

# Vector Fields of the Dynamics of Non-Holonomic Constraint System With Elliptical Configuration Space

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## ABSTRACT

Computational physics can be used to help solve complex dynamics equations, both translational and rotational. The purpose of this study is to obtain differences in the dynamics of mechanical systems with non-holonomic constraints in various flat and curved configuration spaces based on physics computing. In this study the reduction used is a mathematical calculation of the Port-Controlled Hamiltonian System (PCHS) equation, and mechanical system that is used in this study is Stroller (baby carriage). Equation in determining the stroller's dynamic with and without friction that moves in the curved plane with various initial conditions is Poincare's equation which is based on Routhian reduction. The effect of friction can be clearly seen through dynamics and graphical equations on the Stroller. This method can reduce the Stroller motion equation with and without friction that moves on the ball sphere clearly in the form of a set of differential equations. The method of this study are mathematical calculations using physics computing. The product that resulted is dynamic equations and graphs of Stroller equations with and without friction that move in the curved plane in the form of a spherical ball with various initial conditions based on maples.

**Keywords:** Dynamics, Stroller, PCHS Method, Computational.

## 1. INTRODUCTION

Geometric mechanics is study of physics, mathematics and engineering, which contains many research topics. Many ideas and developments in geometric mechanics have played a role in other scientific disciplines to deal with practical problems [1],[2]. Applied geometric mechanics can be found in various fields such as robotics, vehicle dynamics, and locomotive motion in various animal movements involving non holonomic mechanics [3][4].

Throughout history, there have been many scientists who came from various disciplines to study the system of non holonomic mechanics. The non holonomic system was introduced in mechanics by Hertz in 1894 [5–7], which meant that the system experienced constraints that limited the speed of the system particles in the configuration space [8][9]. Constraints are conditions that limit the motion of a mechanical system so that reducing the degrees of freedom. Holonomic constraints always involve the speed of the system and can be written in the form of degree one [5][10]. These constraints are found in the configuration space and do not reduce the degree of freedom and limit the movement of the system

in the configuration space and momentum. The system can be described by diversity (manifold), which is an effort to build the coordinates of a space in the form of a collection of points, lines or functions [11][12]. The configuration space used in the form of Lie groups, then to look for constraints group theory can be employed, but in this study we only need to use differential calculations [13–15].

A stroller is a simple example of a locomotive motion system with no holonomic constraints, but the study of mechanics is not trivial. Stroller is often associated with small three-wheeled vehicles used by children under five years old. We will approach the modelling from the perspective of a controlled Hamiltonian system (Port Controlled Hamiltonian System), because the Port Controlled Hamiltonian System (PCHS) is one way to express system energy firmly [16]. In addition, the Port Controlled Hamiltonian System is a dynamical system that can be described by a set of differential equations. In this study, the Stroller motion equation will be derived through two methods, namely PCHS and the Levi-Civita Connection constrained [7][17].

Systems with non-holonomic constraints can be hidden by establishing or select one of Levi-Civita connections. The purpose hiding of constraint is to eliminate Lagrange multipliers in the equation of motion [18]. This research is an attempt to better understand the system with non-holonomic constraints from the viewpoint of geometric mechanics, which will analyse the problem of motion geometrically [19][20].

Computers in Physics Learning are courses that need to be developed globally. This is done to prepare advanced generations and be aware of the importance of technology in the face of the industrial revolution 4.0 [13][21]. Physics is a branch of science that is growing rapidly following the development of existing technology. Students must be well prepared in facing technological developments. Technology in learning physics can be embedded in the learning process in the classroom. For example in computer in learning physics.

The object motion that will be discussed in this research is the Stroller motion. Given the motion of the Stroller is an example of the motion of objects that can move by translational and rotational [22][23]. In a study of research about Tipped Top (TT) has successfully solved the equations of reverse TT dynamics in the flat plane while describing the equation of motion using computational physics [24][25]. In this study, researchers will analyse other complex and complicated dynamic systems that operate on flat and spherical planes at the same time with and without friction. Researchers will predict the dynamics of the stroller on the surface in the ball with the help of physics computing. Resolving this equation is not easy, because the configuration space that will be passed by the Stroller is a spherical ball which is a curved plane that has elliptical coordinate variables and Stroller coordinates that move using three coordinate systems, so that the total number of general coordinates to be completed is six common coordinates, i.e. one translational coordinate and five rotational coordinates [7][26][27]. In addition, researchers will also analyse and predict Stroller movements with and without friction.

The problem solved in this research is how to analyse the Stroller motion in the computational space based on computational physics computation. This research will apply technology in solving general equations of a motion system in three-dimensional space. Considering the growing development of science and technology in the world of education, the dynamics of objects that have a configuration space that is quite complicated because it consists of translational and rotational motion which is very complicated if solved manually [28]. This research is a solution for lecturers and students in completing complex dynamics of objects thoroughly and precisely. This study aims to analyse the dynamics of the mechanical system on the Stroller with non-holonomic constraints that move in the translation configuration space using physics computing.

The TT motion equation by applying group theory in the form of a rotational group using the Poincare equation in the flat plane has been formulated by Langerhock with physics computation. In addition, previous studies on the dynamics of mechanical systems have only been formulated for TT that operates in the flat plane and the inner surface of the cylinder [12][29]. Therefore, the authors are interested in continuing the research by formulating the dynamics of a system that has more complex movements, namely a stroller that is played in a curved plane in the form of a surface in a ball with fast and without friction. Detailed motion predictions will be analysed using physics computation

## 2. METHOD

This research is a mathematical theoretical study conducted with a review of several libraries of mechanical systems in the case of the Stroller that has been developed previously and mathematical calculations using physics computing, especially based on Maple. This research is composed of four stages with a brief description as follows:

1. Introduction to computation in the form of Maple to physics education students through tutorials on how to use, install, and apply it in calculating equations of three-dimensional space objects.
2. Discussion of studies on general coordinates, configuration space, general force, Euler Lagrange equation, Point care equation, simplification of Point care equation with group theory and PCHS Method, cyclic coordinates and Routhian reduction in students and their application in analysing forces on the Stroller surface on the ball fast using Maple [30].
3. Discussion of Stroller dynamics, Stroller motion equations through the Euler Lagrange equation and reducing the Euler Lagrange equation by using Maple.

## 3. RESULTS AND DISCUSSION

Calculation of the Stroller Constraint is solved by the Port Controlled Hamiltonian System (PCHS) method. Constraints are conditions that limit the movement of a mechanical system thereby reducing both the degrees of freedom and the range of each degree of freedom. Schematic Dynamics Stroller can be seen in Figure 1 below,

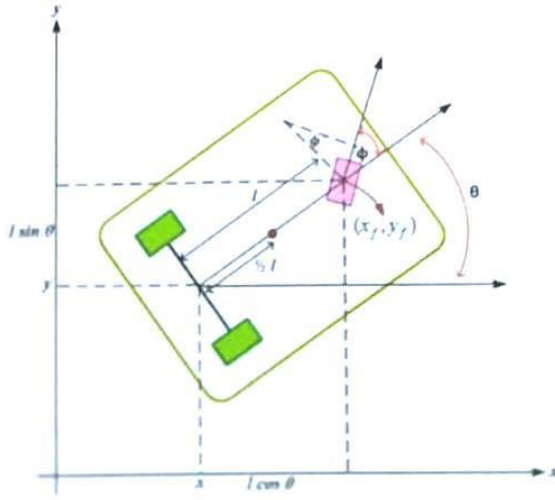


Figure 1. Schematic dynamics stroller

The absolute angle of the front wheels is  $(\theta + \varphi)$ . The front wheels are located on  $(x_{front}, y_{front}) = (x + l \cos \theta, y + l \sin \theta)$  and the front wheel position derivative with respect to time is  $\dot{x}_{front}, \dot{y}_{front}) = (\dot{x} - l \sin \theta \dot{\theta}, \dot{y} + l \cos \theta \dot{\theta})$ . Constraint when not slippage can be written with

$$\begin{aligned} -\sin \theta \dot{x}_{back} + \cos(\theta + \varphi) \dot{y}_{back} &= 0 \\ -\sin(\theta + \varphi) \dot{x}_{front} + \cos(\theta + \varphi) \dot{y}_{front} &= 0 \end{aligned} \quad (1)$$

The constraints for a stroller that moves on a flat surface can be written with

$$\begin{aligned} \tan \theta &= \frac{\sin \theta}{\cos \theta} = \frac{\dot{y}_{back}}{\dot{x}_{back}} \\ \omega^1(q) \dot{q} &= -\sin \theta \dot{x} + \cos \theta \dot{y} = 0 \end{aligned} \quad (2)$$

and

$$\tan(\theta + \varphi) = \frac{\sin(\theta + \varphi)}{\cos(\theta + \varphi)} = \frac{\dot{y}_{front}}{\dot{x}_{front}}$$

$$\omega^2(q) \dot{q} = -(\dot{x} - l \sin \theta \dot{\theta}) \sin(\theta + \varphi) + (\dot{y} + l \cos \theta \dot{\theta}) \cos(\theta + \varphi) = 0$$

$$\omega^2(q) \dot{q} = -\sin(\theta + \varphi) \dot{x} + \cos(\theta + \varphi) \dot{y} + l \cos \varphi \dot{\theta} = 0 \quad (3)$$

if point P has coordinates  $(x, y, z)$  in a Cartesian coordinate system, then that point has coordinates  $(\xi, \eta, \mu)$ , such that

$$\begin{aligned} x &= a \cosh \xi \cos \eta \cos \mu \\ y &= a \cosh \xi \cos \eta \sin \mu \\ z &= a \sinh \xi \sin \mu \end{aligned} \quad (4)$$

by using an exact differential,

$$dx = \frac{\partial x}{\partial \xi} d\xi + \frac{\partial x}{\partial \eta} d\eta + \frac{\partial x}{\partial \mu} d\mu$$

$$\begin{aligned} &= -a \cosh \xi \sin \eta \cos \mu d\eta \\ &\quad - a \cosh \xi \cos \eta \sin \mu d\mu \end{aligned} \quad (5)$$

$$\begin{aligned} dy &= \frac{\partial y}{\partial \xi} d\xi + \frac{\partial y}{\partial \eta} d\eta + \frac{\partial y}{\partial \mu} d\mu \\ &= -a \cosh \xi \sin \eta \sin \mu d\eta \\ &\quad + a \cosh \xi \cos \eta \cos \mu d\mu \end{aligned} \quad (6)$$

Equation (5) is substituted into equations (2) and (3), then the transformation of the Cartesian coordinates to the coordinates of the ball spheres, we get the stroller obstacle that moves on the surface of the ball spherical greetings that can be written with

$$\begin{aligned} \omega^1 &= -\sin \theta dx + \cos \theta dy \\ &= a \cosh \xi \sin \eta \sin(\theta - \mu) d\eta \quad (7) \\ \omega^2 &= -\sin(\theta + \varphi) dx + \cos(\theta + \varphi) dy \\ &\quad + l \cos \varphi d\theta \\ &= a \cosh \xi \sin \eta \sin(\theta + \varphi - \mu) d\eta + \\ &\quad a \cosh \xi \cos \eta \cos(\theta + \varphi - \mu) d\mu + \\ &\quad l \cos \varphi d\theta \end{aligned} \quad (8)$$

Mathematical Calculation of the Port-Controlled Hamiltonian System Equation in a Stroller (PCHS) constrained on a Stroller that moves on the surface in the ball has a general coordinate  $q^i = [\eta \ \mu \ \theta \ \varphi]$ , and the coordinates of the momentum  $p_i = [p_\eta \ p_\mu \ p_\theta \ p_\varphi]$ , then it can be written with,

$$H(q, p) = \left[ \frac{p_\eta^2}{2ma\sqrt{\sinh^2 \xi + \sin^2 \eta}} + \frac{p_\mu^2}{2ma \cosh \xi \cosh \eta} + mga \sinh \xi (1 + \sin \eta) \right] \quad (9)$$

While the Stroller constraint equation can be written in the form of a matrix with,

$$[0] = \begin{bmatrix} \cosh \xi \left( \frac{p_\eta \sin \eta \sin(\theta - \mu)}{m \sqrt{\sinh^2 \xi + \sin^2 \eta}} \right) \\ \frac{\cosh \xi}{m} + p_\theta l \cos \varphi \end{bmatrix} \quad (10)$$

When an ellipse is rotated on its short axis, the surface that is swept by an ellipse is a two-dimensional shape called an oblate spheroidal. This coordinate system is called the spherical coordinate system which corresponds to one of its coordinates  $(\xi = \text{constant})$  in the form of a spherical ball. In this coordinate system each point in the cast is marked by three numbers  $(\xi, \eta, \varphi)$  with  $0 \leq \xi \leq \infty, 0 \leq \eta \leq \pi$  and  $0 \leq \mu \leq 2\pi$ . A point P which has coordinates  $(x, y, z)$  in a Cartesian coordinate, has coordinates  $(\xi, \eta, \varphi)$ , such that,

$$\begin{aligned} x &= a \cosh \xi \cos \eta \cos \mu \\ y &= a \cosh \xi \cos \eta \sin \mu \end{aligned}$$

$$z = a \sinh \xi \sin \mu \tag{11}$$

With a positive real number. The spherical ball is displayed in the form of an orthogonal area, which is in the form of a spheroidal spheroidal and single-leaf hyperboloid and flat plane through the z axis with  $\mu = constant$ . The scale factor in the ball is worth,

$$\begin{aligned} h_\xi &= h_\eta = a \sqrt{\sinh^2 \xi + \sin^2 \eta} \\ h_\mu &= a \cosh \xi \cos \eta \end{aligned} \tag{12}$$

The origins of the study of the non holonomic system were explained by Cohen in his 1977 book, that the birth of the theory of the dynamics of the un holonomic system occurred at the time of the acquisition of general analytical formalism included in the Euler-Lagrange equation, which then attracted the attention of leading scientists [6][23]. There are many problems that constitute non holonomic mechanical systems such as robotics, vehicle dynamics, continuous motion, and constrained systems.

In the last thirty years, the concept and use of robots has evolved, both for industry, health, research and in the household. The word robot was introduced in 1921 by Tired in Rossum’s Universal Robots (R. U. R) by describing robotic machines that resemble people, but can work tirelessly. Developing a mathematical basis for understanding the manipulation of robots that behave not holonomically by formulating kinematics, dynamics, and control of robot manipulators [31][32]. Ariska has developed a general mathematical formulation to study the system of non holonomic mechanics [25][33].

Snakeboard and roller racer are closely related to the robot system with non holonomic constraints. Analysing about the snakeboard model, and the dynamics and controlling forces on the snakeboard by using the Lagrange equation [5][13]. Meanwhile, the study of roller racers was discussed by Gray et al [19]. Gray et al in their article they present continuous motion through cyclic variations in the degree of freedom chosen in mechanical systems which are subject to non holonomic constraints, which decompose geometry, mechanics, and controlled motion of the roller racer. Then Krishnaprasad and Tsakiris in 1998 complete the kinematics analysis on roller racers with a system using the Lagrange-d'Alembert motion equation [6][20][31].

Ostrowski explained that a controlled system with un-holonomic constraints, external forces, and symmetry for example in the case of snakeboard and roller racer can be described by constrained connections through the selection of appropriate connections, which are called Ostviški Levi-Civita constraints, et al. Formulating the right way to calculate the Christoffel symbol for constrained Levi-Civita connections, and to find out the influence of external forces on equations of motion. To

add insight into system dynamics [17][29][31]. Ariska propose a Controlled Hamiltonian System which is related to momentum and holonomics . This study tries to display the dynamics of the tricycle that moves on a flat plane and the surface of the spherical ball with a constrained Levi-Civita connection, and find the PCHS equation on the tricycle [17].

Research on the dynamics of non holonomic objects has been discussed in several scientific articles including Gray et al, Ueda et al, Ariska, and Ciocci et al, Gray et al proposed the formulation of Evan's theorem about the partial immersion of the tricycle system and explained the relationship between embedding and patterned holes [7][19][21][33]. Ueda, et.al. suggested the optimal operation of the Tricycle in a limited time with heat conduction losses and described the development of mathematical models for non holonomic systems. The mathematical model of objects is created using the Euler Lagrange framework. The model is implemented using the C MEX function in MATLAB. Ueda, et al. reviewed a controlled system to review a controlled system for all robots that move with wheels both hardware and software, based on electronic controlled techniques [2][12].

The article aims to find the principle limit on the net effect of the heat engine operating process within a limited time constraint, in addition to identifying and modelling the loss mechanism during hot engine operation. The computation is based on Maple 18 which is used to model the Stroller motion equation with the PCHS Method. Fields vector for Stroller that moves on a flat plane can be written with the matrix as follows,

$$\begin{aligned} X_1 &= l \cos \theta \frac{\partial}{\partial x} + l \sin \theta \frac{\partial}{\partial y} + \tan \varphi \frac{\partial}{\partial \varphi} \\ X_2 &= \frac{\partial}{\partial \varphi} \end{aligned} \tag{13}$$

Proof that these two vector fields are orthogonal,

$$\begin{aligned} &\langle\langle X_1, X_2 \rangle\rangle \\ &= [l \cos \theta \quad l \sin \theta \quad \tan \varphi \quad 0] \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & J_f \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \end{aligned}$$

The Vector Field for a Stroller that moves on the surface in an ellipse is,

$$X_1 = l \cos \eta \cos(\theta - \mu) \frac{\partial}{\partial \eta} - l \sin \eta \sin(\theta - \mu) \frac{\partial}{\partial \mu}$$

$$X_2 = \frac{\partial}{\partial \varphi} \tag{14}$$

Proof that these two vector fields are orthogonal,

$$\langle (X_1, X_2) \rangle = [x_1^\eta \quad x_1^\mu \quad x_1^\theta \quad 0] \begin{bmatrix} ma\sqrt{\sinh^2 \xi + \sin^2 \eta} & 0 & 0 & 0 \\ 0 & \cosh \xi + \cos \eta & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & J_f \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ x_2^\varphi \end{bmatrix}$$

so that it is obtained with Maple 18-based physics computation that the PCHS equation is constrained for a stroller that moves on a flat plane with low energy,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \\ \dot{p}_x \\ \dot{p}_y \\ \dot{p}_\theta \\ \dot{p}_\varphi \end{bmatrix} = \begin{bmatrix} \frac{p_x}{m} \\ \frac{p_y}{m} \\ \frac{p_\theta}{J} \\ \frac{p_\varphi}{J} \\ -\lambda_1 \sin \theta - \lambda_2 \sin(\theta + \varphi) \\ -\lambda_1 \cos \theta - \lambda_2 \cos(\theta + \varphi) \\ \lambda_2 l \cos \varphi \\ 0 \end{bmatrix} \tag{15}$$

with an outer variable,

$$y^1 = \frac{p_x}{m} + \frac{p_y}{m} = \frac{1}{m}(p_x + p_y) \tag{16}$$

While the PCHS equation is constrained for a stroller that moves on the surface in the ball sphere is,

$$\dot{\eta} = \frac{p_\eta}{ma\sqrt{\sinh^2 \xi + \sin^2 \eta}}$$

$$\dot{\mu} = \frac{p_\mu}{ma \cosh \xi \cos \eta}$$

$$\dot{\theta} = \frac{p_\theta}{J_f}$$

$$\dot{p}_\eta = \frac{\sin \eta}{2 ma} \left( \frac{p_\eta^2 \cos \eta}{(\sinh^2 \xi + \sin^2 \eta)^{3/2}} - mga \sinh \xi \cos \eta \right)$$

$$\dot{p}_\mu = a \cosh \xi \cos \eta (\lambda_1 \cos(\theta - \mu) + \lambda_2 \cos(\theta + \varphi - \mu))$$

$$\dot{p}_\theta = (\lambda_2 l \cos \varphi)$$

$$\dot{p}_\varphi = 0$$

with an outer variable,

$$y^1 = \frac{1}{ma} \left( \frac{p_\eta \cos \eta}{\sqrt{\sinh^2 \xi + \sin^2 \eta}} + \frac{p_\mu}{\cosh \xi \cos \eta} \right) \tag{17}$$

#### 4. CONCLUSION

Mechanical systems with non holonomic constraints for Stroller that move on a flat plane and on the surface in a ball can be described by the method of the Port Controlled Hamiltonian System (PCHS), which is a dynamic system that can be described by a set of differential equations and the energy system will be stated expressly. The results of this study as a basis for grounding in examining mechanical systems with non holonomic constraints on a stroller that moves in other curvilinear fields with unrestricted energy, and the PCHS equation on a stroller can be used to find dynamic equations for research on other geometric mechanics.

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