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Research Paper

Inventory Model for Deteriorating Pharmaceutical Items with Linear Demand Rate

Indrawati[1,2], Fitri Maya Puspita[2*], Siti Suzlin Supadi[3], Evi Yuliza[2], Krisda Rizki[2]

[1]Mathematics and Natural Sciences Doctoral Study Program, Sriwijaya University, Indralaya, 30662, Indonesia
[2]Mathematics Department, Faculty of Mathematics and Natural Sciences, Sriwijaya University, Indralaya, 30662, Indonesia
[3]Institute of Mathematical Sciences, University of Malaya, Kuala Lumpur, 50603, Malaysia
*Corresponding author: fitrimayapuspita@unsri.ac.id

Abstract

Good management of goods is needed so that the inventory activities of a business can run smoothly as the part of supply chain management which aims to monitor the flow of stock of goods from the purchasing process, and storage to the point of sale. In terms of inventory or supplies of pharmaceutical goods, conditions such as shortages or stockouts must also be considered which are a matter of control, management, and security. In this study, an inventory model is formulated with deterioration or damage to pharmaceutical goods that occurs due to the length of time when the goods are stored with a linear demand level. In the optimal solution, the inventory time occurs when it reaches the zero point (t_1) of 0.34 and the cycle length (T_1) of 0.83 with an average minimum total cost ($\overline{TC}$) of \$445.25 per cycle which is completed by WolframAlpha software. Sensitivity analysis changes the value results in the value of ($\overline{TC}$) which that increases for all parameters. In increasing the linear function variables (a and b), it produces t_1 and T_1 stable values. An increase in the cost of each item damage (D_C) and constant damage rate (θ) produces a t_1 stable value, but the value of T_1 increases. The increase in storage costs (h) results in a decrease in the value of t_1 and T_1. An increase in the cost of shortages (s) results in an increase in the value of t_1 and a decrease in the value of T_1.

Keywords

Inventory Model, Pharmaceutical Goods, Deteriorating, Linear Demand Level, Complete Backlogging

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1. INTRODUCTION

In the pharmaceutical world, a system of distribution and sales provisions is needed that can develop the company's operational development to achieve its company goals (Ahmadi et al., 2022; Karthick and Uthayakumar, 2021; Priyan and Mala, 2020; Savadkoohi et al., 2018). Therefore, in terms of inventory or supplies of pharmaceutical goods, conditions such as shortages or out-of-stock must also be considered which is a matter of control, management, and security. Good management of goods is required so that the inventory activities of a business can run smoothly. Management of related goods with buying, holding, and selling goods (Çömez-Dolgan et al., 2020; He et al., 2023; Limansyah et al., 2020; Parvathi and Gajalakshmi, 2013; de Paula Vidal et al., 2022). If in inventory model, the uncertainty is involved, then the fuzzy inventory model is needed to obtain the optimal policies of the goods (Susanti et al., 2023), and also, the optimal allocation regarding multi-period model was conducted by Alimuddin et al. (2023).

According to Andiraja and Agustina (2020), there are two problem factors examined in formulating the inventory model (Lee et al., 2020; Rizqi and Khairunisa, 2020), namely deterioration or damage to goods and the level of demand for goods (Kumar et al., 2023; Pakhira et al., 2020; Priyan and Mala, 2020; Tiwari et al., 2018) and perishable which is unusability beyond a determined expiry date (Fan and Ou, 2023; Gioia and Minner, 2023; Mohamadi et al., 2024; Shah et al., 2023; Silbermayr and Waitz, 2024; Zhou et al., 2023). Deteriorating in inventory usually occurs due to the length of time when the goods are stored which causes losses, where there is a complete backlogging condition that occurs because the customer does not want to wait for orders to come and move to another company or the customer is willing to wait until the goods are available (Kumar et al., 2023; Lin and Wang, 2018; Pramanik and Maiti, 2019; Priyan and Mala, 2020; Tiwari et al., 2018).

In a time-dependent inventory model, demand plays an important role in the healthcare industry (Maddikunta et al., 2022; Uthayakumar and Tharani, 2018, because demand levels are in a dynamic state. Pharmaceutical goods which are often known as drugs whose items are easily damaged become a problem faced by the pharmaceutical supply system in overcoming shortages and loss of profits. A small proportion of shortages are unfulfilled customer's requests from pharmaceutical suppliers (Uthayakumar and Karuppasamy, 2016), resulting in shortage costs.

A sensitivity analysis is a analysis that is needed to find out which variables are more influential in achieving accurate results from the developed model (Kumar et al., 2023; Limansyah et al., 2020; Stechlinski et al., 2019) and to see changes in the output of the model obtained (Fachri, et.al. 2019). Many studies have developed pharmaceutical inventory models for items by presenting different concepts, one of which was carried out by Uthayakumar and Tharani (2018) in developing inventory models for pharmaceutical goods damage with demand depending on quadratic time in complete backlogging (Braglia et al., 2019; Duary et al., 2022). Braglia et al. (2019) results mainly show that the model of a single-product, single-location inventory system provided is the best way to obtain the optimal policy because of the detailed explanation of the sensitivity analysis is also be done. However, the research did not explain in detail how to solve the numerical experimentation for the steps taken. Duary et al. (2022) explained a model for deteriorating items under capacity constraints and partially backlogged shortages where suppliers offer discounts on the prices. However, the research did not conduct sensitivity analysis to show some choices that can be made by manufacturers if they adopt the policy.

2. EXPERIMENTAL SECTION

2.1 Research Procedure

The steps taken in this research are as follows:

1. Determine and define the notations and assumptions to formulate inventory models for the deterioration of pharmaceutical goods. The following are the notation and assumptions:

D_C : The cost of each deteriorated item.
h : The inventory holding cost per unit per unit of time.
H : Total cost of holding inventory.
$I(t)$: Amount of inventory at t.
s : The shortages cost per unit per unit of time.
S : Total cost of shortages.
θ : The deterioration rate for available items (on-hand); $0 < \theta < 1$.
T_1 : The length of the cycle.
t_1 : Inventory time when it reaches zero point.
t_1^* and T_1^* : The optimal points.
$(\overline{TC})$: The average total cost of pharmaceutical inventory per unit time.

There are several assumptions used in the formation of a decreasing inventory model namely as follows:

1. The rate of deterioration is constant where $0 < \theta < 1$.
2. Shortages allowed and complete backlogging.
3. The lead time rate is equal to zero.
4. During periods of shortages, the level of backlogging is a variable that depends on the length of waiting time for the next filling with waiting time $(t_1 \leq t \leq T_1)$.

One of the objectives of the inventory model is to obtain a minimum cost in determining the cost of inventory. In general, the total inventory cost is affected by several other costs, such as purchasing costs, ordering costs, storage costs, deterioration costs, and shortage costs. However, other costs that can affect the total inventory cost, which may have a relatively small effect and can be ignored. The following is the meaning of the components of the total inventory cost as follows:

a. Purchase Cost.

The purchase cost is the price per unit if the item is purchased from outside or the production cost per unit if it is produced within the company (Tarigan et al., 2020). Cost per unit will always be part of the cost of items in stock or simply the cost incurred to pay for materials or items that have been ordered.

b. Ordering Cost.

Ordering costs are costs incurred in connection with ordering materials/goods, from the time that the order is placed until the goods are available in the warehouse (Afnaria et al., 2020; Limansyah et al., 2020).

c. Inventory Shortage Cost.

Inventory shortage costs are costs incurred due to the unavailability of goods at the time needed. inventory shortage costs are not a real cost, but a missed opportunity cost. Included in the shortage costs are additional administrative costs, costs of delayed receipt of profits, disruption of the production or distribution process, additional expenses, costs of losing customers, and so on (Tarigan et al., 2020).

d. Holding Cost.

Storage costs are costs related to internal storage certain period or costs incurred related to the holding of goods inventory. Storage costs also concern obsolete goods in the warehouse which include storage costs, warehouse rental costs, warehousing administrative costs, warehousing executive salaries, electricity costs, capital costs embedded in inventories, insurance costs, or damage costs.

e. Deteriorating Cost.

Deteriorating costs are costs incurred because materials/goods are stored for too long or there is damage, resulting in a decrease in the quality of the material/goods. The cost of inventory loss is also called the cost of damage due to deterioration (Soraya, 2016).

f. Formulate inventory models for deterioration and demand level.

Deterioration is defined as decay, breakage, evaporation and loss of product utility. Deteriorating pharmaceutical goods is matter to be addressed in the health care system. So, medicine plays an important role in patient care so it needs good planning. At the beginning of the inventory cycle the number of items will achieve the maximum supply as Equation (1) shown as follows (Uthayakumar and Tharani, 2018):

$$I(0) = Q \tag{1}$$

with :

$I(0)$: Total current inventory$t = 0$.
Q : The maximum inventory level for the order cycle.

Inventory of goods will continue to experience reduction by assuming that the level of demand is a linear function of time. The general form of a linear function at the demand level (Afnaria et al., 2020) is as follows :

$$D(t) = a + bt \tag{2}$$

with :

$D(t)$: Demand varies with time.
t : Time.
a : A constant at the request level.
b : Variable coefficient t at the level of demand.

2. Perform numerical calculations on data previously used by Uthayakumar and Tharani (2018), to test the model formulation with WolframAlpha software.
3. Conduct a sensitivity analysis.

Sensitivity analysis is an analysis used to determine the effect of changing a parameter while maintaining other parameters. Table 1 shows the parameters to be changed.

Table 1. Sensitivity Analysis Parameters

Parameter	Variation
θ	0.001 0.002 0.003
a	100 101 102
b	50 51 52
D_C	3 4 5
h	10 11 12
s	7 8 9

4. Provide an interpretation of the model obtained.

3. RESULTS AND DISCUSSION

3.1 Mathematical Formulation of Inventory Models

Consider the declining inventory model with linear demand in Equation (2) where inventory will decrease to $t = t_1$, as demand and damage to the goods occur. At time $t_1 = 0$, shortages are allowed in the interval so that $[t_1, T_1]$ complete backlogged occurs.

Based on the notation and assumptions above, the inventory system can be described in the differential equation as follows. For the amount of inventory in the interval $[0, T_1]$ then

$$\begin{aligned} \frac{\partial I(t)}{\partial t} &= -D(t) - \theta l(t), \quad 0 \le t \le t_1 \\ \frac{\partial I(t)}{\partial t} &= -a - bt - \theta l(t), \quad 0 \le t \le t_1 \end{aligned} \tag{3}$$

The amount of inventory when there is a shortage in the interval $[t_1, T_1\ ]$ is formulated as follows.

$$\begin{aligned} \frac{\partial I(t)}{\partial t} &= -\ D(t), \quad t_1 \le t \le T_1 \\ \frac{\partial I(t)}{\partial t} &= -\ (a + bt), \quad T_1 < t \le T_2 \end{aligned} \tag{4}$$

with boundary conditions $I(0) = Q$ and $I(t_1) = 0$

3.2 Deteriorating Pharmaceutical Goods with Linear Demand

In Equation (3) and Equation (4) the inventory level model is obtained in the $[0, t_1]$ and $[t_1, T_1]$ intervals as follows.

$$I(t) = -\frac{a}{\theta} - \frac{b}{\theta}(t - \frac{1}{\theta}) + e^{\theta(t_1 - t)}\left(\frac{a}{\theta} + \frac{b}{\theta}(t_1 - \frac{1}{\theta})\right),\ 0 \le t \le t_1 \tag{5}$$

$$I(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2),\ t_1 \le t \le T_1 \tag{6}$$

If the amount of inventory at $t = 0$. then to calculate Q as follows.

$$\begin{aligned} &I(0) = Q \\ &Q = -\frac{a}{\theta} - \frac{b}{\theta}\left(0 - \frac{1}{\theta}\right) + e^{\theta(t_1 - 0)}\left(\frac{a}{\theta} + \frac{b}{\theta}\left(t_1 - \frac{1}{\theta}\right)\right), 0 \le t \le t_1 \\ &\Leftrightarrow Q = -\frac{a}{\theta} + \frac{b}{\theta^2} + e^{\theta t_1}\left(\frac{a}{\theta} + \frac{b}{\theta}\left(t_1 - \frac{1}{\theta}\right)\right), 0 \le t \le t_1 \end{aligned} \tag{7}$$

Total requests in the $[0, t_1]$ interval is explained as follows.

$$\int_0^{t_1} D(t)\,dt = \int_0^{t_1} (a + bt)\,dt = at_1 + \frac{bt_1^2}{2} \tag{8}$$

The total number of goods that experience deterioration in the interval $[0, t_1]$ or D_T is as follows.

$$\begin{aligned} D_T &= Q - \int_0^{t_1} D(t)\,dt \\ \Leftrightarrow D_T &= Q - \left(\frac{at_1 + bt_1^2}{2}\right) \\ \Leftrightarrow D_T &= \left(-\frac{a}{\theta} + \frac{b}{\theta^2} + e^{\theta t_1}\left(\frac{a}{\theta} + \frac{b}{\theta}\left(t_1 - \frac{1}{\theta}\right)\right)\right) - \left(\frac{at_1 + bt_1^2}{2}\right) \end{aligned} \tag{9}$$

Total storage (H) at $[0, t_1]$ is as follows.

$$\begin{aligned} H &= \int_0^{t_1} I(t)\,dt = \int_0^{t_1} I(t)\,dt \\ \Leftrightarrow H &= \left\{-\frac{a}{\theta^2} - \frac{at_1}{\theta} - \frac{bt_1{}^2}{2\theta} + \frac{b}{\theta^3} + e^{\theta t_1}\left(\frac{a}{\theta^2} + \frac{b}{\theta^2}\left(t_1 - \frac{1}{\theta}\right)\right)\right\} \end{aligned} \tag{10}$$

Total shortages in the interval $[t_1, T_1]$ are as follows.

$$\begin{aligned} S &= -\int_{t_1}^{T_1} I(t)\,dt \\ \Leftrightarrow S &= \left(\frac{a}{2}\left(T_1^2 + t_1^2 - 2t_1T_1\right) + \frac{b}{6}\left(T_1^3 + 2t_1^3 - 3t_1^2T_1\right)\right) \end{aligned} \tag{11}$$

3.3 Average Total of Pharmaceutical Inventory Cost

The costs included in this model are the purchase price, shipping costs, storage costs, deterioration costs, and storage cost shortages. Thus, the average total cost of pharmaceutical inventory in $[0, T_1]$ per item per unit time $(\overline{TC})$ (Afnaria et al., 2020) is as follows.

$$\begin{aligned} \overline{TC} &= \frac{1}{T_1}\left(A + hH + D_cD_T + sS\right) \\ \Leftrightarrow \overline{TC} &= \frac{1}{T_1}\left\{A + h.\left\{-\frac{a}{\theta^2} - \frac{at_1}{\theta} - \frac{bt_1{}^2}{2\theta} + \frac{b}{\theta^3} + e^{\theta t_1}\left(\frac{a}{\theta^2} + \frac{b}{\theta^2}\left(t_1 - \frac{1}{\theta}\right)\right)\right\}\right. \\ &\quad + D_c.\left\{\left(-\frac{a}{\theta} + \frac{b}{\theta^2} + e^{\theta t_1}\left(\frac{a}{\theta} + \frac{b}{\theta}\left(t_1 - \frac{1}{\theta}\right)\right)\right) - \left(at_1 + \frac{bt_1{}^2}{2}\right)\right\} \\ &\quad \left. + s.\left(\frac{a}{2}\left(T_1^2 + t_1{}^2 - 2t_1T_1\right) + \frac{b}{6}\left(T_1^3 + 2t_1{}^3 - 3t_1^2T_1\right)\right)\right\} \end{aligned} \tag{12}$$

The first order derivative with TC respect to t_1 and T_1 is as follows.

$$\frac{\partial(\overline{TC})}{\partial t_1} = \frac{1}{T_1}\left(h\frac{\partial H}{\partial t_1} + D_c\frac{\partial D_T}{\partial t_1} + s\frac{\partial S}{\partial t_1}\right) \tag{13}$$

$$\frac{\partial(\overline{TC})}{\partial T_1} = -\frac{1}{T_1^2}A - \frac{1}{T_1^2}hH - \frac{1}{T_1^2}D_cD_T - \frac{1}{T_1^2}sS + \frac{1}{T}s\frac{\partial S}{\partial T_1} \tag{14}$$

because $\frac{\partial D_T}{\partial T_1} = \frac{\partial H}{\partial T_1} = 0$, then

$$\frac{\partial D_T}{\partial t_1} = (a + bt_1)(e^{\theta t_1} - 1) \tag{15}$$

$$\frac{\partial H}{\partial t_1} = \left(\frac{a + bt_1}{\theta}\right)(e^{\theta t_1} - 1) \tag{16}$$

$$\frac{\partial S}{\partial t_1} = (a + bt_1)(t_1 - T_1) \tag{17}$$

$$\Leftrightarrow \frac{\partial S}{\partial T_1} = a(T_1 - t_1) + \frac{b}{2}(T_1^2 - t_1^2) \tag{18}$$

Substituting in Equation (15)-(17) to Equation (13), and Equation (18) to Equation (14), we obtain the following.

$$\frac{\partial(\overline{TC})}{\partial t_1} = \frac{1}{T_1}\left(\left(\frac{a + bt_1}{\theta}\right)(e^{\theta t_1} - 1) + (a + bt_1)(e^{\theta t_1} - 1) + (a + bt_1)(t_1 - T_1)\right) \tag{19}$$

$$\begin{aligned} \frac{\partial(\overline{TC})}{\partial T_1} &= -\frac{1}{T_1^2}A - \frac{1}{T_1^2}hH - \frac{1}{T_1^2}D_cD_T - \frac{1}{T_1^2}sS + \frac{1}{T}s\frac{dS}{dT_1} \\ \frac{\partial(\overline{TC})}{\partial T_1} &= -\frac{1}{T_1^2}A - \frac{1}{T_1^2}h\left(-\frac{a}{\theta^2} - \frac{at_1}{\theta} - \frac{bt_1^2}{2\theta} + \frac{b}{\theta^3} + e^{\theta t_1}\left(\frac{a}{\theta^2} + \frac{b}{\theta^2}\left(t_1 - \frac{1}{\theta}\right)\right)\right) \\ &\quad - \frac{1}{T_1^2}D_c\left(\left(-\frac{a}{\theta} + \frac{b}{\theta^2} + e^{\theta t_1}\left(\frac{a}{\theta} + \frac{b}{\theta}\left(t_1 - \frac{1}{\theta}\right)\right)\right) - \left(at_1 + \frac{bt_1^2}{2}\right)\right) \\ &\quad - \frac{1}{T_1^2}s\left(\left(\frac{a}{2}\left(T_1^2 + t_1^2 - 2t_1T_1\right) + \frac{b}{6}\left(T_1^3 + 2t_1^3 - 3t_1^2T_1\right)\right)\right) \\ &\quad + \frac{1}{T}s\left(a\,(T_1 - t_1) + \frac{b}{2}\left(T_1^2 - t_1^2\right)\right) \end{aligned} \tag{20}$$

The optimal value of t_1 and T_1 is denoted by t_1^*, T_1^* which is obtained by satisfying the necessary conditions to minimize the average total cost function

$$\frac{\partial(\overline{TC})}{\partial t_1} = 0, \text{ and } \frac{\partial(\overline{TC})}{\partial T_1} = 0 \tag{21}$$

3.4 Numerical Calculation

In numerical calculations, an algorithm is needed according to Uthayakumar and Tharani (2018), as follows:

Step 1 : Set T_1 =1, in the equation $\frac{\partial(\overline{TC})}{\partial t_1} = 0$, and obtain t_1.
Step 2 : Substitute t_1 in the equation $\frac{\partial(\overline{TC})}{\partial T_1} = 0$, and get T_1.
Step 3 : Substitute T_1 in the equation $\frac{\partial(\overline{TC})}{\partial t_1} = 0$, and get t_1.
Step 4 : Repeat Steps 2 and 3 until there is no change in achieving the value of t_1 and T_1.
Step 5 : Calculate $\overline{TC}$ by substituting optimal values of t_1 and T_1.

On Numerical calculations, use data from Uthayakumar and Tharani (2018) to test the model formulation. Consider an inventory system with parameters A = \$200 per order, D_c = \$3 per item, h = \$10 per unit per unit time, s = \$7 per unit per unit time, a = 100, b = 50, θ = 0.001, T_1^*, and $\overline{TC}$ by using Equations (19)-(21).

Iteration 1

Step 1: Set $T_1 = 1$ in the equation $\frac{\partial \overline{TC}}{\partial t_1} = 0$ and determine t_1.

$$\frac{1}{T_1}\left(h\left(\frac{a+bt_1}{\theta}\right)(e^{\theta t_1}-1)+D_c(a+bt_1)(e^{\theta t_1}-1)+s(a+bt_1)(t_1-T_1)\right)=0$$

$$\Leftrightarrow \left(10\left(\frac{100+50t_1}{0.001}\right)\left(e^{0.001t_1}-1\right)+3\,(100+50t_1)\left(e^{0.001t_1}-1\right)+7\,(100+50t_1)\,(t_1-1)\right)=0$$

$$\Leftrightarrow 350t_1^2+500150e^{0.001t_1}t_1-499800t_1+1000300e^{0.001t_1}-1001000=0$$

$$\Leftrightarrow t_1 = 0.4116$$

Step 2: Substitute t_1 in the equation $\frac{\partial \overline{TC}}{\partial T_1} = 0$ and determine T_1 with value $t_1 = 0.4116$.

$$\begin{aligned}
&-\frac{1}{T_1^2}A-\frac{1}{T_1^2}h\left(-\frac{a}{\theta^2}-\frac{at_1}{\theta}-\frac{bt_1{}^2}{2\theta}+\frac{b}{\theta^3}+e^{\theta t_1}\left(\frac{a}{\theta^2}+\frac{b}{\theta^2}(t_1-\frac{1}{\theta})\right)\right)\\
&-\frac{1}{T_1^2}D_c\left(\left(-\frac{a}{\theta}+\frac{b}{\theta^2}+e^{\theta t_1}\left(\frac{a}{\theta}+\frac{b}{\theta}(t_1-\frac{1}{\theta})\right)\right)-\left(at_1+\frac{bt_1{}^2}{2}\right)\right)\\
&-\frac{1}{T_1^2}s\left(\frac{a}{2}(T_1^2+t_1^2-2t_1T_1)+\frac{b}{6}(T_1^3+2t_1^3-3t_1^2T_1)\right)\\
&+\frac{1}{T_1}s\left(a(T_1-t_1)+\frac{b}{2}(T_1^2-t_1^2)\right)=0
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow -\frac{1}{T_1^2}(200)-\frac{1}{T_1^2}\cdot 10\left(-\frac{100}{0.001^2}-\frac{100\cdot 0.4116}{0.001}-\frac{50\cdot(0.4116)^2}{2\cdot 0.001}\right.\\
&\left.+\frac{50}{0.001^3}+e^{0.001\cdot 0.4116}\left(\frac{100}{0.001^2}+\frac{50}{0.001^2}\left(0.4116-\frac{1}{0.001}\right)\right)\right)\\
&-\frac{1}{T_1^2}\cdot 3\left(-\frac{100}{0.001}+\frac{50}{0.001^2}+e^{0.001\cdot 0.4116}\left(\frac{100}{0.001}+\frac{50}{0.001}\left(0.4116-\frac{1}{0.001}\right)\right)\right)\\
&-\left(100\,(0.4116)+\left(\frac{50(0.4116)^2}{2}\right)\right)\\
&-\frac{1}{T_1^2}\cdot 7\left(\left(\frac{100}{2}\left(T_1^2+(0.4116)^2-2\,(0.4116)\,T_1\right)\right.\right.\\
&\left.\left.+\frac{50}{6}\left(T_1^3+2(0.4116)^3-3(0.4116)^2T_1\right)\right)\right)\\
&+\frac{1}{T_1}7\left(100\,(T_1-(0.4116))+\frac{50}{2}\left(T_1^2-(0.4116)^2\right)\right)=0
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow -\frac{200}{T_1^2}-\frac{96.3425}{T_1^2}-\frac{0.0289028}{T_1^2}\\
&-\frac{58.3333T_1^3-350T_1^2+317.768T_1-67.4304}{T_1^2}\\
&+175T_1-\frac{317.768}{T_1}+700=0\\
&\Leftrightarrow T_1 = 0.804423
\end{aligned}$$

Step 3: Substitute T_1 in the equation $\frac{\partial \overline{TC}}{\partial t_1} = 0$ and get t_1. We have $T_1 = 0.804423$.

$$\begin{aligned}
&\frac{1}{T_1}\left(h\left(\frac{a+bt_1}{\theta}\right)(e^{\theta t_1}-1)+D_c(a+bt_1)(e^{\theta t_1}-1)\right.\\
&\left.+s(a+bt_1)(t_1-T_1)\right)=0
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \frac{1}{0.804423}\left(10\left(\frac{100+50t_1}{0.001}\right)(e^{0.001t_1}-1)+3(100+50t_1)(e^{0.001t_1}-1)\right.\\
&\left.+7(100+50t_1)(t_1-0.804423)\right)=0\\
&\Leftrightarrow t_1 = 0.331142.
\end{aligned}$$

Step 4: Repeat Steps 2 and 3 until there is no change in reaching the value of t_1 and T_1. using the same formula until iteration 6, as follows.

For Iteration 2,

$T_1 = 0.824744$

$t_1 = 0.339507$

For Iteration 3,

$T_1 = 0.831538$

$t_1 = 0.342303$

For Iteration 4,

$T_1 = 0.833836$

$t_1 = 0.34325$

For Iteration 5,

$T_1 = 0.834617$

$t_1 = 0.34357$

For Iteration 6

$T_1 = 0.834882$.

Due to the almost constant value of T_1 which is nearly 0.83, and the t_1 value of 0.34, we complete the iterations with those values. On the results of the values t_1 and T_1, it can be stated that the solution has converged, where each value t_1 and T_1 is close to each other. So that the values of t_1^* and T_1^* (optimal value) is 0.34 and 0.83, then the iteration calculation is stopped.

Step 5: Calculate $\overline{TC}$ by substituting the values of t_1 and T_1.

$$\begin{aligned}
\overline{TC} = \frac{1}{T_1}\Big\{&A+h\left\{-\frac{a}{\theta^2}-\frac{at_1}{\theta}-\frac{bt_1{}^2}{2\theta}+\frac{b}{\theta^3}+e^{\theta t_1}\left(\frac{a}{\theta^2}+\frac{b}{\theta^2}(t_1-\frac{1}{\theta})\right)\right\}\\
&+D_c.\left\{-\frac{a}{\theta}+\frac{b}{\theta^2}+e^{\theta t_1}\left(\frac{a}{\theta}+\frac{b}{\theta}(t_1-\frac{1}{\theta})\right)-\left(at_1+\frac{bt_1{}^2}{2}\right)\right\}\\
&+s\left(\frac{a}{2}(T_1^2+t_1^2-2t_1T_1)+\frac{b}{6}(T_1^3+2t_1^3-3t_1^2T_1)\right)\Big\}
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \overline{TC} = \frac{1}{0.83}\Big\{200+10\Big\{-\frac{100}{(0.001)^2}-\frac{100\cdot 0.34}{(0.001)}-\frac{50\cdot(0.34)^2}{2\cdot(0.001)}\\
&+\frac{50}{(0.001)^3}+e^{0.001\cdot 0.34}\left(\frac{100}{(0.001)^2}+\frac{50}{(0.001)^2}\left((0.34)-\frac{1}{(0.001)}\right)\right)\Big\}
\end{aligned}$$

$$\begin{aligned}
&+3\Big\{\left(-\frac{100}{(0.001)}+\frac{50}{(0.001)^2}+e^{0.001\cdot 0.34}\left(\frac{100}{(0.001)}+\frac{50}{(0.001)}\left((0.34)-\frac{1}{(0.001)}\right)\right)\right)\\
&-\left(100\cdot 0.34+\frac{50\cdot(0.34)^2}{2}\right)\Big\}
\end{aligned}$$

$$\begin{aligned}
&7\left(\frac{100}{2}\left((0.83)^2+(0.34)^2-2\cdot(0.34)(0.83)\right)\right.\\
&\left.+\frac{50}{6}\left((0.83)^3+2\cdot(0.34)^3-3\cdot(0.34)^2\cdot(0.83)\right)\right)\Big\}
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \overline{TC} = \frac{1}{0.83}(200+64.358+0.0193074+105.1838)\\
&\Leftrightarrow \overline{TC} = 445.2543
\end{aligned}$$

$\overline{TC}$ obtained of 445.2543 which shows that the average minimum total cost per cycle.

3.5 Sensitivity Analysis Calculations

After this sensitivity analysis, the same calculation is carried out as to calculate the value of t_1, T_1, and $\overline{TC}$ by changing each parameter and maintaining the other parameters according to Table 1. The results of the sensitivity analysis are obtained in Table 2.

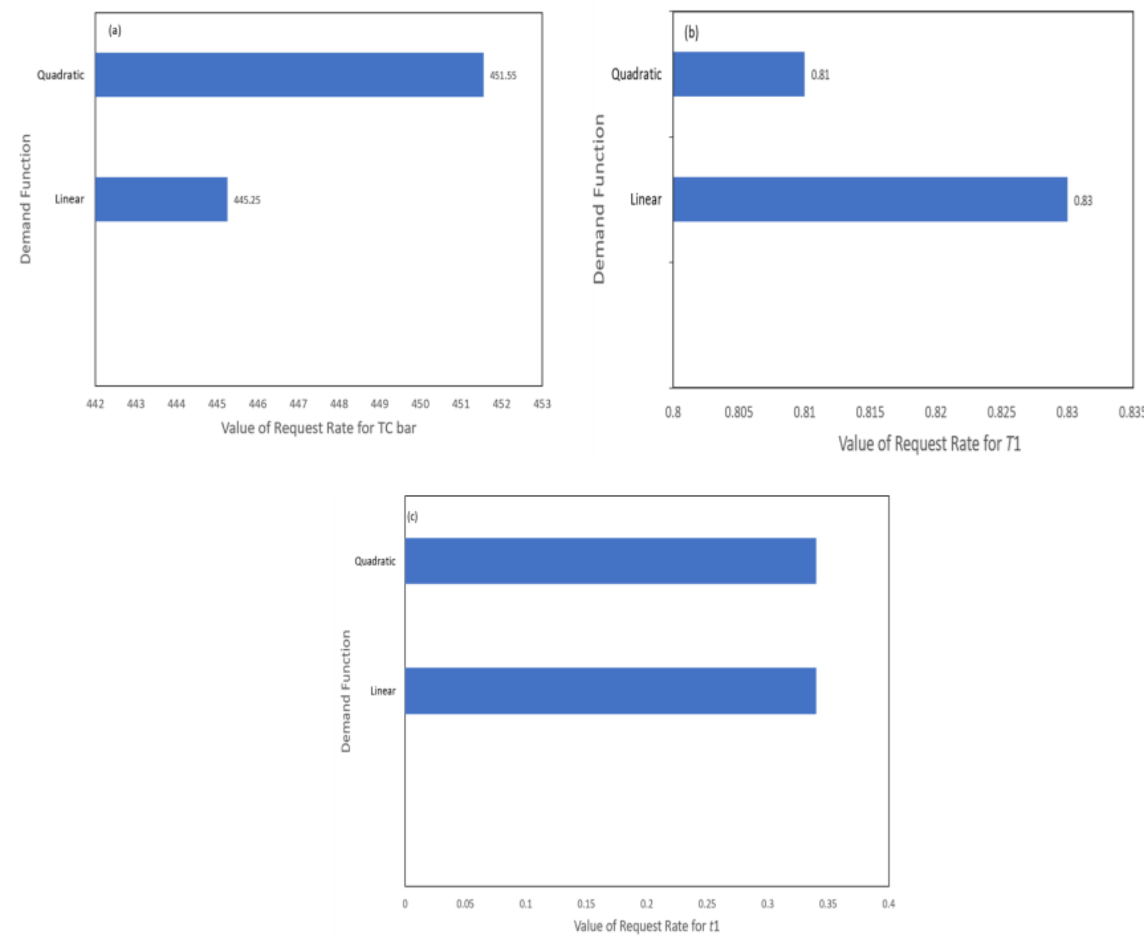

Figure 1. Comparison of Request Rate for Linear and Quadratic level of demands; (1.a) in terms of $\overline{TC}$; (1.b) in terms of T_1; (1.c) in terms of t_1, respectively.

Based on Table 2, the following conclusions are obtained.

1. Increasing values of a and b will generate values of t_1 and T_1 which are stable at 0.34 and 0.83, but at the same time the value $\overline{TC}$ increases.

2. Increasing values of D_C and θ will generate value of t_1 which is stable at 0.34, but at the same time the value T_1 and $\overline{TC}$ increases.

3. Increasing value of h results in the decrease in values of t_1 and T_1 by 0.02 and 0.01, but at the same time the value $\overline{TC}$ increases.

4. An increase value of s results in a decrease in value of T_1 equal to 0.02, but at the same time, values of t_1 and $\overline{TC}$ increase.

It is shown that the value of $\overline{TC}$ experiences a difference of around 0.38%, which means that there is no significant difference when value a varies from 100 to 102, so the results obtained are optimal.

When θ value varies from 0.001 to 0.003, it shows that the valueof $\overline{TC}$ experiences a difference of around 0.02%, which means there is no significant difference, so the results obtained are optimal. When the value of b varies from 50 to 52, it shows that the value $\overline{TC}$ experiences a difference of around 0.15%, which means there is no significant difference, so the results obtained are optimal. If D_C varies from 3 to 5 then the value $\overline{TC}$ experiences a difference of around 0.01%, which means there is no significant difference, so the results obtained are optimal.

If the value of h varies from 10 to 12 then the value $\overline{TC}$ experiences a difference of around 1.63%, which means there is no significant difference, so again the results obtained are optimal. Optimal solution when s value varies from 7 to 9 which explains that the value $\overline{TC}$ experiences a difference of around 3.14%, and means there is no significant difference.

The results obtained by using the level of demand on the

Table 2. Sensitivity Analysis Results

Parameter	Variation	t_1	T_1	$\overline{TC}$
θ	0.001	0.34	0.83	445.25
	0.002	0.34	0.84	445.33
	0.003	0.34	0.84	445.35
a	100	0.34	0.83	445.25
	101	0.34	0.83	446.96
	102	0.34	0.83	448.67
b	50	0.34	0.83	445.25
	51	0.34	0.83	445.92
	52	0.34	0.83	446.59
D_c	3	0.34	0.83	445.25
	4	0.34	0.84	445.30
	5	0.34	0.84	445.31
h	10	0.34	0.83	445.25
	11	0.32	0.82	452.61
	12	0.30	0.81	459.25
s	7	0.34	0.83	445.25
	8	0.36	0.81	462.05
	9	0.37	0.78	476.58

linear function are compared with the results using the level of demand on the quadratic function that has been done by Uthayakumar and Tharani (2018) as in Figure 1.

Based on Figure 1, it can be seen that the results of $\overline{TC}$, T_1 and t_1 are not too different. The level of demand on the linear function can be said to be better than the quadratic function because the $\overline{TC}$ minimum yield is \$445.25 per cycle.

4. CONCLUSION

Based on the results and discussion in the inventory model for deteriorating pharmaceutical goods with a linear demand level, it can be concluded that this model starts at the time the inventory is valued Q and decreases over time in the $[0, t_1]$ for one time in one cycle, so that when shortages occur there is a waiting time until the next order can be placed (T_1) with the assumption that the lead time is zero. This causes the order to arrive immediately after it is ordered. The inventory model obtained is as follows.

$$\overline{TC} = \frac{1}{T_1}(A + hH + D_c D_T + sS)$$

In the optimal solution, t_1 and T_1 equal to 0.34 and 0.83 with an average minimum total cost of \$445.25 per cycle. Sensitivity analysis changes in value results in the value ($\overline{TC}$) to increase for all parameters. In increasing a and b, it produces t_1 and T_1 stable values. At an increase in D_C and θ, it produces a t_1 stable value, but the value of T_1 increases. The increase in h results in a decrease in the value of t_1 and T_1. An increase in s results in an increase in the value of t_1 and a decrease in the value of T_1. For further work, it is suggested to also consider the quadratic demand and no backlogging to pursue a better inventory model to gain by modeling the pharmaceutical items.

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