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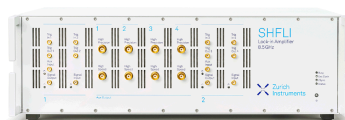
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# Mathematics Teachers' Perception on Mathematical Proof

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**Abstract.** This paper aims to describe mathematics teachers' perception of mathematical proof. Proving in mathematics for the teacher is crucial as the foundation of developing students' skills in generating correct proof to some conjectures which require good mathematical reasoning. It has been highlighted in some works of literature that mathematical reasoning which is closely related to proving in mathematics also contributes to the development of a chosen set of 21st-century skills. A total number 152 of Indonesian in-service mathematics teachers were given a survey concerning their perception of mathematical proving. The description of the result consists of five important categories including the process of mathematical proof, thinking ability in mathematical proof, appropriate school level in learning proof, topic, and understanding in a mathematical proof. It is found that Indonesian in-service mathematics teachers had a positive attitude towards mathematical proof as a teaching approach in their classrooms, as the opportunity for students to improve helped students improve their thinking ability.

## INTRODUCTION

Proving in learning mathematics is one of the skills students need to acquire. It contributes to the development of students' mathematical thinking. Cognitive ability in proving includes the ability to analyze, synthesize, and evaluate, not just memory factual knowledge or simple application of various formulas or principles [1]. Students are expected to be able to reason well and express the results of their reasoning written, systematic, and strict (rigorous). This activity occurs when carrying out the process evidence [1,3]. In mathematics, the proof is a series of logical argument that explains the truth of a statement. These arguments can come from the premise of the statement itself, other theorems, definitions, and finally can be derived from the postulate that the system mathematics is derived. What is meant is logical here, are all the steps on each argument must be justified by step previously. Students must be able to defend whether or not a generalization is correct. As students' progress through school, their understanding of what constitutes a valid and faulty justification should lead to more formal justifications. These formal justifications, as they are referred to, give mathematical arguments for a proposition, which are frequently comparable to proofs or a large amount of evidence. As a component of proofs, a result, prompting reasons is a crucial step in the proof process. The view of what qualifies and disqualifies a justification is shaped by their education. Mathematics teachers play important role in shaping students' mathematics skills in proving. In this paper, a survey is conducted to see the perception of mathematics teachers about proving in mathematics. The explanation on how this perception reflects their success in training students' mathematical skills in proving will be elaborated on later. A highlight of [2] is that making judgments regarding the validity of the information that bombards students by examining their quantitative and logical implications is part of mathematical reasoning (both deductive and inductive). Mathematical reasoning which is closely related to proving in mathematics also contributes to the development of a chosen set of 21st-century skills in this context.

However, some studies reported issues in learning proof in mathematics. A report from [3] find that in classroom mathematics of pre-service teachers, about a third of those misinterpreted proof as purely empirical verification, wrong algebraic operation, and other misleading arguments in presenting proof in geometry subject for grade 5 to 10. Another study also found that students, as well as teachers in mathematics classrooms presented irrelevant and inaccurate arguments orally in proving, also in this study, shows that the written proof cannot be categorized or be associated with arguments that approximate the standard of proof [4]. So the truth of all the premises on every

deduction has been proven or given as an assumption. According to [5], evidence plays a role: 1) to verify that a statement is true, 2) for explaining why a statement is right, 3) to communicate mathematical knowledge, 4) to find or invent new math, or 5) systematize statements into a system axiomatic. Mathematical proofs are used for the following purposes: Prove the results, inform and persuade others, find and place the results. The result is a deduction system and to maintain Generated generalization [1,6]. In the process of proof, first and foremost is the correctness of the assertion. There are some stages of mathematical proving, firstly is the need to check (accuracy phase); Next, explain why the claim is correct (explanatory phase) and the third phase is abstracted by the control of generalization conditions [7]. Generating correct proof to some conjectures require good mathematical reasoning. Mathematical reasoning is defined differently in curriculum documents, which further suggests its importance. It covers some other specific terms including induction, deduction, and adaptive reasoning which highlight the focus on mathematical aspects of an object or event, the conjectures about the object and event, and the inferences based on the relationship between those aspects [8].

## METHOD

This survey involved 152 teachers attending in service certification training. Data were collected using a questionnaire that we conducted. The goal of the survey was to investigate Indonesian in-service mathematics teacher's perception on mathematical proof consisting 5 important categories including : 1) The process of mathematical proof, 2) Thinking ability in mathematical proof, 3) Appropriate School level for learning proof, 4) Topic in mathematical proof, and 5) Understanding on mathematical proof.

### *The process of mathematical proof*

There were 4 items in the survey to measure mathematics teacher's perception on mathematical proof related to the process of mathematical proof ("The proof is the foundation of mathematics"; "The proof is hard be understood"; "The process of mathematical proof is exhausting"; and "A mathematical proof is a sequence of arguments that show the truth of a mathematical statement"). The researchers used a 4-point Likert-type scale to rate these four items from 1 (strongly disagree) to 4 (strongly agree).

### *Thinking ability in mathematical proof*

There were 6 items in the survey to measure mathematics teacher's perception on mathematical proof related to thinking ability in mathematical proof ("Mathematical proof can improve reasoning skills"; "Mathematical proofs can improve understanding of mathematical concepts"; "Mathematical proof is a form of problem solving"; "Mathematical proofs can improve argumentation"; and "Mathematical proof is a high level of logic"; "Mathematical proof is a high order thinking skill"). The researchers used a 4-point Likert-type scale to rate these six items from 1 (strongly disagree) to 4 (strongly agree).

### *Appropriate School level for learning proof*

There were 2 items in the survey to measure mathematics teacher's perception on mathematical proof related to appropriate school level for learning proof ("Mathematical proof is not suitable for junior high school"; "Mathematics proof learning is suitable for high school students") The researchers used a 4-point Likert-type scale to rate these two items from 1 (strongly disagree) to 4 (strongly agree).

### *Topic in mathematical proof*

There were 2 items in the survey to measure mathematics teacher's perception on mathematical proof related to topic in mathematical proof ("Proofs are connected to each other"; "Only geometry is suitable for learning proofs."); . The researchers used a 4-point Likert-type scale to rate these two items from 1 (strongly disagree) to 4 (strongly agree).

### *Understanding on mathematical proof*

There were 2 items in the survey to measure mathematics teacher's perception on mathematical proof related to topic in mathematical proof ("Mathematical proofs are facts that support the truth of a mathematical statement"; "The truth

of a mathematical statement can be demonstrated by some supporting examples.” The researchers used a 4-point Likert-type scale to rate these two items from 1 (strongly disagree) to 4 (strongly agree).

## RESULT AND DISCUSSION

### Result

#### Respondents' Profile

**TABLE 1.** The detail of respondents' profile

| No. | Criteria                      | Result  |
|-----|-------------------------------|---|
| 1.  | Gender                        | Male = 47 teachers<br>Female = 105 teachers   |
| 2.  | Range of Age                  | Minimum = 22 years<br>Maximum = 54 years  |
| 3.  | Teaching Experience           | Minimum = 1 year<br>Maximum = 24 years  |
| 4.  | Level of school in teaching   | 1. Primary school (2 teachers)<br>2. Junior High School (69 teachers)<br>3. Ordinary secondary school (59 teachers)<br>4. Vocational secondary school (22 teachers)   |
| 5.  | Region of school distribution | 1. Aceh ( 4 teachers)<br>2. South Sumatera (51 teachers)<br>3. Lampung (3 teachers)<br>4. Bengkulu (9 teachers)<br>5. Jakarta (6 teachers)<br>6. Others (81 teachers) |

#### The process of mathematical proof

**TABLE 2.** The percentage of responses to questionnaire (%)

| Statement   | Strongly Disagree | Disagree | Agree | Strongly Agree |
|---|-------------------|----------|-------|----------------|
| The proof is the foundation of mathematics  | 0                 | 1        | 27    | 72             |
| The proof is hard to be understood  | 9                 | 39       | 43    | 9              |
| The process of mathematical proof is exhausting   | 14                | 42       | 30    | 14             |
| A mathematical proof is a sequence of arguments that show the truth of a mathematical statement | 0                 | 3        | 31    | 66             |

#### Thinking ability in mathematical proof

**TABLE 3.** The percentage of responses to questionnaire (%)

| Statement  | Strongly Disagree | Disagree | Agree | Strongly Agree |
|--|-------------------|----------|-------|----------------|
| Mathematical proof can improve reasoning skills                        | 0                 | 1        | 19    | 80             |
| Mathematical proofs can improve understanding of mathematical concepts | 0                 | 0        | 17    | 83             |
| Mathematical proof is a form of problemsolving                         | 0                 | 4        | 31    | 65             |
| Mathematical proofs can improve argumentation                          | 0                 | 2        | 35    | 63             |
| Mathematical proof is a high level of logic                            | 1                 | 7        | 31    | 61             |
| Mathematical proof is a high order thinking skill                      | 1                 | 9        | 31    | 59             |

Appropriate School level for learning proof

**TABLE 4.** The percentage of responses to questionnaire (%)

| Statement   | Strongly Disagree | Disagree | Agree | Strongly Agree |
|---|-------------------|----------|-------|----------------|
| Mathematical proof is not suitable for junior high school       | 20                | 51       | 22    | 7              |
| Mathematics proof learning is suitable for high school students | 1                 | 11       | 43    | 45             |

Topic in mathematical proof

**TABLE 5.** The percentage of responses to questionnaire (%)

| Statement                                     | Strongly Disagree | Disagree | Agree | Strongly Agree |
|---|-------------------|----------|-------|----------------|
| Proofs are connected to each other            | 0                 | 1        | 45    | 55             |
| Only geometry is suitable for learning proofs | 25                | 59       | 11    | 5              |

Understanding on mathematical proof

**TABLE 6.** The percentage of responses to questionnaire (%)

| Statement   | Strongly Disagree | Disagree | Agree | Strongly Agree |
|---|-------------------|----------|-------|----------------|
| Mathematical proofs are facts that support the truth of a mathematical statement      | 1                 | 1        | 27    | 71             |
| The truth of a mathematical statement can be demonstrated by some supporting examples | 2                 | 4        | 40    | 54             |

### Discussion

According to the result from table 2 to table 6, we can see the percentage of responses to the questionnaire related to the teacher's perception on mathematical proof. In table 2, the statements are related to the process of mathematical proof, 72% of respondents agree a lot that the proof is the foundation of mathematics, and 66% of respondents agree a lot that a mathematical proof is a sequence of arguments that show the truth of a mathematical statement. It is clear that more than half of the respondents have a positive perception regarding the process of mathematical proof. On the other hand, 43% of respondents agree that the proof is hard to be understood but 42% of respondents disagree that the process of mathematical proof is exhausting.

Related to the thinking ability in mathematical proof, most respondents agree with a lot that mathematical proof can improve some skills such as reasoning skills, understanding of mathematical concepts, argumentation, high-level thinking, and high order thinking skills. Mathematical proof is an important skill in advanced mathematical thinking as they help to substantiate. Whether the statement is true or false, and why. And such work helps to gain insight understand the mathematical ideas involved and Helps students understand concepts and their relationships. It then connects the mathematical proofs, definitions, statements, and processes appropriately to obtain the desired result. Therefore, the process of mathematical proof sharpens students' understanding of logical thinking [9].

Proofs are a fundamental part of mathematics. Researchers argue that evidence should be included in mathematics education at all levels. The main reason is that, since proofs play a central role in mathematics, we do not think it is reasonable to introduce proofs only at the high school level and expect students to immediately appreciate them [10,11]. This argument for introducing proofs at all school levels does not necessarily mean that proofs are integrated into the mathematical experience of all students, including elementary school students [9]. According to the result of

the survey, mathematics teachers disagree that mathematical proof is not suitable for junior high school (51 percent), but they agree that mathematics proof learning is suitable for high school students.

Considering the topic, mathematical proofs are required in now no longer simplest geometry, but additionally algebra and trigonometry. Otherwise, students will now no longer have the ability to recognize the contents of widespread mathematical formulas [12]. According to the result of this research, most of the teachers agree a lot that proof are connected to each other (55 percent) and only geometry is a suitable topic for learning mathematical proof.

As Knuth points out [10,11], these suggestions are too dependent on teachers, requiring a well understanding of the role and nature of mathematical proof. For the last table about understanding on mathematical proof, most of the teachers strongly agree that mathematical proof is facts that support the truth of a mathematical statement (71 percent), and the truth of a mathematical statement can be demonstrated by some supporting examples (54 percent). Based on this result, it seems that the teacher's understanding on mathematical proof is not basically clear. The study result shows parallelism with other studies [13-15], which determined that teachers do not set out a clear view for a mathematical proof. Since we know that mathematical proof is a kind of deductive process. NCTM believes that the coaching of mathematical proofs ought to observe the subsequent stages: 1) Recognition of deductive reasoning and proofs is one of the ideas of mathematics, 2) The introduction and exploration of diverse mathematical conjectures, 3) Developing and comparing diverse mathematical inferences and proofs, and 4) Selecting and making use of diverse sorts of deductive reasoning and proofs.

This survey is limited only to teachers' perception on mathematical proof. It is still unknown how the teachers use proof in their teaching practices. Therefore, another survey is needed to figure out how they use proof in their teaching practices and how these perceptions affect it.

## CONCLUSION

One such factor is the knowledge and beliefs of teachers which are covered as teachers' perception. It has been identified as an important factor determining the classroom practice of teachers. Overall, Indonesian in-service mathematics teachers had a positive attitude towards mathematical proof as a teaching approach in their classrooms, as the opportunity for students to improve helped students improve their thinking ability. In addition, teachers confirmed that the use of mathematical proof in mathematics education is suitable for high school student not junior student. They assume that proof needs a higher level of mathematics. Even though, they perceive mathematical proof positively, they still consider examples as a proof. This means that they do not fully understand the concept of mathematical proof. That is the reason that teachers tend to skip mathematical proof in their teaching [16]

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