

$$G(s) = C(sI - A)^{-1}B + D$$

$$= [1 \ 0] \left\{ \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \right\}^{-1} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} + D$$

$$= [1 \ 0] \begin{bmatrix} s & -1 \\ \frac{k}{m} & s + \frac{b}{m} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$\begin{bmatrix} s & -1 \\ \frac{k}{m} & s + \frac{b}{m} \end{bmatrix}^{-1} = \frac{1}{s^2 + \frac{b}{m}s + \frac{k}{m}} \begin{bmatrix} s + \frac{b}{m} & 1 \\ -\frac{k}{m} & s \end{bmatrix}$$

$$G(s) = \frac{1}{ms^2 + bs + k}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \cdot u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$y(s) = G(s) \cdot u(s)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \frac{1}{ms^2 + bs + k} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} \frac{1}{m}s^2 + bs + k \cdot u_1 \\ \frac{1}{m}s^2 + bs + k \cdot u_2 \\ \vdots \\ \frac{1}{m}s^2 + bs + k \cdot u_m \end{bmatrix}$$

Ex. 2-3

$$G(s) = C(sI - A)^{-1}B + D$$

$$\frac{Y(s)}{U(s)} = \left(\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} -14 & -56 & -160 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0 \right)$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s+14 & 56 & 160 \\ 1 & 3 & 0 \\ 0 & -1 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \frac{1}{(s^3 + 14s^2 - 160) - (56s)} \begin{bmatrix} s^2 + 14s - 56 & -3 & -1-3 \\ -56s - 160 & s^2 + 14s & s+4 \\ -160s & -160 & s^2 + 14s - 56 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \left[\begin{array}{l} \frac{s^2 + 14s - 56}{s^3 + 14s^2 - 56s - 160} \\ \frac{-56s - 160}{s^3 + 14s^2 - 56s - 160} \\ \frac{-160s}{s^3 + 14s^2 - 56s - 160} \end{array} \right]$$

$$G(s) = \frac{-56s - 160}{s^3 + 14s^2 - 56s - 160}$$