Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Utilization of physics computation based on maple in determining the dynamics of tippe top

To cite this article: M Ariska *et al* 2019 *J. Phys.: Conf. Ser.* **1166** 012009

Utilization of physics computation based on maple in determining the dynamics of tippe top

M Ariska[1], H Akhsan[1], M Muslim[1]

[1] Physics Education departement, Universitas Sriwijaya, Palembang, Indonesia

*Corresponding author's email: ariskamelly@yahoo.co.id.

Abstract. Physics computing can be used to help solve complex dynamic equations, both translation and rotation. The purpose of this research is to obtain the tippe top's dynamic equation using computational physics based maple. The equation of tippe top motion has been reduced by the Routhian reduction method with the Poincare equation with computational assistance. Computation has also been carried out in the search for numerical solutions of tippe top dynamics using the Maple program. Dynamics of tippe top can use a decrease in the Poincaré equation. However, the Poincaré equation requires that quasi coordinates be found from quasi velocity, whereas for the case of reverse topping dynamics cannot be found an exact solution of quasi coordinates of quasi velocity. Therefore, the equation of tippe top must be reduced so that the reverse tippe top problem can be solved. This method can reduce the equation of tippe top motion moving in the flat plane clearly in the form of a set of differential equations. This research has reduced the equation of top tippe motion on the inner surface of the tube and solve the equation of tippe top motion by utilizing physics computation based maple. The findings of this study are equations of tippe top in 3D space in the form of differential equations which can be clearly described using computing.

1. Introduction

Motion Tippe top in various arena an example of the daily motion of rigid body systems with non-holonomik constraints, but the studies are not simple mechanics. Tippe top is a kind of tops that have at truncated spherical shape with a small stem as a handle and can reverse itself in a state of spin. When the ball is rotated at a high-speed corner on a flat surface, then it will turn Tippe top spin on the stalks before. This phenomenon is called inversion [1, 2]. In previous studies, the equation of Tippe top back to top formulated engaged in the flat by using various methods such as Eular equation and Maxwell-Bloch equations. Authors interested in formulating the dynamics of Tippe top if played well in the field of flat and curved areas of the surface in a tube [3]. The author will first review the dynamics of Tippe top in a horizontal plane with the Poincaré equation and then proceed to review the dynamics of a Tippe top surface moves in the tube. Poincaré equations chosen by the authors because this equation can formulate the dynamics of complex systems such as the system moves at the same translational rotation [4]. Rotational dynamics difficult formulated with Eular-Lagrange equation for the rotational dynamics contain angular velocityis generally not a direct derivative of the general coordinates. This is due to the rotation of the generator is not commutative, so that therotational dynamics is difficult if done with Eular-Lagrange equation. Poincaré equations chosen by the authors because this equation can formulate the dynamics behind with clear tops [5].

Moreover, Poincaré equations can describe the dynamic system in the form of systems of differential equations. This thesis is an attempt to under stand the motion of Tippe top by using group the or in the simplification of the equations of motion Tippe Top via Poincaré equations [6]. The purpose of this research is to reduce the equations of motion tippe top on the configuration space flat and curve via the Poincaré equation and understand the dynamics of tippe top on the configuration space flat and curve. The origin of the movement tippe top research described in a 1890 book by John Perry are experimenting with turn in ground stones which are found in the beach. Perry explained that this round stone has a center of mass which does not coincide with the geometrical center of the stone. When the stone is rotated, the center of mass is higher away from the soil surface [7, 8]. A description of the movement of the tippe top start poured in several scientific articles in the 1950s, including by Pliskin in 1953 which states that the friction interaction at tippe top of the floor instrumental in tippe top's lap. While Synge 1952 explains that the phenomenon is due to the movement of the tippe top instability dynamics without involving friction. Further more, Del Campo in 1955 describes in detail the mathematical calculations regarding the role of friction in tippe top. Del Campo concluded that influencing of friction evention tippe top inversion [9, 10].

2. Methods

This study is a theoretical mathematical study. The study was conducted by a review of some literature about the mechanical systems on tippe top case that has been previously developed and mathematical calculations. Poincaré equation can be written by,

$$\frac{d}{dt}\left(\frac{\partial \bar{T}}{\partial s^i}\right) - c^r{}_{li}(q)s^l\frac{\partial \bar{T}}{\partial s^r} - \frac{\partial \bar{T}}{\partial \sigma^i} = S_i. \tag{1}$$

However, this equation requires that the discovery of quasi velocity as a direct derivative of the time of the coordinate quasi. Meanwhile, in the case of tippe top's quasi velocity is not owned directly derived from a cyclic coordinate [11]. Therefore, Poincaré equation used in this study to analyze the dynamics of tippe top on a flat surface and the surface of the tube is Poincaré equation that is based on the reduction Routhian, which can be written as follows

$$\frac{d}{dt}\frac{\partial R}{\partial v^\rho} - \sum_{\mu=2}^{n}\sum_{\lambda=2}^{n} c^\lambda{}_{\mu\rho}v^\mu\frac{\partial R}{\partial v^\lambda} - \sum_{\mu=2}^{n} c^\lambda{}_{\mu\rho}v^\mu\beta_1 - X_\rho R = 0. \tag{2}$$

In classical mechanics, the movement of rigid bodies is generally described by two analogous vector equations: $\mathrm{F} = \mathrm{dp/dt}$ for translation of the centre of mass and $\mathrm{M} = \mathrm{dL/dt}$ for rotation around the centre of mass, with F the total external force, p the momentum, M the total moment of external forces, and L the angular momentum. We consider the intriguing movement of the tippe top. It consists of a spherical body and a cylindrical stem, with the centre of mass (CM) displaced from the centre c of the sphere (see Fig. 1). When initially put into rotation around its axis of symmetry $\hat{e}_3$ vertical, the stem gradually moves downwards and finally the top flips over into a stable vertical rotation on the stem. Apparently the rotation has changed sign, while vector L has preserved its original vertical position. Further, CM has moved upwards at the cost of a decrease in magnitude of L [12].

This unexpected behaviour is explained by the action of a friction force Fat the (slipping) contact point of the top with the surface. F causes a moment M, which can be imagined to have vector components $\mathrm{M}_{n,n'}$ and M_3, the latter along the axis of symmetry $\hat{e}_3$ [13]. Likewise, the angular momentum L has components $\mathrm{L}_{n,n'}$ and L_3. In the beginning, $\mathrm{L}_3 = \mathrm{L}$ and $\mathrm{L}_{n,n'} = 0$. Then, due to instability, F origin ates and the resulting M_3 tends to decrease L_3, while $\mathrm{M}_{n,n'}$ starts to increase $\mathrm{L}_{n,n'}$.

As L remains constan, the angle θ of the top's inclination will grow to fulfil proper vector addition.When $\theta = \frac{\pi}{2}$, L_3 = 0and $L_{n,n'} = L$ [14].

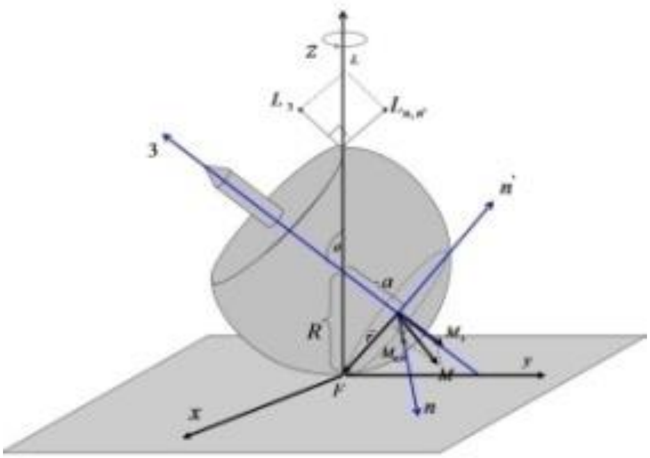

Figure 1. Coordinats of Tippe Top

Then the rotation along $\hat{\boldsymbol{e}}_3$ changes sign and, again through the action of $\mathbf{M}_{\boldsymbol{n},\boldsymbol{n}'}$ and $\mathbf{M}_3$. $\mathbf{L}_3$ starts to grow at the cost of $\mathbf{L}_{\boldsymbol{n},\boldsymbol{n}}$'. Finally, the stem will scrape the surface (see Fig. 1) and through the action of a new frictional force **F'** with moment **M'** the top will lift itself up and strive towards a stable, though extinguishing, rotation on the stem. In fact, the component $\mathbf{L}_{\boldsymbol{n},\boldsymbol{n}'}$ is extinguished by the new $\mathbf{M}_{\boldsymbol{n},\boldsymbol{n}'}$ and $\mathbf{L}_3$ finally becomes equal to **L** [15, 16].

During the inversion process takes place at the center of mass of the Tippe top elevated. This suggests rotational kinetic energy decreases during the inversion takes place as a result of potential energy has increased, so that the total angular velocity and total angular momentum decreases during the inversion process. This process can be seen in Figure 2 which shows the Tippe top inversion process [17, 18].

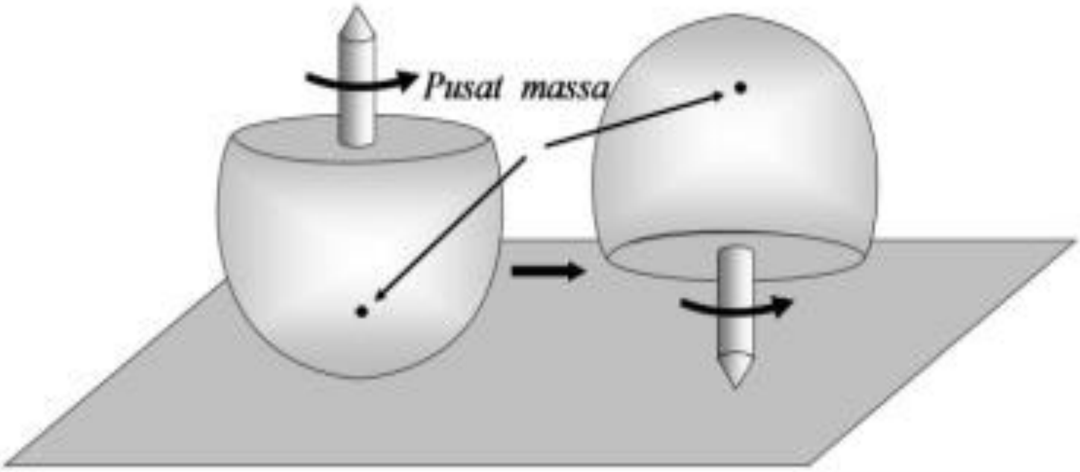

Figure 2. Centre of Mass Tippe Top

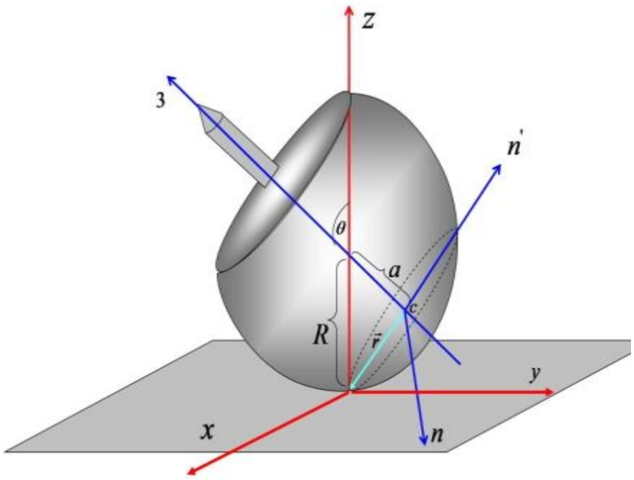

Figure 3. Tippe top's Motion

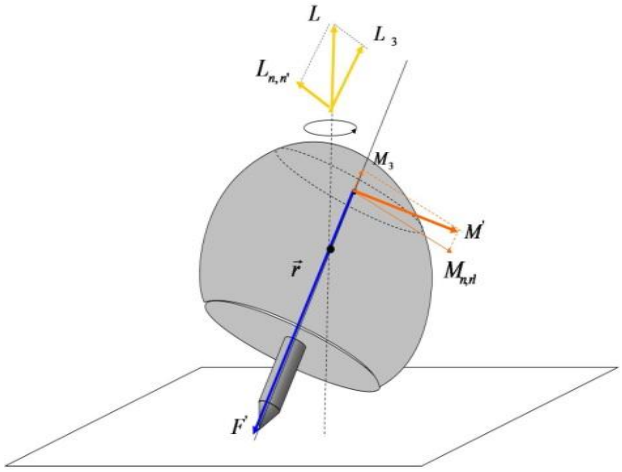

Figure 4. Tippe top Inverting

3. Results and Discussion

3.1. The dynamics of a Tippe top moves in a horizontal Plane Through the Poincaré Equation

The moment of the total general force on the tippe top moving in a horizontal plane is expressed in one-form are :

$$|\mathrm{F}_N| = mg + m\ddot{z} = mg + ma(\dot{\theta}^2 \cos\theta + \ddot{\theta}\sin\theta). \tag{3}$$

Numerical solution of the equations of motion Tippe top to coordinate $\theta(t), \phi(t), \dot{\theta}(t), \dot{\phi}(t), \dot{x}(t)$, and $\dot{y}(t)$, which has the following initial conditions :
$I_n = I_{n'} = I = 45\ gr.cm^2, I_3 = 50, \alpha = 60^0, m_k = 1\ gr, m_t = 3\ gr, m_b = 13\ gr, m_{total} = 17\ gr, a = 0{,}6\ cm, R = 1{,}3\ cm, D = 2{,}6\ cm$. Value of the initial condition $\theta(0) = 0{,}1\ rad, \phi(0) = 0, \dot{\phi}(0) = \dot{\theta}(0) = 0$, and $\dot{x}(0) = \dot{y}(0) = 0, \beta_1 = 2500\ gr.m^2.rad/s$ and assumed $\mu = 0{,}3$.
Graphic images obtained are:

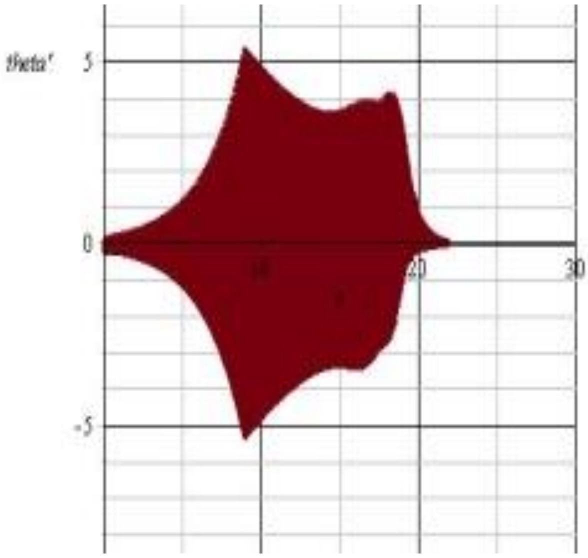

Figure 5. Graph $\dot{\theta}$ to t

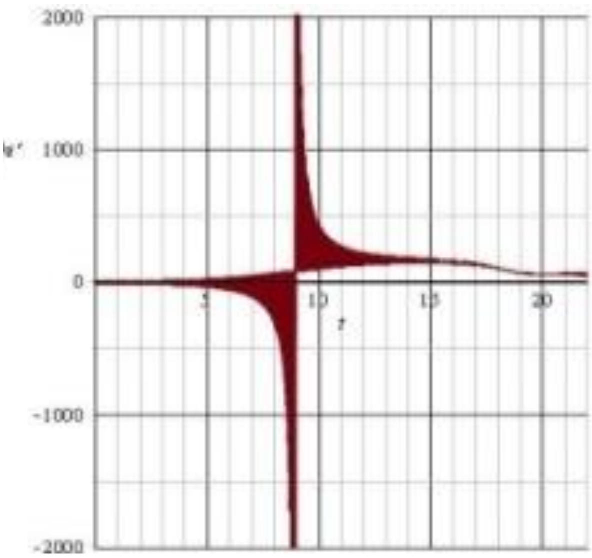

Figure 6. Graph $\dot{\Phi}$ to t

3.2. The Dynamics 0f A Tippe Top Moves In The Tube Inner Surface Through The Poincaré Equation.
Dynamics of Tippe top can be analyzed by assuming that the radius of the tube is much greater than the radius tops behind ($r \gg R$). Several dynamics defined in Tippe top axis in the tube as follows:

a. Axis is fixed to the space (X, Y, Z).
b. The axis of the stem at the point of contact Tippe top (X', Y', Z').
c. Axis which origin at in the mass of Tippe top, and always remain on tippe top $(1,2,3)$.
d. The axis of the stem in the center of mass tippe top (x, y, z) [19, 20].

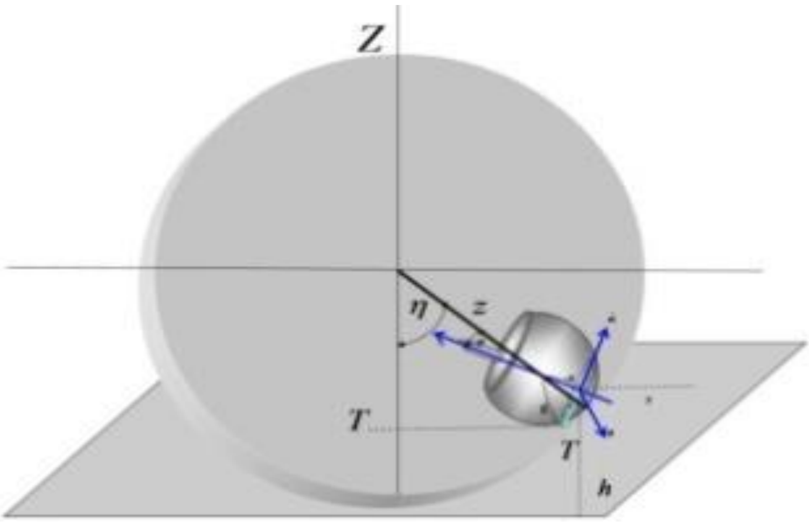

Figure 7. Tippe top Played in the Tube

3.3. Potential Energy Tippe Top Played In The Tube
Potential energy tippe top played in the tube,

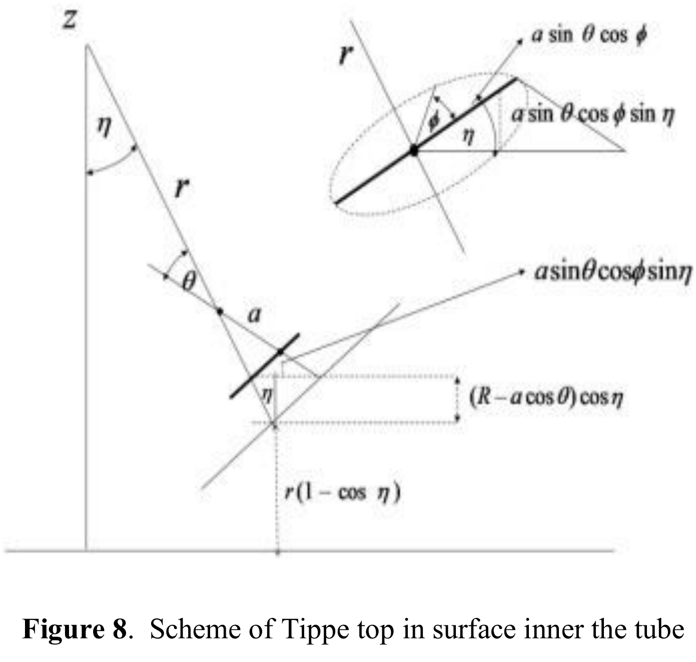

Figure 8. Scheme of Tippe top in surface inner the tube

In Figure 8 Tippe top has a potential energy of

$$U = mgh$$
$$= mg(r(1-\cos\eta) + (R - a\cos\theta)\cos\eta + a\sin\theta\cos\phi\sin\eta) \quad (4)$$

The moment of the general style tops behind the move son the surface of the tube, can be written according to

$$\mathbf{Q} = \mathbf{F}_f \cdot \hat{e}_X dX + \left(-r\hat{e}_z \times \vec{F}_f\right) \cdot \hat{e}_X d\eta + \left(\vec{r}_p \times \vec{F}_f\right) \cdot (\hat{e}_x d\theta + \hat{e}_z d\phi + \hat{e}_3 d\psi). \quad (5)$$

Based on the equations, graph stops dynamics behind in the tube with afriction surface can be described as follows

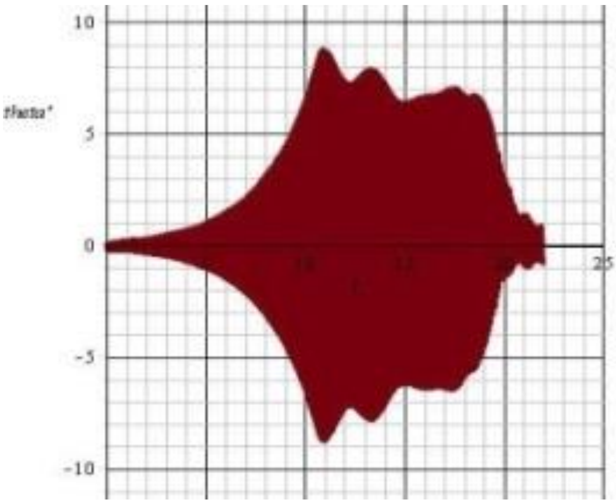

Figure 9. Graph TT in tube

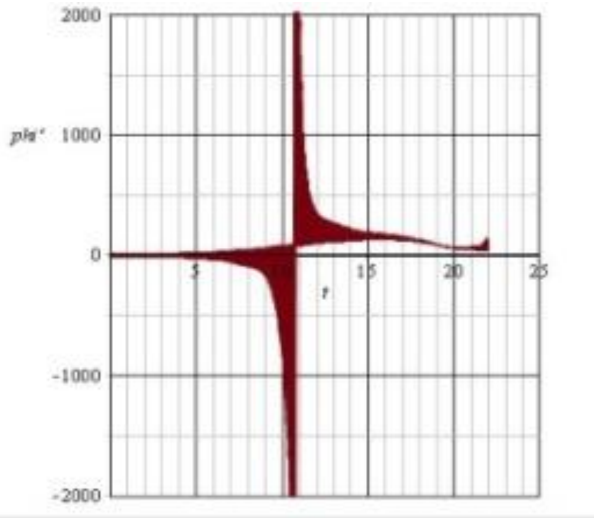

Figure 10 Graph of $\dot{\Phi}$ to t

Based on the Figure 9 it can be seen that the dynamics of turning in a top surface of the rod in tube tops more random than turning on a flat surface and the reversal process requires a longer time is 22 second while in one second flat surface 20 second. Likewise with the initial condition when the tilt rod is rotated back tops also have a difference between turning in a top surface of the tube and in the horizontal plane. Behind the flat tops in the field of stem tilt boundaries while playing behind a top that flips is θ (0) = 0.9 rad. Meanwhile, a top trunk tilt boundary moving through the tube surface is θ (0) = 0.2 rad, if the slope is more than 0.2 rad turning in a top surface of the tube cannot be flipped.

4. Conclusion

Based on the study of the dynamics of Tippe Top via Poincaré equation on configuration space flat and tube the following conclusions can be drawn: (1) Mechanical system with constraints for Tippe top holonomic not engaged in the flat and the tube surface can be described by equations of Poincaré, the system dynamics can be described by a set of differential equations and the energy system clearly stated; (2) Tippe top that moves in a horizontal plane with friction can be seen in Figure 5 and Figure 6 which states that after Tippe top spin for a few seconds then slowly. Tippe top will experience a reversal, then after $\theta(t)$ approach angle of π, Tippe top will spin stabilized with trunk without precision on axis $\hat{e}_z$; (3) Tippe top moves with friction in the tube inner surface can be seen in Figure 9 and Figure 10 which states that the tops behind will experience a reversal after spinning for 22 seconds, then after $\theta(t)$ form an angle of π, turning a top will spin stabilized with trunk without precision on axis $\hat{e}_z$. So, after flipping through a top will spin with precision and tippe top stem turning in steady state.

References

[1] Ciocci MC, Malengier B, Langerock B and Grimonprez B. 2011. *Journal of Applied Mathematics*. **2012** (*4*) :15.
[2] Talman, Richard. 1999. *Geometric Mechanics*. Wiley-VCH Verlag GmbH and Co. KgaA.
[3] Rosyid, M. F., 2009, *Keragaman Licin untuk Fisikawan (Penghantar Hitung Tensor),* Diktat Kuliah Metode FisikaTeoritik pada Program S2 Fisika UGM, Jurusan Fisika FMIPA UGM, Yogyakarta.
[4] Goldstein, H., 1980, *Classical Mechanics*, Addison-Wesley, Cambridge.
[5] Karapetyan A.V., 1994 *J. Appl. Phys*. **5**(*8*), 387.
[6] Holm, D. D., Schmah, T. and Stoica, C., 2009, *Geometry Mechaanics and Symetry from Finite to Infinite Dimensions*, Oxford University Press, New York.
[7] Hall, Brian C. 2003. *Lie Groups, Lie algebras, and representations*-Verlag.
[8] Langerock, Bavo., 2007. *arXiv:0704.1221v1[math.DS]*. **8**(*1*):78.
[9] Bou-rabee NM, Marsden JE and Romero LA. 2004. *SIAM Journal Applied Dynamical Systems*; **3**(*3*): 352.
[10] Rauch-Wojciechowski S, Sköldstam M and Glad T. 2005. *SIAM Journal.* **10**(*4*): 333.
[11] Moritz B, Whiting E and Sorkine-hornung O. 2008. *J. Appl. Phys*. **7**(*9*), 527.
[12] Marsden JE and Romero LA. 2008. *SIAM Journal Applied Dynamical Systems.* **50**(*2*): 431.
[13] Cohen,C. M. 1977. *Am. J. Phys.***45***(3)*.12.
[14] Fowles and Cassiday. 2005. *Analytical Mechanics*. Thomson Learning. Inc. United States of America.
[15] Gray CG and Nickel BG. 2000. *American Journal of Physics*; **68**(*9*): 821.
[16] Yue Chan . 2010. *Journal of Intelligent Transportation Systems*. **79**(*5*) :56.
[17] Blankenstein G. 2003. *J. of Robotics Research*; **12**(*3*):197.
[18] Broer H, Krauskopf B, Vegter G and Simó C. 2001. *Global Analysis of Dynamical Systems*. **17** (*7*): 96.
[19] Zhou Y, Sueda S and Matusik W. Boxelization . 2006. *J. Appl. Math. Mech*. **8** (*2*) : 387.
[20] Ueda T, Sasaki K and Watanabe S. 2005. *Society for Industrial and Applied Mathematics*. **4**(July): 52.