therefore will generate optimal H_A and H_B, then $H_A = xA^{x-1}B^{1-x}$ and $H_B = (1-x)A^x B^{-x}$ with maximum profit $\sum_i [A^x B^{(1-x)}]$. Based on this case, the obtained Lemma 3.

Lemma 3: If the ISP uses a two-part tariff pricing scheme, the best price will be $H_A = xA^{x-1}B^{1-x}$ and $H_B = (1-x)A^x B^{-x}$ for consumers with optimal profit $\sum_i [A^x B^{(1-x)}]$.

3.2 High-end and Low-end Heterogeneous Consumers

Heterogeneous consumers are assumed to be high-end consumers (type 1) and n low-end consumers (type 2). The willingness of heterogeneous consumers in paying for pricing schemes is a consideration for service providers in providing services, therefore heterogeneous consumers have an upper limit of A in peak hours and B in peak hours, $x_1 > x_2$ and $y_1 > y_2$.

Optimization problem of consumers:

$$\underset{A_i, B_i, C_i}{\text{Max}} \; {A_i}^x {B_i}^{(1-x)} - H_A A_i - H_B B_i - HC_i \tag{10}$$

Subject to:

$$A_i \leq \overline{A_\iota} C_i \tag{11}$$
$$B_i \leq \overline{B_\iota} C_i \tag{12}$$
$$ {A_i}^{x_i} {B_i}^{(1-x_i)} - H_A A_i - H_B B_i - HC_i \geq 0 \tag{13}$$
$$C_i = 0 \text{ or } 1 \tag{14}$$

Eq.(10) stated tht ISP wnts to maximize their revenue in dealing with high-end and low-end consumers subject to some restrictins imposed in Eq.)11)-(14)

Optimization problems of providers:

$$\underset{H, H_A, H_B}{\text{Max}} \; m(H_A A_1^* + H_B B_1^* + HC_1^*) + n(H_A A_2^* + H_B B_2^* + HC_2^*) \tag{15}$$

with

$$(A_1^*, B_1^*, C_1^*) = argmax \; {A_i}^x {B_i}^{(1-x)} - H_A A_i - H_B B_i - HC_i$$

Subject to (11)-(14). Eq. (15) explain the maximum satisfaction would like to be achieved by the consumers subject to some conditions that have to be satisfied, stated in Eq. (11)-(14).

Case 4. For service providers who choose to use a flat-fee pricing scheme, it will be set $H_A = 0, H_B = 0$ and $H > 0$ where the price set by the ISP does not affect the time of use. This means that those who join will get the maximum consumption rate at $A_1 = \bar{A}$, $A_2 = \bar{A}$ and $B_1 = \bar{B}$, $B_2 = \bar{B}$ for its level of satisfaction. The price that isp charges to obtain the maximum level of satisfaction of high–end consumers is $H \leq \bar{A}^{x_1}\bar{B}^{1-x_1}$ and low-end consumers is $H \leq \bar{A}^{x_2}\bar{B}^{1-x_2}$. With high-end cost determinations following low-end costs, it is assumed that $(m)\bar{A}^{x_1}\bar{B}^{1-x_1} \leq (m+n)\bar{A}^{x_2}\bar{B}^{1-x_2}$. This means that ISPs get the maximum benefit when using the $H = \bar{A}^{x_2}\bar{B}^{1-x_2}$.

Optimization Problems of Providers:

$\underset{H}{\text{Max}} \; m(HC_1^*) + n(HC_2^*)$
$= m(\bar{A}^{x_2}\bar{B}^{1-x_2}) + n(\bar{A}^{x_2}\bar{B}^{1-x_2})$
$= (m+n)[\bar{A}^{x_2}\bar{B}^{1-x_2}]$

Thus, the maximum benefit of ISPs with *flat-fee* is $(m+n)[\bar{A}^{x_2}\bar{B}^{1-x_2}]$, with m the number of high-end consumers and n the number of low-end consumers. Based on this case obtained Lemma 4.

Lemma 4: If the ISP chooses a flat-fee pricing scheme, the price is obtained $\bar{A}^{x_2}\bar{B}^{1-x_2}$ for consumers with optimum profit $(m+n)[\bar{A}^{x_2}\bar{B}^{1-x_2}]$.

Case 5. For service providers who choose to use usage-based pricing schemes, it will be set as $H_A > 0, H_B > 0$ dan $H = 0$. Therefore, optimization of high-end heterogeneous consumer problems is as follows:

$$\underset{A,B,C}{\text{Max}}\, {A_1}^{a_1}{B_1}^{(1-a_1)} - H_A A_1 - H_B B_1 - HC_1 \tag{16}$$

To maximize the price, differentiation A and B will be done in the Eq. (16) as follows:

$$\begin{aligned}&\frac{\partial({A_1}^{x_1}{B_1}^{(1-x_1)} - H_A A_1 - H_B B_1)}{\partial A_1} = 0,\\ &\text{then } x_1 {A_1}^{x_1-1}\,{B_1}^{(1-x_1)} = H_x\\ &\Leftrightarrow {A_1}^{x_1-1} = \frac{H_A}{x_1 {B_1}^{(1-x_1)}}\\ &\Leftrightarrow {A_1}^{*} = \left(\frac{H_A}{x_1 {B_1}^{(1-x_1)}}\right)^{\frac{1}{x_1-1}}\end{aligned} \tag{17}$$

Where Eq. (17) and Eq. (18) states the process to find the local optimal toward its one of variable and

$$\begin{aligned}&\frac{\partial({A_1}^{x_1}{B_1}^{(1-x_1)} - H_A A_1 - H_B B_1)}{\partial A_1} = 0,\\ &\text{then } (1-x_1){A_1}^{x_1}{B_1}^{-x_1} = H_B\\ &\Leftrightarrow {B_1}^{-x_1} = \frac{H_B}{(1-x_1){A_1}^{x_1}}\\ &\Leftrightarrow {B_1}^{*} = \left(\frac{H_B}{(1-x_1){A_1}^{x_1}}\right)^{-\frac{1}{x_1}}\end{aligned} \tag{18}$$

Optimization Problems of Providers:

$$\underset{A,B,C}{\text{Max}}\, {A_2}^{a_2}{B_2}^{(1-a_2)} - H_A A_2 - H_B B_2 \tag{19}$$

To maximize the price in Eq. (19) differentiation is carried out as in Eq.(20)-(21) follows:

$$\begin{aligned}&\frac{\partial({A_1}^{x_2}{B_1}^{(1-x_2)} - H_A A_2 - H_B B_2)}{\partial A_2} = 0,\\ &\text{Then } x_2 {A_2}^{(x_2-1)}\,{B_1}^{(1-x_2)} = H_A\\ &\Leftrightarrow {A_2}^{x_2-1} = \frac{H_2}{x_2 {B_2}^{(1-x_2)}} \Leftrightarrow {A_2}^{*} = \left(\frac{H_2}{x_2 {B_2}^{(1-x_2)}}\right)^{\frac{1}{x_2-1}}\end{aligned} \tag{20}$$

and

$$\begin{aligned}&\frac{\partial({A_2}^{x_2}{B_2}^{(1-x_2)} - H_A A_2 - H_B B_2)}{\partial A_2} = 0,\\ &\text{then } (1-x_2){A_2}^{x_2}{B_2}^{(-x_2)} = H_B\\ &\Leftrightarrow {B_2}^{-x_2} = \frac{H_B}{(1-x_2){A_2}^{x_2}} \Leftrightarrow {B_2}^{*} = \left(\frac{H_B}{(1-x_2){A_2}^{x_2}}\right)^{-\frac{1}{x_2}}\end{aligned} \tag{21}$$

Optimization Problems of Providers:

$$
\begin{aligned}
&\max_{P_X,P_Y} m(H_A A_1{}^* + H_B B_1{}^*) + n(H_A A_2{}^* + H_B B_2{}^*) \\
&\quad = \max_{H_A,H_B}(m+n)\left[H_A\left(\frac{H_A{}^{(\frac{1}{x_2-1})}}{x_2{}^{(\frac{1}{x_2-1})}B_2{}^{(\frac{1-x_2}{x_2-1})}}\right) + H_B\left(\frac{H_B{}^{(-\frac{1}{x_2})}}{(1-x_2)^{(-\frac{1}{x_2})}A_2{}^{(-\frac{x}{x})}}\right)\right] \\
&\quad = \max_{H_A,H_B}(m+n)\left[\left(\frac{H_A{}^{(1+\frac{1}{x_2-1})}}{x_2{}^{(\frac{1}{x_2-1})}B_2{}^{(\frac{1-x_2}{x_2-1})}}\right) + \left(\frac{H_B{}^{(1-\frac{1}{x_2})}}{(1-x_2)^{(-\frac{1}{x_2})}A_2{}^{(-1)}}\right)\right] \\
&\quad = \max_{H_A,H_B}(m+n)\left[\left(\frac{(x_2A_2{}^{(x_2-1)}B_2{}^{(1-x_2)})^{(1+\frac{1}{x_2-1})}}{x_2{}^{(\frac{1}{x_2-1})}B_2{}^{(\frac{1-x_2}{x_2-1})}}\right) + \left(\frac{((1-x_2)A_2{}^{x_2}B_2{}^{-x_2})^{(1-\frac{1}{x_2})}}{(1-x_2)^{(-\frac{1}{x_2})}A_2{}^{(-1)}}\right)\right] \\
&\quad = \max_{H_A,H_B}(m+n)\left[\, x_2A_2{}^{x_2}\, B_2{}^{(1-x_2)} + (1-x_2)A_2{}^{x_2}B_2{}^{(1-x_2)}\right] \\
&\quad = \max_{H_A,H_B}(m+n)\left[A_2{}^{x_2}B_2{}^{(1-x_2)}\right]
\end{aligned}
$$

With this, the maximum profit of ISPs with usage based is $(m+n)\left[\, A_2{}^{x_2}\, B_2{}^{(1-x_2)}\right]$, with m the number of high-end consumers and n the number of low-end consumers. Based on this case obtained Lemma 5.

Lemma 5: If the ISP chooses a usage-based pricing scheme, then the minimum price is obtained $H_A = x_2A_2{}^{(x_2-1)}B_2{}^{(1-x_2)}$ and $H_B = (1-x_2)A_2{}^{x_2}B_2{}^{x_2}$. For consumers with optimal profit $(m+n)\left[\, A_2{}^{x_2}\, B_2{}^{(1-x_2)}\right]$.

Case 6. For service providers who choose to use a two-part tariff pricing scheme, it will be set $H_A > 0, H_B > 0$, and $H > 0$. To maximize the price of consumer demand, it is set $x_1 > x_2$, so that $a_1(m) < a_2(m+n) \Leftrightarrow a_1 < \frac{a_2(m+n)}{m}$. This allows the ISP to set prices on $H_A = x_2A_2{}^{(x_2-1)}B_2{}^{(1-x_2)}$, $H_B = (1-x_2)A_2{}^{x_2}B_2{}^{x_2}$ and $H = A_1{}^{x_1}B_1{}^{(1-x_1)} - A_2{}^{x_2}\, B_2{}^{(1-x_2)}$.
Optimization problems of Providers:

$$
\begin{aligned}
&\operatorname*{Max}_{P_X,P_Y} m(H_A A_1{}^* + H_B B_1{}^* + HC_1{}^*) + n(H_A A_2{}^* + H_B B_2{}^* + HC_2{}^*) \\
&= \max_{H_A,H_B}(m+n)\left[A_2{}^{x_2}B_2{}^{(1-x_2)} + A_2{}^{x_2}B_2{}^{(1-x_2)} - \; A_2{}^{x_2}\, B_2{}^{(1-x_2)}\right] \\
&= \max_{H_A,H_B}(m+n)\left[A_2{}^{x_2}B_2{}^{(1-x_2)}\right]
\end{aligned}
$$

With this, the maximum pricing of ISPs with two-part tariff is $(m+n)\left[\, A_2{}^{x_2}\, B_2{}^{(1-x_2)}\right]$. Based on this case obtained Lemma 6.

Lemma 6: If the ISP chooses a two part tariff pricing scheme, then the ISP can provide a minimum price for consumers $H_A = x_2A_2{}^{(x_2-1)}B_2{}^{(1-x_2)}$ and $H_B = (1-x_2)A_2{}^{x_2}B_2{}^{x_2}$, with optimal profit $(m+n)\left[\, A_2{}^{x_2}\, B_2{}^{(1-x_2)}\right]$.

3.3 Heterogeneous High-demand and Low-demand Consumers

For heterogeneous high-demand and low-demand consumers, it is assumed that there are two types of consumers based on their consumption level. However, because ISPs are considered unable to distinguish, for three lemmas in high-demand and low-demand heterogeneous consumers will use a case proof similar to high-end and low-end heterogeneous consumers. Based on this, the following results are obtained:

Lemma 7: If the ISP chooses a flat-fee pricing scheme, it is obtained $H = \bar{A}^{x_2}\bar{B}^{1-x_2}$ with optimal profit $(m+n)[\bar{A}^{x_2}\bar{B}^{1-x_2}]$.

Lemma 8: If the ISP chooses a usage-based pricing scheme, then the optimal price is obtained $H_A = x_2 A_2^{(x_2-1)} B_2^{(1-x_2)}$ during peak hours and $H_B = (1-x_2) A_2^{x_2} B_2^{x_2}$ at peak hours with optimal profit $(m+n)\left[A_2^{x_2} B_2^{(1-x_2)} \right]$.

Lemma 9: If the ISP chooses a two part tariff pricing scheme, it is obtained $H_A = x_2 A_2^{(x_2-1)} B_2^{(1-x_2)}$, $H_B = (1-x_2) A_2^{x_2} B_2^{x_2}$ and $H = A_1^{x_1} B_1^{(1-x_1)} - A_2^{x_2} B_2^{(1-x_2)}$ subscription fee that can be used to achieve optimal profit on $(m+n)\left[A_2^{x_2} B_2^{(1-x_2)} \right]$.

Based on the analysis of cases 1 - case 9 to see the most optimal profit in each type of consumer, it will be compared with the evidence using sisfo traffic data. Furthermore, Table 1-3 are parameter values for homogeneous, high-end and low-end heterogeneous, and high-demand and low-demand heterogeneous customers, respectively.

TABLE 1. Parameter Values for Homogeneous Consumers

Parameter	**Pricing Scheme**		
	Flat-fee	**Usage-Based**	**Two-Part Tariff**
x	4	4	4
y	3	3	3
$\bar{A}$	512.79	512.79	512.79
$\bar{B}$	112.90	112.90	112.90

TABLE 2. Parameter Values for High-end and Low-end Heterogeneous Consumers

Parameter	**Pricing Scheme**		
	Flat-fee	**Usage-Based**	**Two-Part Tariff**
x_1	4	4	4
x_2	3	3	3
y_1	3	3	3
y_2	2	2	2
$\bar{A}_1$	512.79	512.79	512.79
$\bar{A}_1$	298.67	298.67	298.67
$\bar{B}_2$	112.90	112.90	112.90
$\bar{B}_2$	109.31	109.31	109.31

TABLE 3. Parameter Values for High-demand and Low-demand Heterogeneous Consumers

Parameter	**Pricing Scheme**		
	Flat-fee	**Usage-Based**	**Two-Part Tariff**
x_1	3	3	3
x_2	3	3	3
y_1	2	2	2
y_2	2	2	2
$\bar{A}_1$	512.79	512.79	512.79
$\bar{A}_1$	298.67	298.67	298.67
$\bar{B}_2$	112.90	112.90	112.90
$\bar{B}_2$	109.31	109.31	109.31

With
$\bar{A} = \bar{A}_1$ is the highest level consumption at peak hours in kbps
$\bar{A}_2$ is the second-highest level consumption at peak hours in kpbs
$\bar{B} = \bar{B}_1$ is the highest level consumption at off-peak hours in kbps
$\bar{B}_2$ is the second-highest level consumption at off-peak hours in kbps

$m = n = 1$ where *m* is the number of high-end or high-demand customers, and *n* is the number of low-end or low-demand customers.

Table 4 shows the recapitulation of three pricing schemes among three kinds of consumers.

TABLE 4. The Comparisons among Three Pricing Schemes

Pricing Strategy	Homogeneous	High-end and Low-end	High-demand and Low-demand
Flat-fee	$\sum_i [A^x B^{(1-x)}]$ $= \sum_i [(512{,}79)^4 (112{,}90)^{1-4}]$ $= \sum_i [48{,}048.116]$	$(m+n)[\bar{A}^{x_2} \bar{B}^{1-x_2}]$ $= (1+1)[(298{,}67)^3 (109{,}31)^{1-3}]$ $= 4{,}459.488$	$(m+n)[\, A_2{}^{x_2}\, B_2{}^{1-x_2}]$ $= (1+1)[(298{,}67)^3 (109{,}31)^{1-3}$ $= 4{,}459.488$
Usage-based	$\sum_i [A^x B^{(1-x)}]$ $= \sum_i [(512{,}79)^4 (112{,}90)^{1-4}]$ $= \sum_i [48{,}048.116]$	$(m+n)[\bar{A}^{x_2} \bar{B}^{1-x_2}]$ $= (1+1)[(298{,}67)^3 (109{,}31)^{1-3}]$ $= 4{,}459.488$	$(m+n)[\, A_2{}^{x_2}\, B_2{}^{1-x_2}]$ $= (1+1)[(298{,}67)^3 (109{,}31)^{1-3}$ $= 4{,}459.488$
Two-part tariff	$\sum_i [A^x B^{(1-x)}]$ $= \sum_i [(512{,}79)^4 (112{,}90)^{1-4}]$ $= \sum_i [48{,}048.116]$	$(m+n)[\bar{A}^{x_2} \bar{B}^{1-x_2}]$ $= (1+1)[(298{,}67)^3 (109{,}31)^{1-3}]$ $= 4{,}459.488$	$(m+n)[\, A_2{}^{x_2}\, B_2{}^{1-x_2}]$ $= (1+1)[(298{,}67)^3 (109{,}31)^{1-3}$ $= 4{,}459.488$

For homogeneous consumer types, the maximum profit obtained in the three pricing schemes is the same, namely 48048.116/kbps, for high-end and low-end consumers, the maximum profit obtained in the three pricing schemes is the same, namely 4459.488/kbps, and for high-end consumers. -demand and low-demand profits obtained in the three pricing schemes are the same, namely 4459.488/kbps.

Based on Table 4, it can be concluded that by using the utility function of independent goods, ISPs will get maximum profit on homogeneous consumer types compared to other types of consumers with a profit of 48048.116/kbps.

CONCLUSION

From the result, it can be concluded that in the application of the utility function of independent goods, the three pricing schemes produce the same optimal profit for each type of consumer. However, ISPs will get the highest profit on homogeneous consumer types compared to the application of high-end and low-end heterogeneous consumer types and high-demand and low-demand heterogeneous consumers.

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