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Research Article

Utilization of Maple-based Physics Computation in Determining the Dynamics of Tippe Top

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Abstract

Tippe top is an example of simple moving system of rigid body with non-holonomic constraint, but the analysis of this system is not simple. A tippe top equation has been derived with Routhian reduction method and Poincaré equation, and physics computation in finding numeric solution of the dynamics of the tippe top has also been utilized by using Maple program. However, the Poincaré equation required that quasi-coordinate of the quasi-velocity is found, while in the case of the dynamics of tippe top, there is not any exact solution of the quasi-coordinate of the quasi-velocity was found. Therefore, the tippe top equation should be reduced to solve the problem. In this research, Routhian reduction was employed so that the Routhian reduction-based Poincaré equation was used to derive the tippe top equation. The method was able to derive a tippe top equation on a flat plane and tube inner surface clearly represented differential equations.

Keywords: *mechanics, Poincaré equation, Maple, physics computation*

Pemanfaatan Komputasi Fisika Berbasis Maple dalam Menyelesaikan Dinamika Tippe Top

Abstrak

Komputasi fisika dapat digunakan dalam membantu menyelesaikan persamaan dinamika benda yang kompleks, baik translasi maupun rotasi. Tujuan penelitian ini adalah mendapatkan persamaan dinamika gasing balik dengan menggunakan komputasi fisika berbasis maple. Persamaan gerak gasing balik telah diturunkaan dengan metode reduksi Routhian dengan persamaan Poincare dengan bantuan komputasi, dan telah pula dilakukan komputasi dalam pencarian solusi numerik dinamika gasing balik dengan menggunakan program Maple. Dalam penelitian ini reduksi yang digunakan adalah reduksi Routhian, sehingga persamaan yang digunakan dalam menentukan persamaan gerak gasing balik adalah persamaan Poincaré yang didasari oleh reduksi Routhian. Metode ini dapat menurunkan persamaan gerak gasing balik yang bergerak di bidang datar dengan jelas berupa himpunan persamaan diferensial. Penelitian ini dapat dilanjutkan dengan menyelesaikan persamaan dinamika gaing balik di bidang melengkung seperti tabung da bola. Tujuan penelitian ini adalah menyelesaikan persamaan gerak gasing balik dengan memanfaatkkan komputasi fisika berbasis maple. Hasil temuan penelitian ini adalah persamaan gerak gasing balik *pada ruang 3D berupa persamaan diferensial yang dapat digambarkan dengan jelas menggunakan komputasi.*

Kata Kunci: *Mekanika, Persamaan Poincaré, Komputasi Fisika, Maple*

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I. INTRODUCTION

Rosyid reveals that a diversity is basically a slippery curve or surface or similar objects with a higher dimension. Dimensional real diversity n is a topological space that is locally homeomorphic with R^n, meaning that each point in diversity has an environment similar to an environment in R^n [1]. The state of diversity in a space depends on the space topology. Diversity is seen as a topological space as there is an open set that will be used as an environment (or coordinate domain) [2, 3]. Diversity can also be said to be a space that locally resembles a Euclidean space with certain dimensions.

The sub-diversity of M in R^n, denoted by TM, is a combination of all tangent spaces on M, that is,

$$TM = \bigcup_{x \in M} T_x M \tag{1}$$

The tangent projection is the mapping $\pi : TM \rightarrow M$ given by $(x, v) \mapsto x$, which is the projection that carries the tangent vector to its base point [4, 5]. Symplectic geometry is very different geometry which is used in mechanics in Hamilton's formulation. Symplectic geometry can still be expanded to Poisson geometry. Vector fields are basically ordinary differential equations. Vector field, generally tensor and differential form used in all branches of differential geometry [6].

Fowles states that choosing the right general coordinates will be easier to solve mechanical system problems [7]. A mechanical system can be expressed in various coordinate systems, such as in the case of a mathematical pendulum with two holonomic constraints, so that the degree of freedom is one, polar coordinates can be chosen with the coordinate θ. When choosing a coordinate system, the domain for the coordinate system must also be considered, so that all the points that may be occupied by particles can be expressed by the coordinate system. The Lagrange equation is a reduced Poincaré equation. The Poincaré equation is an announcement for the Lagrange equation. When the Lagrange equation is difficult to apply for certain cases, the Poincaré method can be used by paying attention to the symmetry of the system being reviewed [7]. Back-up motion in various arenas is a daily example of rigid body motion systems with non-holonomic constraints. In the study of mechanics, however, the motion systems are not simple. The tippe top, sometimes called as a reverse top, is a type of top which has a spherical shape cut with a small rod as a handle and can flip itself in a rotating state. When the part of the ball is rotated with a high angular velocity on the surface of the flat plane, then the tippe top will turn around the part of the stem. This phenomenon is called inverse [8, 9].

In the previous study, the equations of reverse spinning were formulated for tippe top moving in a flat plane using various methods such as the Euler equation and the Maxwell-Bloch equation [9]. In this research, it is preferable to formulate the tippe top motions when it is played on a flat plane using the Poincaré equation and draw the motions using Maple 18 based physics computing. Initially, the motions of the tippe top in a flat plane should be reviewed with the Poincaré equation and proceed to review the motions on the inner surface of the tube. The Poincaré equation is chosen by the authors as this equation is believed to formulate the dynamics of a complex motion system, such as translational and rotational systems, this is also reinforced by Holm [6] who asserted that the rotational dynamics are difficult to formulate with the Euler-Lagrange equation because the rotational dynamics have angular velocity which is generally not time-derivative directly from the general coordinates. In addition, rotation generator is not commutative, so that the rotational dynamics are difficult if solved by the Euler-Lagrange equation [6, 10].

The Poincaré equation is chosen by the author because this equation can formulate the reverse gas dynamics clearly. In addition, the Poincaré equation can describe a dynamic system in the form of a differential equation system. This research is an attempt to understand the reverse motion by using group theory in simplifying the equations of the reverse motion through the Poincaré equation. The purpose of this study is to reduce the equations of tippe top motions in 3D space on a flat plane through the Poincaré equation and understand the motions of tippe top which moves on a flat plane using computational physics [11, 12].

The origin of research on reverse gasification was described in a book back in 1890 which was written by Cohen [13] who experimented with turning round stones found on the beach. Cole explained that round stone has a center of mass that does not coincide with the center of the geometry of the stone [13]. When the stone is rotated, the center of mass becomes higher away from the surface of the ground. Explanations about the tippe top movement began to be stated in a number of scientific articles since the 1950s, including by Pliskin who stated that the interaction of frictions on the tippe top on the floor plays an important role in reverse spinning [14]. The phenomenon of reverse gasping is the result of dynamics instability without involving frictions. Furthermore, it was developed by Cohen [13] who explained in detail with mathematical calculations regarding the role of friction on the tippe top. Cohen concluded that friction affects the reversal of the tippe top [15].

In the previous studies, there was no clear graphic images of the dynamics of tippe top with various initial conditions when the tippe top was initially rotated [1]. This study provides a clear picture of the dynamics of the tippe top with various slope angles. The results show the graphs that illustrate the dynamics of the tippe top with the Poincare equation clearly and detail analysis in every second with various slope angles. Previous studies did not include a clear picture of detailed dynamics of the tippe top per second [8, 9].

This research will review the tippe top dynamics using the Poincare equation with Maple 18 based physics computing. The researcher will review the reverse and rotational top movement using five general coordinates, namely two general coordinates in translational dynamics and three general coordinates for rotational dynamics. The reverse gas movement will be changed from the initial condition in the form of a tilt angle when the first top is played. Turning dynamics will be observed using Maple 18 based physics computing.

II. RESEARCH METHOD

This research is a theoretical mathematical study with the utilization of physics computations. Computing was carried out with the help of Maple 18. The research was carried out with a review of several studies regarding mechanical systems in the case of previously developed tippe top and mathematical calculations.

Golstein reveals that the Poincare equation can be written with [4],

$$\left(\frac{\partial \bar{T}}{\partial s^i}\right) - c^r{}_{li}(q)s^l \frac{\partial \bar{T}}{\partial s^r} - \frac{\partial \bar{T}}{\partial \sigma^i} = S_i \quad (2)$$

However, this equation requires quasi velocity be found as a direct time-derivative from the temporary quasi coordinate. Therefore, the Poincare equation used in this study to analyze the dynamics of the tippetopon a flat plane is the Poincare equation which is based on the Routhian reduction [2], which can be written as follows:

$$\frac{d}{dt}\frac{\partial R}{\partial v^\rho} - \sum_{\mu=2}^{n}\sum_{\lambda=2}^{n} c^\lambda{}_{\mu\rho} v^\mu \frac{\partial R}{\partial v^\lambda} = 0 \quad (3)$$

III. RESULTS AND DISCUSSION

The rigid body motion is represented by two vector equations, namely $F = dp/dt$ for translation at the center of mass, and $M = dL/dt$ for rotation around the center of mass, with F total is external force. Tippe top consists of a ball and a cylindrical rod with the center of mass being able to move from the center of c on the ball, meaning that it can be straight under the center of geometry or straight above the center of geometry [10, 16].

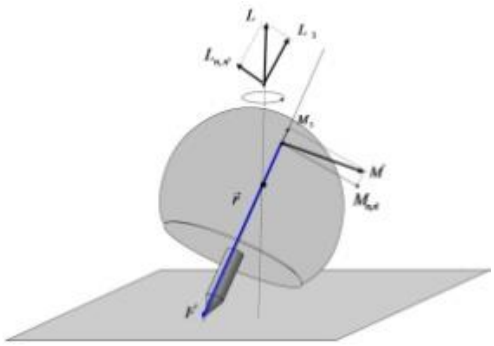

Figure 1. Tippe top reversal process

Initially the top spinning around the symmetry axis is e^3 vertically, then the top spinning rod slowly moves down and finally the head flips up and rotates vertically with the reverse trunk. Rotation changes the reverse direction, while vector L stays in the original vertical position. Furthermore, the center of mass moves upwards due to the decrease in the value of L. This is due to the action of friction F on the top contact point back to the table.

The friction force F causes the appearance of the force M, which can be imagined to have the vector components $M_{(n, n')}$ and M_3 along the symmetry axis e^3. Similarly, angular momentum L has components $L_{(n, n')}$ and L_3. Initially, $L_3 = L$ and $L_{(n, n')} = 0$, due to instability, the friction force produces M_3 which decreases L_3, while $M_{(n, n')}$ starts increasing $L_{(n, n')}$. Since L holds the constant, the angle θ that is the tilt angle of the top will continue to expand, and when $\theta = \pi/2$, $L_3 = 0$ and $L_{(n, n')} = L$. Then the rotation along the 3 axis changes direction, and because the actions $M_{(n, n')}$ and M_3, L_3 starts to increase due to the decrease in $L_{(n, n')}$. Finally, the rod touches the table due to the action of the new force friction force and moment, which is F' with the torque moment M' which makes the tippetop can lift itself stably. The component $L_{(n, n')}$ is delayed by the new $M_{(n, n')}$ and finally L_3 becomes the same as L [17, 18].

During the inversion process, the center of mass takes place on the top of the tippe top. This states that rotational kinetic energy decreases during inversion, and as a result, the potential energy has increased, so that the total angular velocity and total angular momentum decreases during the inverse process [19]. Figure 1 shows the process which shows the inversion process in reverse [20].

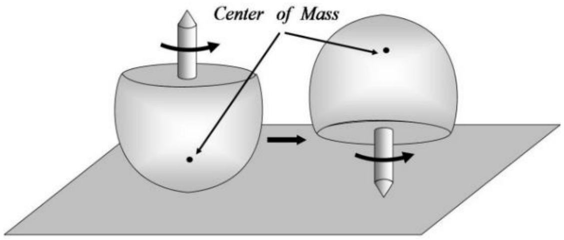

Figure 2. The Center Point of the Mass on the Tippe Top

Routhian is defined by

$$R(q^2, \dots\dots, q^n; \beta_1, v^2, \dots\dots, v^n) = T - U - v^1 \frac{\partial T}{\partial v^1} \quad (4)$$

by referring to the Routhian reduction [2], Poincaré equation is written by maintaining the following variables:

$$\frac{d}{dt}\frac{\partial R}{\partial v^\rho} - \sum_{\mu=2}^{n}\sum_{\lambda=2}^{n} c^\lambda{}_{\mu\rho} v^\mu \frac{\partial R}{\partial v^\lambda} - \sum_{\mu=2}^{n} c^\lambda{}_{\mu\rho} v^\mu \beta_1 - X_\rho R = 0,\ \rho = 2, \dots, n. \quad (5)$$

Completion of the Equations of Motion in the Tippe Top on a Flat Plane

The total general force moment on the tippe top moving in a flat plane is expressed in the following formula:

$$\mathbf{S} = \mathbf{F}_x\, dx + \mathbf{F}_y\, dy + (\vec{r} \times \vec{\mathbf{F}})_\theta d\theta + (\vec{r} \times \vec{\mathbf{F}})_\phi d\phi + (\vec{r} \times \vec{\mathbf{F}})_\psi\, d\psi \quad (6)$$

$$\mathbf{S_x} = -\mu|F_N|\,(\dot{x} - \sin\phi\,\dot{\theta}(R - a\cos\theta) + \sin\theta\cos\phi\,(R\dot{\psi} + a\dot{\phi})) \quad (7)$$

$$\mathbf{S_y} = -\mu|F_N|\,(\dot{y} + \cos\phi\,\dot{\theta}(R - a\cos\theta) + \sin\phi\sin\theta\,(R\dot{\psi} + a\dot{\phi})) \quad (8)$$

$$\mathbf{S_\theta} = -\mu|F_N|(R - a\cos\theta)((R\dot{\psi} + a\dot{\phi})\sin\theta\cos\phi + \dot{\theta}\sin\phi\,(a\cos\theta - R)) \quad (9)$$

$$\mathbf{S_\phi} = -\mu|F_N|(a - R\sin\theta)\sin\theta\left((\cos\phi\dot{x}\,\dot{y}) + \sin\theta\,(a\dot{\psi} + R\dot{\phi})\right) \quad (10)$$

$$\mathbf{S_\phi} = -\mu|F_N|(a - R\sin\theta)\sin\theta\left((\cos\phi\dot{x}\,\dot{y}) + \sin\theta\,(a\dot{\psi} + R\dot{\phi})\right) \quad (11)$$

with Lagrangian tippe top on a flat plane as follows:

$$R = \frac{1}{2}\left(I\dot{\theta}^2 + I\sin^2\theta\,\dot{\phi}^2\right) - mg(R - a\cos\theta) - \frac{(\beta_1)^2}{2I_3} + \beta_1\dot{\phi}\cos\theta \quad (12)$$

with

$$\beta_1 = I_3(\dot{\psi} + \dot{\phi}\cos\theta) \quad (13)$$

The tippe top which is played in a flat plane without friction is as follows:

$$\ddot{\theta} = \frac{\sin\theta}{I}\left(\cos\theta\,(I\dot{\phi}^2) + (2\beta_1\dot{\phi} + mga)\right) \quad (14)$$

$$\ddot{\phi} = \frac{\beta_1}{I}\dot{\theta}\csc\theta\,(1 - \csc\theta) - \frac{2I}{I}\dot{\theta}\dot{\phi}\cot\theta \quad (15)$$

$$\ddot{x} = 0 \text{ and } \ddot{y} = 0 \quad (16)$$

Reverse motion equation if there is an external force in the form of a reverse force which is a force of constraint is as follows:

$$\ddot{\theta} = \frac{\sin\theta}{I}\left(2\dot{\phi}\beta_1 + \cos\theta\, I\dot{\phi}^2 + mga\right) - \frac{\mu|F_N|\dot{x}}{I}(R - a\cos\theta) \quad (17)$$

$$\ddot{\phi} = \frac{1}{I\sin\theta}\left(-\mu|F_N|\dot{y}(a - R\sin\theta) - \beta_1\dot{\theta}(1 - \csc\theta) - 2I\dot{\theta}\dot{\phi}\cos\theta\right) \quad (18)$$

$$\ddot{x} = -\mu\frac{|F_N|}{m}\left(\dot{x} + \left(R\dot{\psi} + a\dot{\phi}\right)\sin\theta\cos\phi + \dot{\theta}\sin\phi\,(a\cos\theta - R)\right) \quad (19)$$

$$\ddot{y} = -\mu\frac{|F_N|}{m}\left(\dot{y} + \left(R\dot{\psi} + a\dot{\phi}\right)\sin\phi\sin\phi + \dot{\theta}\sin\phi\,(R - a\cos\theta)\right) \quad (20)$$

with normal force as follows:

$$|F_N| = mg + m\ddot{z} = mg + ma(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta) \quad (21)$$

Numerical solution of the equations of the reverse motion for coordinates $\theta(t), \phi(t), \dot{\theta}(t), \dot{\phi}(t), \dot{x}(t)$, and $\dot{y}(t)$, with the initial conditions tippe top is in the Table 1.

Table 1. Initial Conditions of Tippe Top

$I_n=I_{n'}=I$ (gr.cm^2)	I_3 (gr.cm^2)	m_{total} (g)	R (cm)	D (cm)
45	50	13	1.3	2.6

The value of initial conditions based on previous studies [1,9,13] is in the Table 2.

Table 2. Initial Conditions Based on Previous Studies

$\theta(t)$ rad	$\phi(0)$ rad	$\dot{\phi}(0)$ rad/s	$\dot{\theta}(0)$ rad/s	$\dot{x}(0)$ cm/s	$\dot{y}(0)$ cm/s	β_1 gm^2rad/s	μ
0.1	0	0	0	0	0	2,500	0.3

The obtained graph is shown in Figure 3.

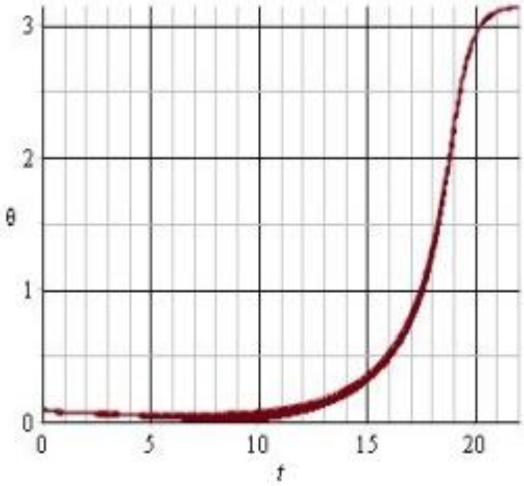

Figure 3. Relationship Between Angle (θ) and Time (t)

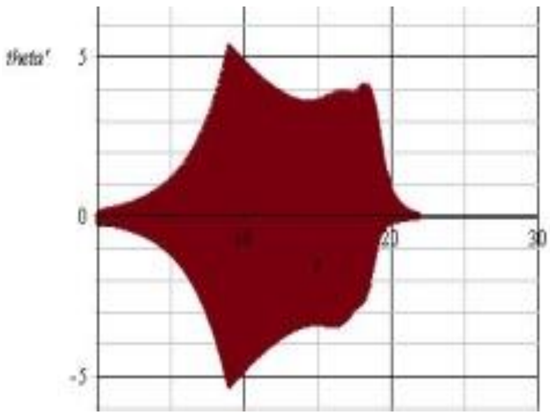

Figure 4. Relationship Between Angular Velocity ($\dot{\theta}$) and Time (t)

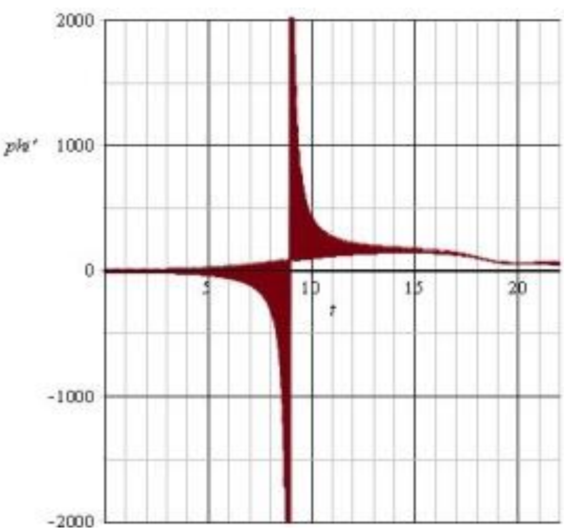

Figure 5. Relationship Between Angle ($\dot{\phi}$) and Time (t)

Based on equations 6, 7, 8, and 9 can illustrate the graphs of the relationship between angle *θ (t)* with time and angular velocity $\dot{\theta}(t)$ with respect to time and graph $\dot{\phi}(t)$. In the three images, it can be seen that the tippe top occurs at the 9th second and stops spinning. The tippe top process occurs in accordance with the predetermined initial requirements. This shows that differential equations that have been derived are true and can be proved computationally by using Maple. The purpose of solving the dynamics system by describing the numerical equations of the tippe top is to compare the results of the solutions that have been calculated with the nature of the qualitative analysis on the tippe top equation studied in this study.

The numerical solution of the equations of reverse motion for the coordinate *θ(t)* which has the initial conditions based on Table 1 and Table 2 shows the main characteristic of the reverse reversal. In the graph it can be seen that the tippe top is spinning then slowly it will experience a reversal at 20 seconds, after *θ(t)* forms an angle of *π*, the tippe top will rotate with its stem stably without precision towards the *z*-axis where the speed of precision and speed the angle *θ(t)* is zero. So, after reversing the top of the tippe top, it will rotate with its stem without precision and experience a steady state.

Based on Figures 3, 4, and 5, it shows that the mechanical system with a non-holistic constraint for tippe top moving in a flat plane can be described by the Poincaré equation, which is a dynamic system that can be described by a set of differential equations and system energy is clearly stated. On the graph showing the tippe top moving on a flat surface without friction can be seen in Figures 3 and 4 which state that if the surface of the flat where the tippe top is moving has no friction (slippery), the tippe top will not reverse. This is expressed by the angular velocity *θ* and the constant angular velocity which illustrate that the tippe top is in a constant stable rotating state with very little precision on the e_z axis and there is no translation movement from the tippe top. As for the tippe top that moves on a flat plane with friction can be seen through Figure 5 which states that after the spinning top rotates for a few seconds, slowly the tippe top will experience a reversal. After *θ(t)* approaches an angle of *π*, the tippe top will rotate stably with the stem without precision on the e_z.

In addition to the previous studies which stated that the dynamics of tippe top were solved by the Euler and Routhian Reduction equations, this study the shows how the dynamics of the tippe top was successfully solved by the Poincare equation which is derived from the Euler-Lagrange equation with the assistance of Maple-based physics computing. The authors are successful in describing the top spin dynamics with the initial conditions that have been determined with various coordinate points in accordance with the configuration space of the tippe top that moves in a flat plane.

IV. CONCLUSION

Based on research on the use of Maple-based physics computation in formulating the dynamics of tippe top, the following conclusions can be drawn: (1) mechanical systems with a non-holistic constraint for tippe top moving on a flat plane can be described by the Poincaré equation, which is a system of dynamics that can be described by a set of differential equations and system energy as clearly stated; and (2) Based on the results of the previous studies on similar object, the dynamics tippe top were solved by the Euler and Routhian Reduction equations. In this study the dynamics of the tippe top dynamics are successfully solved by the Poincare equation which is derived from the Euler-Lagrange equation with the assistance of Maple-based computation. The author succeeded in describing the dynamics of top motions with the initial conditions that have been determined with various coordinate points in accordance with the tippe top configuration space that moves in a flat plane.

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