ANALYZING STUDENTS' THINKING IN MATHEMATICAL PROBLEM SOLVING USING VYGOTSKIAN SOCIOCULTURAL THEORY

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ABSTRACT

Objective: This study aims to investigate students' thinking in solving mathematical problem solving (MPS) using the Vygotskian sociocultural theory (SCT) in the form of mathematical objects.

Method: This qualitative study involved 34 high school students in Palembang, 20 female and 14 male students. Data were obtained from students' work and video recordings when students solved problems. The analysis of students' work was reviewed from the semiotic system to see the students' semiotics, including languages (natural and alphanumeric), concepts and propositions, procedures, and arguments. The video recordings were analyzed to assess communication and collaboration among students during problem-solving activities.

Research Findings and Discussions: Analyzing students' work and video recordings revealed significant insights into their problem-solving strategies and sociocultural interactions. Through semiotic analysis, it was possible to observe how students expressed and communicated mathematical ideas. Moreover, examining video recordings elucidated communication and collaboration dynamics within the classroom context. These findings shed light on the effectiveness of integrating Mathematical Problem Solving (MPS) with Socio-Cultural Theory (SCT) principles in fostering algebraic thinking and enhancing students' mathematical abilities.

Research Implication: The findings of this study hold implications for both research and practice in mathematics education. They provide valuable insights for designing instructional approaches that promote collaborative problem-solving and facilitate the development of algebraic thinking skills. Additionally, the application of Vygotskian SCT in mathematics classrooms offers a promising avenue for cultivating a supportive sociocultural environment conducive to learning.

Originality/Value: This study contributes to the existing literature by offering a comprehensive exploration of MPS through the lens of Vygotskian SCT, mainly focusing on using mathematical objects as mediational tools. By examining students' semiotic systems and sociocultural interactions, this research enhances our understanding of the complex processes involved in mathematical problem-solving and underscores the importance of sociocultural factors in shaping students' mathematical experiences.

Keywords: Collaboration, Communication, Mathematical Problem Solving, Vygotskian's Sociocultural.

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ANALISANDO O PENSAMENTO DOS ALUNOS NA RESOLUÇÃO DE PROBLEMAS MATEMÁTICOS USANDO A TEORIA SOCIOCULTURAL VYGOTSKIANA

RESUMO

Objetivo: Este estudo visa investigar o pensamento dos alunos na resolução de problemas matemáticos (MPS) usando a teoria sociocultural Vigotskiana (SCT) na forma de objetos matemáticos.

Método: Este estudo qualitativo envolveu 34 estudantes do ensino médio em Palembang, 20 estudantes do sexo feminino e 14 do sexo masculino. Os dados foram obtidos a partir do trabalho dos alunos e gravações de vídeo quando os alunos resolveram problemas. A análise do trabalho dos alunos foi revista a partir do sistema semiótico para ver a semiótica dos alunos, incluindo línguas (naturais e alfanuméricas), conceitos e proposições, procedimentos e argumentos. As gravações de vídeo foram analisadas para avaliar a comunicação e a colaboração entre estudantes durante atividades de solução de problemas.

Resultados de Pesquisa e Discussões: Analisar o trabalho dos alunos e gravações de vídeo revelou percepções significativas sobre suas estratégias de resolução de problemas e interações socioculturais. Através da análise semiótica, foi possível observar como os alunos expressavam e comunicavam ideias matemáticas. Além disso, o exame de gravações de vídeo elucidou a comunicação e a dinâmica da colaboração no contexto da sala de aula. Estas descobertas lançam luz sobre a eficácia da integração dos princípios de Resolução de Problemas Matemáticos (MPS) com a Teoria Sociocultural (SCT) na promoção do pensamento algébrico e no reforço das habilidades matemáticas dos alunos.

Implicação da Pesquisa: Os resultados deste estudo têm implicações tanto para a pesquisa quanto para a prática na educação matemática. Eles fornecem informações valiosas para projetar abordagens instrucionais que promovem a resolução colaborativa de problemas e facilitam o desenvolvimento de habilidades de pensamento algébrico. Além disso, a aplicação da SCT vigotskiana em salas de aula de matemática oferece uma avenida promissora para cultivar um ambiente sociocultural propício à aprendizagem.

Originalidade/valor: Este estudo contribui para a literatura existente, oferecendo uma exploração abrangente de MPS através das lentes da SCT Vigotskiana, concentrando-se principalmente no uso de objetos matemáticos como ferramentas mediacionais. Ao examinar os sistemas semióticos dos alunos e as interações socioculturais, esta pesquisa melhora nossa compreensão dos processos complexos envolvidos na resolução de problemas matemáticos e ressalta a importância dos fatores socioculturais na formação das experiências matemáticas dos alunos.

Palavras-chave: Colaboração, Comunicação, Resolução de Problemas Matemáticos, Sociocultural de Vigotski.

ANALIZAR EL PENSAMIENTO DE LOS ESTUDIANTES EN LA RESOLUCIÓN DE PROBLEMAS MATEMÁTICOS UTILIZANDO LA TEORÍA SOCIOCULTURAL VYGOTSKIANA

RESUMEN

Objetivo: Este estudio tiene como objetivo investigar el pensamiento de los estudiantes en la resolución de problemas matemáticos (MPS) utilizando la teoría sociocultural de Vygotskian (SCT) en forma de objetos matemáticos.

Método: En este estudio cualitativo participaron 34 estudiantes de secundaria de Palembang, 20 mujeres y 14 hombres. Los datos se obtuvieron del trabajo de los estudiantes y las grabaciones de video cuando los estudiantes resolvieron problemas. El análisis del trabajo de los estudiantes se revisó desde el sistema semiótico para ver la semiótica de los estudiantes, incluidos los idiomas (naturales y alfanuméricos), conceptos y propuestas, procedimientos y argumentos. Las grabaciones de video fueron analizadas para evaluar la comunicación y colaboración entre los estudiantes durante las actividades de resolución de problemas.

Hallazgos y discusiones de la investigación: El análisis del trabajo de los estudiantes y las grabaciones de video revelaron información significativa sobre sus estrategias de resolución de problemas e interacciones socioculturales. A través del análisis semiótico, fue posible observar cómo los estudiantes expresaban y comunicaban ideas matemáticas. Además, el examen de las grabaciones de video aclaró las dinámicas de comunicación y colaboración dentro del contexto del aula. Estos hallazgos arrojan luz sobre la efectividad de integrar los principios de Resolución de Problemas Matemáticos (MPS) con la Teoría Socio-Cultural (SCT) para fomentar el pensamiento algebraico y mejorar las habilidades matemáticas de los estudiantes.

Implicación de la investigación: Los hallazgos de este estudio tienen implicaciones tanto para la investigación como para la práctica en la educación matemática. Proporcionan información valiosa para diseñar enfoques instruccionales que promueven la resolución colaborativa de problemas y facilitan el desarrollo de habilidades de pensamiento algebraico. Adicionalmente, la aplicación de la TSC Vygotskiana en las aulas de matemáticas ofrece una vía prometedora para cultivar un entorno sociocultural propicio al aprendizaje.

Originalidad/Valor: Este estudio contribuye a la literatura existente al ofrecer una exploración integral de MPS a través de la lente de SCT Vygotskiano, centrándose principalmente en el uso de objetos matemáticos como herramientas mediacionales. Al examinar los sistemas semióticos y las interacciones socioculturales de los estudiantes, esta investigación mejora nuestra comprensión de los procesos complejos involucrados en la resolución de problemas matemáticos y subraya la importancia de los factores socioculturales en la configuración de las experiencias matemáticas de los estudiantes.

Palabras clave: Colaboración, Comunicación, Resolución de Problemas Matemáticos, Sociocultural de Vygotskian.

1 INTRODUCTION

Mathematical problem solving (MPS) plays an important role in learning mathematics (Klang et al., 2021; Utemov et al., 2020) in which the students use their skill and knowledge in solving the problems with unknown solutions (Chirinda & Barmby, 2018). MPS can rationalize the use of mathematics concepts that are learned in various kinds of problems, so it can deepen student problem solving skills (Jäder et al., 2020; Zelenina et al., 2021). There are four characteristics that must be shown by the students during MPS, namely students' prior knowledge, problem solving strategy, metacognitive skill, and students' self-confidence (Schoenfeld, 2016). Therefore, investigation is needed to see whether or not the four skills have been applied by the students. The result of the investigation can be used as evaluation materials in improving problem solving skills (Güner & Erbay, 2021). The more information about what students know and how they think, the more opportunities that can be created for student success (Cail & Hwang, 2002; Spangenberg & Pithmajor, 2020;).

Cail and Hwang (2002) investigated US and Chinese students' abilities in completing MPS. The results revealed that US students use the strategy of making pictures, guessing and testing, and making all possibilities which are called concrete strategies. Meanwhile, Chinese students use an abstract strategy where they solve problems by modeling problems. Grenell et al. (2022) investigated students' strategies and metacognitive skill in solving problems about equation. The result revealed that there was a misconception of students about equation. Suseelan et al. (2022) investigated the mistakes made by low-ability students in solving word problems. The results of the investigation revealed that students' procedural skills need to be improved (Jusoh et al., 2023; Indrianto et al., 2024).

Pramuditya et al. (2022) investigated characteristics of Indonesian students' MPS abilitites in open ended based virtual reality game learning. The result revealed that students in low category still lack in identyfing the adequact of data for problem solving so they could not solve the problem. However, overall students can use one at least one strategy when solve the problem, that is guess and check. Thanheiser & Melhuish (2023) analyze students' thinking in mathematically productive teaching routines, namely student-focused learning. Of the studies that have been discussed, there has been no in-depth research on what each student's writing means. In fact, mathematics is related to symbols, variables, and signs which are communication tools in mathematics (Claudia et al., 2021; Ukobizaba et al., 2021). This is all related to semiotics, for this reason this study uses semiotics in analyzing students' thinking.

Semiotics discusses something from various objects like gestures, words, the mathematical figures, body position, perceptual activity, and rhythm (Presmeg et al., 2016). One of the theories in semiotics is Vigotskian semiotic, that is social learning theory discussing how the actual teaching and learning process happens (Radford & Sabena, 2015; Setiawati & Handrianto, 2023; Munir et al., 2024). This approach is part of Vygotskian's Sociocultural Theory (SCT). Vygotskian's SCT proposes that a student's cognitive level can be improved through collaboration with other students who have higher skill compared to him/her (Vygotsky, 1978). This theory defines sociocultural in a very general term, that is social interaction between individuals, custom, culture, and language used to communicate and share ideas (Gauvain, 2019). In Vygotskian's SCT are communication and collaboration (Bature & Atweh, 2019).

Windsor (2010) states that a class environment that supports students to communicate and collaborate can facilitate algebraic thinking. Communication and collaboration in a class is the students' ability in communicating ideas both oral and written and also teamwork among students in solving problems (Chalkiadaki, 2018). The existence of communication and collaboration can be key to successful learning (Kerrigan et al., 2021). A good communication and collaboration is influenced by student awareness, and the given problems must include problem solving (Campbell et al., 2022). Also, communication and collaboration is useful for completing, exploring, and investigating ideas and opinions, and sharpening ideas to persuade other (Díaz et al., 2020; Hidayat & Aripin, 2023).

This research aims to see students' thinking in MPS using Vygotskian's SCT in the form of mathematical objects (languages, concepts, prepositions, procedures, and arguments). In this study, collaboration and communication become important elements in sociocultural learning to improve student problem solving skills.

2 THEORETICAL FRAMEWORK

The sociocultural theory (SCT) developed by Vygotsky serves as the basis for this investigation. This theory proposes that cultural tools and social interactions benefit cognitive development (Vygotsky, 1978; Gauvain, 2019). In accordance with the zone of proximal development, which is a fundamental premise of SCT, Vygotsky (1978) asserts that students were able to acquire a more profound understanding by collaborating with their classmates who had a higher level of expertise or by receiving assistance from their teachers.

SCT concepts have been the subject of an investigation by researchers in mathematics education. These researchers have explored several ways in which the principles of SCT might potentially enhance students' ability to solve mathematical problems (Campbell et al., 2022; Chalkiadaki, 2018; Díaz et al., 2020; Kerrigan et al., 2021). Researchers Windsor (2010) and Bature and Atweh (2019) discovered that students' conceptual understanding and algebraic thinking improved when they collaborated and interacted with one another to find solutions to difficulties.

Previous research in mathematics education has focused on a variety of topics, including the application of problem-solving strategies, metacognitive skills, and the distinction between conceptual and procedural knowledge (Baker et al., 2002; Cail & Hwang, 2002; Grenell et al., 2022; Hiebert, 1990; Spangenberg & Pithmajor, 2020). This research contributes to the prior corpus of work. The goal of this study was to conduct a complete investigation into how students' mathematical thinking could be highlighted during activities that include problem-solving. Semiotic analysis and sociocultural concepts will work together to carry out this investigation (Husin et al., 2023; Shamkhi et al., 2023). In order to do this, it presents a synthesis of many theoretical perspectives.

Through the lenses of Vygotsky's sociocultural theory and mathematical object semiotic analysis, this study investigated how students solved mathematical problems. Students worked together in small groups to find solutions to problems, and the sociocultural aspects of the problem-solving process were analyzed by observing how the students communicated and worked together. According to Vygotsky (1978), the SCT focuses on social processes as a means of learning. The video recordings demonstrate the participants engaging with one another, communicating with one another, and co-constructing knowledge (Do-Carmo et al., 2024; Waty et al., 2024).

The researchers coded and analyzed indicators of collaboration (such as listening to the ideas of others or sharing resources) and level of communication (such as the number of questions asked or ideas proposed) in order to gain an understanding of how the sociocultural classroom environment aided or hindered the problem-solving efforts of the students (Campbell et al., 2022; Kerrigan et al., 2021). According to Vygotsky (1978), particular focus was given to situations in which peers with higher levels of competence assisted peers with lower levels of competence within their zone of proximal development.

Simultaneously, mathematical semiotics was utilized to analyze pupils' written work to ascertain how they used various mathematical objects, including languages, concepts, procedures, and arguments (Borji et al., 2018; Claudia et al., 2021). According to the findings of a study conducted by Presmeg et al. (2016) and Radford and Sabena (2015), this semiotic analysis may be able to understand better how students employ symbols, representations, and reasoning processes in conjunction with the problem-solving process.

Borji et al. (2018) discovered that the capacity of students to communicate their thoughts was disclosed by assessing their use of natural language, numerical expressions, and diagrammatic representations. All of these elements showed different semiotic registers, which was something that was shown by the analysis. We may discover their strengths and shortcomings in problem-solving if we analyze their conceptual knowledge, principle applications, and heuristic strategies (Hiebert, 1990; Baker et al., 2002).

By incorporating sociocultural and semiotic analytical viewpoints, this study aimed to address a gap in our knowledge of the mathematical thinking of pupils. In contrast to the sociocultural perspective, which emphasized the interpersonal dynamics and cultural instruments that assisted learning, the semiotic approach shone attention on the cognitive processes and meaning-making involved in the problem representations and solutions that students come up with (Anas et al., 2023; Tannoubi et al., 2023).

Combining these two theoretical approaches helped researchers find a link between how students solved problems, such as metacognitive techniques, algebraic reasoning, and problem-solving strategies, and the specific social and semiotic processes that helped or hurt their mathematical development. The findings of this extensive research may serve as a guide for creating new instructional strategies that will encourage students to collaborate in situations abundant in semiotic information to solve issues and comprehend complex concepts.

3 METHODOLOGY

3.1. PARTICIPANTS

This exploratory type of mix method research with sequential was conducted by doing qualitative research continued by quantitative research. The participants of this research were chosen by using purposive sampling from 20 female students and 14 male students divided into 7 groups with heterogeneous abilities. Therefore, the participants decided by the researcher were S6, S9, S10 and S15.

3.2 DATA COLLECTION

In the qualitative research stage, data source of this research was from students' work, direct observation and video records when students solved problems and also interviews. Students' work consists of problem solving done in 45 minutes. The problem solving question was "A father makes two types of table and a type of chair with a total number 21 items. Table type A has six legs, table type B has four legs, and the chair has three legs. If the total number of tables' and chairs' legs is 100 legs, how many 3-leg-chairs made by father are there?". The aim of this question is to see students' semiotics in solving the problem that include languages (natural and alphanumeric), concepts, propositions (principle), procedures, and arguments. Furthermore, through observation activities from the result of the video record, the researcher observed how the students' thinking process in solving mathematical problems and also observed the communication and collaboration behaviour that happened in social culture in the class. The researchers also really need accurate and relevant information, therefore the researcher did field study with unstructured interviews on the participants that had been chosen based on this research needs.

In the quantitative research stage, the data sources were the score of students' work in solving the problems, and the existence of collaboration and communication during students working in social culture in the class.

3.3 DATA ANALYSIS

The data analysis was divided into two parts that were the data analysis from students' work and data analysis from the video record. The data from students' work was reduced by grouping students' answers into mathematical objects. The mathematical object can be seen on Table 1. Data reduction result was used to describe students' semiotics.

Table 1

Mathematical Object

Mathematical object	Description
Languages	
- *Natural languages*	Including *terms* written without numbers
- *Alphanumeric languages*	Including mathematical expression, notation, or variable
Concepts	Including the definition and description from the concept used in solving the problems
Propositions (Principles)	Including statement about the concepts
Procedures	Including the steps used in solving the problems
Arguments	Including statement to validate or explain proposition and procedures

Source: Borji et al., 2018

While the video record was analysed from students' communication and collaboration in the class. The indicators of communication used the indicators developed by Campbell et al. (2022) and the indicators of collaboration used the indicators developed by Liljedah (2020).

Table 2

The Indicators of Students' Communication and Collaboration

Code	Variables	Indicators
Communication		
CM_1	*Talking turns*	The number of talking turns of each student
CM_2	*Questions*	The number of questions given to the group members
CM_3	*New solution ideas*	The number of ideas proposed by students
Collaboration		
CL_1	*Open to other's idea*	Listening to other students' idea
CL_2	*Respectful of others*	Giving information to each other
CL_3	*Actively inclusive*	Talking alternately
CL_4	*Sharing the marker*	Working together

Furthermore, a correlation test was done to see whether there was correlation between communication and students' work result, collaboration and students' work result, and communication and collaboration itself.

3.4 PROCEDURES

In general, there was communication and collaboration in problem solving activities. These activities include problem solving questions. Moreover, good communication and collaboration is influenced by SCT. The theoretical framework of this research is available in figure 1 as followed.

Figure 1

Theoretical Framework

4 RESULTS AND DISCUSSION

4.1 RESULTS

Activities done in this research reveal students' mathematical thinking when they are discussing in social culture in the class using semiotics analysis through mathematical objects. In social culture in the class, the researchers rely on the students' understanding and knowledge in groups observed by the help of video recording, although sometimes the researchers also took a note on an observational sheet, when students worked in groups having difficulties in solving the problem solving questions. The researchers also explained what the researchers learned through mathematical objects that appeared when students tried to give explanations written on their answer sheet. Through this exploration, the researcher also learned more about social culture in the class that students' collaboration and communication are the effort of students to discuss and think in solving the problems. It gave insight to the researcher about the kind of problem given to students and how the researcher could help the students' difficulties when interacting with their thoughts and interacting with their group friends' thoughts. The researchers focused on the students' answer sheet at mathematical object part because it was the main point of how students thought in solving problem solving questions.

4.1.1 Mathematical Object: Languages

Students' thoughts outlined in writing can be seen from the mathematical object. In alphanumerical language cases, students thought for example a is the number of type A tables so students write $6 \times a$, then b is the number of type B tables so students write $4 \times b$, and c is a chair so students write $3 \times c$. Not only alphanumeric language, the visible semiotic system is natural language which reaffirms the multiplication result is the number of legs which can be seen in figure 2.

Figure 2

The Answer of S10

It can be seen that S10 only focused on the sum of the multiplication result. Meanwhile, there are two conditions namely the number of things made by father is 21 and the total number of the legs is 100. Thus, the answer of S10 is not appropriate because it does not fulfill the problem condition.

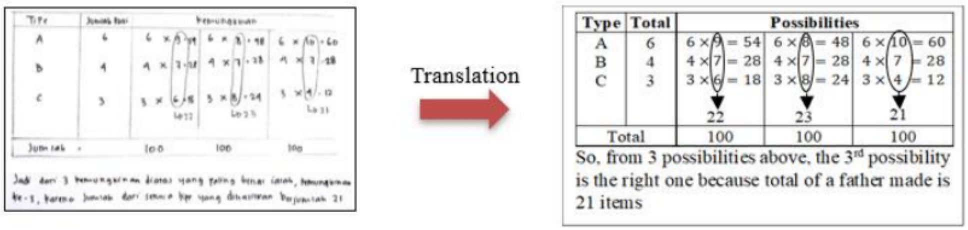
4.1.2 Mathematical Object: Concepts and Principle

Mathematical objects in the form of concepts and propositions (principles) cover the big picture of the concepts studied in solving math problems, concepts are introduced through definitions, descriptions, explicitly. The concept used by students is the concept of a three-variable linear sales system. While propositions are statements about these concepts. Propositions are usually given in the form of statements, which can also be used to examine a concept (Borji et al., 2018). Student concepts and principles can be seen in Figure 3.

From figure 3, S6 used table to write available information in the problem. S6 also used variables where A is as type A table, B as type B table, and C as chair. Summation operation is symbolized with a circle and an arrow. The available sentence also explains that from those three possibilities made, the one that meets the problems condition is the the third possibility in which the number of table and chair made by father is 21 and the number of the legs is 100. Thus, students are proper in using system of three linear equation concept.

Figure 3

The Answer of S6



The principles included in system of three linear equation are in line with the principles of algebra, including the summation or subtraction operations that can be performed on similar variables. In addition, multiplication or division operations can be performed on all variables. These principles are a concern for students in carrying out operations to solve problems. In addition, you can also use the commutative, associative and distributive principles of multiplication over addition. Another principle that can be done is to do the multiplication first, then do the summation operation.

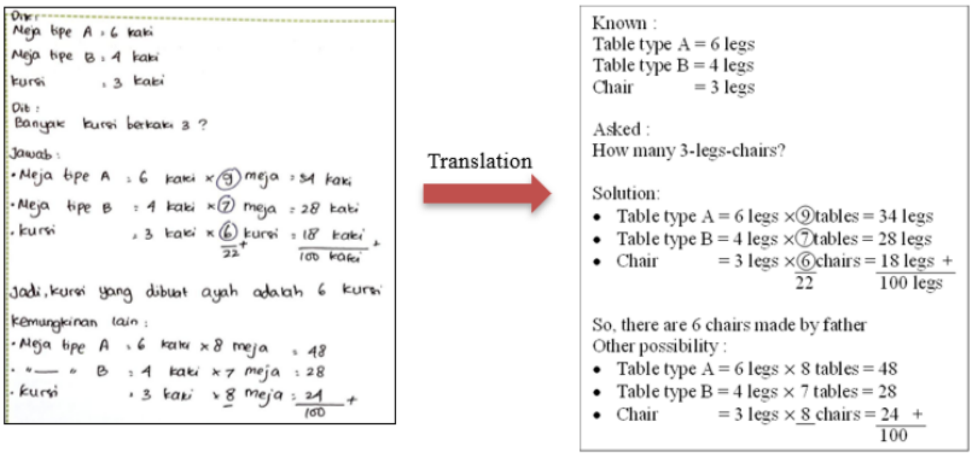
S6 student answers show possible answers to the problems sought. In finding a solution, students use the multiplication principle before doing the summation of the multiplication results. From this step, the number of table and chair legs is 100. Then, the student summed the table and chair quantity.

4.1.3 Mathematical Object: Procedures

Procedure in mathematics objects refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skills in performing problem solving flexibly, efficiently and effectively (Kilpatrick et al., 2001). The students' procedures in completing the MPS can be seen in Figure 4. Judging from the S9 answer, procedurally the students did not experience any difficulties. The first thing to do is to understand the problem where students write down what students know and ask. Furthermore, in determining and implementing plans students use a guessing strategy where students guess 9 type A tables, 7 type B tables, and 6 chairs. Students multiply these guesses by the number of legs on each table and chair. After that, students sum up the results of the multiplication and it is found that all the legs are 100 which are appropriate to the condition of the problem. On looking back, students apparently found other possible answers. However, these two answers were actually inaccurate because they did not meet the requirements that all the tables and chairs that Father had made were 21. However, overall the students' procedures for doing them were correct.

Figure 4

The Answer of S9



4.1.4 Mathematical Object: Arguments

Argument includes the statements to validate and explain propositions. Argument is used to emphasize whether the answer obtained is correct or not by explaining back using their language, so the student can be sure whether their work on that question have finished or not yet. In this analysis, the researcher compared the identification results through utterance from the interview and picture made by students. The researcher did the interview to S15 to see his argument in solving the problem and compared to picture 5 made by students.

R : "What is it meant by these boxes?"

S15 : "The boxes are used as the things made by father, Ms. Then these lines are the legs."

R : "How many boxes were made?"

S15 : "There are 21"

R : "How do you make lines into 100 precisely?"

S15 : "First give three lines for each box Ms, so there are already 63 lines and the rests are 37. Then, each boxes is given one more line. Therefore 21 lines were added, the rests are 16 lines divided to each boxes, 2 lines for each. Therefore there are 8 boxes got two additional 2 lines"

R : "And then how?"

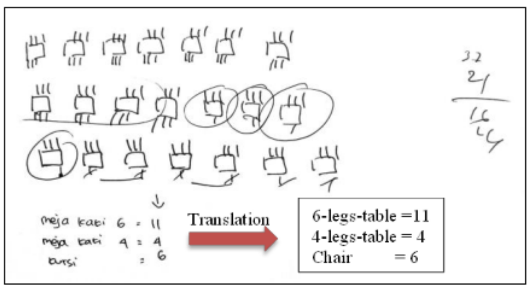
S15 : "Because it is known that there are only 6 legs, 4 legs and 3 legs and also because there must be 3-legs-thing and the number of the legs must be even. Therefore it was taken 1 line from each boxes, so the there are already 6 chairs"

R : "Then?"

S15 : "The lines taken were combined to the available boxing with 4 lines already, divided into 3 boxes. Boxes with 6 lines are 11, boxes with 4 lines are 4, thus the total number is 100."

Figure 5

The Answer of S15



4.1.5 Communication and Collaboration

In general, in each group there are 1-2 students who lack of communication and don't even participate in group discussions. Table 3 shows the results of the correlation test between variables.

Table 3

Correlation Test

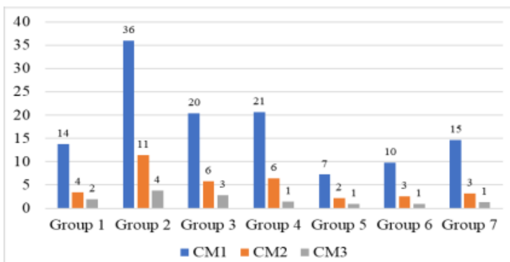
	Communication	*Collaboration*	*Student's score*
Communication	1		
Collaboration	**0.483984872**	1	
Student's score	**0.035842535**	**0.267041687**	1

From table 3, it can be seen that there is a relationship between the variables even though it is low. Communication that occurs in each group is different. The number of speaking turns for each student in the group varies. In group 2, on average each student spoke 36 times, they focused on finding answers by discussing with each other. Whereas in group 5, on average the students spoke 7 times where they tried to solve it themselves first and when they were in a condition they could not solve the problem individually, then they started talking in groups. However, they are a group that can solve problems the fastest among other groups.

During the discussion, students ask each other questions. Not too many questions were asked by students in each group, as well as the ideas they proposed. However, the ideas were actually conveyed in the discussions. Sometimes these ideas do not get a solution, but it is from this idea that communication and collaboration in groups occur. The average number of occurrences of each communication indicator can be seen in Figure 6.

Figure 6

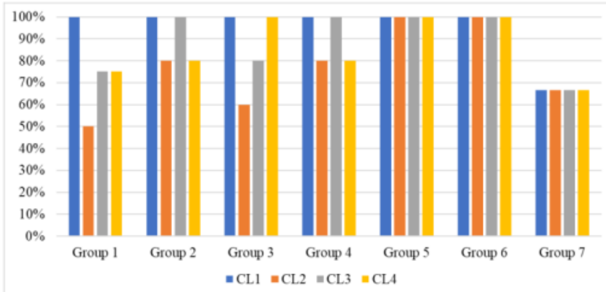
The Average of Communication Indicator Occurrences



Collaboration occurs because of communication among students. When students state their ideas, other students listen. Each student shares information with each other where they share ideas and information about problems when some ignore the conditions of the problem. Each student also did not interrupt the conversation, they started talking when the others had finished speaking. It is from this exchange of ideas that cooperation occurs among students in groups. The percentage of collaboration indicators appearing can be seen in Figure 7.

Figure 7

The Average of Communication Indicator Occurrences



4.2 DISCUSSIONS

4.2.1 Mathematical Object: Languages

For mathematical object language, two aspects are seen, namely, alphanumeric language and natural language. This section also includes the use of symbols and tables in the responses or answers given by students. Borji et al. (2018) proposed further about specific elements in language, namely terms, expressions, notations, and graphics. The use of language that will be discussed in this study is the use of written language, not in gestures or other non-verbal language. The use of alphanumeric students is also influenced by their mathematical abilities. As seen in the S10 answer, he wrote "$6 \times a \rightarrow 5$ items = 30 legs". Mathematically, sentences written like this do not follow the correct rules for writing mathematical equations. Using arrows in the middle of an algebraic expression can be very confusing and meaningless. However, here S10 tries to make sense that "5 items" is the value of the variable "a". Variables should be defined separately, then substituted back into the initial sentence to reduce the use of inaccurate arrows. Students have a tendency to translate their natural expressions into their own language by mixing symbols and numbers that are not yet mathematically correct. The natural language used by students, for example, is the use of the word "items". The term is usually used to indicate the quantity of a particular object. Halliday (2003) mentions that school students make actions, for example counting objects by naming the objects one by one and giving names to these objects. In the case of S10, students preferred to label "pieces" as an object name, even though they already knew that the object being discussed was a table.

4.2.2 Mathematical Object: Concepts and Principle

The concept that is owned by a student has a correlation to the student's ability to give reasons in situations that involve the definition of a concept, the correlation between concepts or both in the process of solving mathematical problems. The students' concept of system of three linear equation can be seen from the fulfillment of the problem conditions where the number of tables and chairs that Father made was 21 and the number of legs was 100. Even though S6 didn't make an equation, he used tables to make it easier to guess. This is in line with the results of Zwanch (2022), students prefer to make guesses on word problems that can be modeled. To check whether the concept is correct, S6 uses the principle of multiplication then summation. S6 multiplies the number of table and chair legs by guessing. Then he sums up the results of the multiplication. Baker et.al (2002) states that students demonstrate conceptual knowledge when they are able to identify, label, and generate examples. Examples of concepts given are using, connecting, manipulating and differentiating concepts; identify and apply laws; knowing and applying facts and definitions; compare, contrast and integrate these concepts and rules; identify, interpret, and apply signs, symbols, and terminology used to represent concepts or interpret assumptions and relationships involving mathematical concepts. From S6's answer it also appears that there is a connection between abstract mentality that cannot be explained or a connection to Mathematical concepts, S6 requires manipulation of abstract images in mathematics with what is around it to make it easier for them to learn mathematical concepts. S6 also adds question requirements first, before planning a strategy for solving them, then S6 makes connections between the information provided and what to look for, after that S6 solves the problem. This is what was revealed by Polya that the aspects that will affect students' problem-solving abilities are skills that students can master before solving problems such as skills in understanding mathematical terms, connecting ideas, planning appropriate strategies, reading and analyzing diagrams and algorithms.

4.2.3 Mathematical Object: Procedures

Procedures about mathematics are knowledge of the rules or methods used to complete mathematical tasks (Walle, 2006). At this stage an analysis of how students' procedural knowledge is carried out by looking at the step by step that students do in carrying out assignments. The use of symbols is a useful way of conveying mathematical ideas to others. However, skills in using procedures will not help develop conceptual knowledge related to these procedures (Hiebert, 1990). Procedural rules should not be taught without concepts. Procedural knowledge, including skill proficiency should be included, although it should not be considered as important as conceptual understanding (Walle, 2006).

4.2.4 Mathematical Object: Arguments

The argument is a series of statements that have the expression of a conclusion statement. From the results of the students' work, interviews with S15 and S6 work it appears that the arguments given by students in solving problems are valid because the statements substituted for hypotheses are correct, so the conclusions are also correct. This is in line with the results of Nordin & Boistrup (2018) which revealed that interviews were conducted to find out in detail the students' arguments.

4.2.5 Communication and Collaboration

In each group, there are students who dominate the conversation where they have more knowledge than the others. They come up with an idea and explain to others about the idea. This is in line with the results of Campbell et al. (2022) that the smarter students dominate the group. The more turns to speak for each student sometimes cannot be a benchmark for good communication, as happened in group 5. They focused on working individually first and then discussing it with their group. Thus, each student in the group received more varied answers than the group of students who directly discussed the problem as a group. Even so, the collaboration that existed in group 5 was very good so that they were able to solve the problem properly. This is in line with Kerrigan et al. (2021) where the key to success in learning is good collaboration.

5 CONCLUSION

This study contributes in describing how students complete the MPS using Vygotskian's SCT in the form of mathematical objects. In particular, through the semiotic system we can observe how students think in solving problems and the social relations that occur in class. The use of MPS in the classroom can facilitate students to communicate and collaborate in solving problems. The findings emphasize the importance of knowing sociocultural in the classroom to see students' thinking in solving problems. The findings of this study also show that if students already have conceptual knowledge, students are able to complete more solutions correctly. After being given a conceptual knowledge guide and further explanation of the problem being solved, students can demonstrate procedural knowledge enhancement. This also explains that conceptual knowledge guides the modification of procedural knowledge in solving new problems because students who have initial conceptual knowledge are more motivated to modify existing procedural knowledge in solving new problems. Whereas with the first procedural theory that students have, where students solve problems procedurally will encourage the construction of conceptual knowledge after having experience in solving problems. First, using the conceptual in generating procedural will strengthen the conceptual and facilitate it to reappear in the future. Second, procedural improvements by providing more challenging problems and requiring more completion processes will also encourage conceptual improvements. Third, in improving student procedural, students will be explained about misconceptions in the finishing procedure. It will also influence and assist students in improving their conceptual and reducing their misconceptions. Fourth, in improving student procedural, students will be explained about misconceptions in the finishing procedure. It will also influence and assist students in improving their conceptual and reducing their misconceptions. The last step in improving student procedural is to encourage students to explain the solutions they are working on and indirectly cause students to apply the basic conceptual in the procedures they use.

Although the way students express their thoughts orally is easier to understand than in writing, both have an equally important role. Students' thoughts that are written down help us understand the strategies they use. Meanwhile, students' thoughts which are expressed orally through class discussions can find out implicit thoughts when written is difficult to understand and students are given more opportunities in small group discussions and big class discussions. When students' thinking is less visible, semiotic analysis needs to be done to see how students think. The semiotics used in other studies are mathematical objects. The originality of this study is the use of a combination of mathematical objects and semiotic systems that are influenced by socioculture in the classroom. These results can be used for other research as material for designing a lesson that can support algebraic thinking. The conclusion of this study proposes that the learning approach based on Vygotsky's sociocultural theory can help improve students' thinking in solving mathematical problems. Interaction between students and teachers, as well as between a student and other students, is very important in facilitating student thinking. Engaging students in problem-solving activities that involve discussion and collaboration with their peers can enrich their understanding of mathematical concepts and help them to develop better problem-solving skills. This research contributes to the development of science in the field of mathematics education, especially in digging deeper into the influence of Vygotsky's sociocultural theory on students' thinking in solving mathematical problems. This research can provide benefits for teachers and education practitioners in improving the quality of mathematics learning in the classroom. Teachers can adopt a learning approach based on Vygotsky's sociocultural theory to help improve students' thinking in solving mathematical problems. Improving students' understanding and mathematical skills can have a positive impact on social and economic development in society. Thus, this research contributes to society in producing students who are more skilled in solving mathematical problems and can help them in their daily life and in their future.

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