

# 9.Exploring Applications of Lagranges Equations in Technology 2024\_12.pdf

*by Mutiara Arieny*

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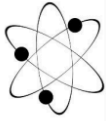
**Submission date:** 05-May-2025 01:28PM (UTC+0700)

**Submission ID:** 2666608462

**File name:** 9.Exploring\_Applications\_of\_Lagranges\_Equations\_in\_Technology\_2024\_12.pdf (374.36K)

**Word count:** 4487

**Character count:** 27030



## Exploring Applications of Lagrange's Equations in Technology: A Systematic Literature Review

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Received : June 22, 2024

Received in revised form: Oktober 30, 2024

Accepted : November 02, 2024

Online : December 29, 2024

**Abstract** – Lagrange's equation is a formula in analytical mechanics used to solve problems with physical system dynamics. It allows mathematical modeling to simplify complex mechanical problems by changing the coordinate system, thus providing a deeper understanding of motion. In this research, a literature study was conducted using the Systemic Literature Review (SLR) method from 30 data sources, 24 of which were indexed by Scopus. A total of 11 articles have been reviewed with a focus on the application of Lagrange's equation in various technologies. The review results show that Lagrange multipliers provide a powerful tool for optimizing energy flow within complex smart grids. The benefits extend beyond smart grids. Lagrange's equations are a powerful mathematical tool applicable to various engineering challenges that involve finding optimal solutions under constraints.

**Keywords:** Lagrange, mechanics, optimization, technology, physics.

### Introduction

Lagrange's equation is one of the fundamental methods in analytical mechanics used to solve problems of the dynamics of physical systems, which was introduced by Joseph-Louis Lagrange in the 18th century. This method allows mathematical modeling of mechanical systems using the energy principle (Irschik & Holl, 2015). This method simplifies complex mechanical problems by changing the coordinate system, thereby providing a deeper understanding of motion (Ariska et al., 2020a). The application of Lagrange's equations has expanded to a variety of disciplines, from robotics physical simulations to advanced control systems.

In the context of robotics, Lagrange's equations are used to design and control robot movements. These equations help in modeling robot dynamics more accurately and efficiently. For example, a more in-depth Lagrange equation analysis of robot arm movements can produce more precise and responsive control (Xu, 2020). So, robots can be optimized for specific tasks such as moving goods, assembling components, or surgical operations that require high precision.

In addition to robotics, Lagrange's equations are used in physical simulations to model and predict the behavior of dynamic systems under various conditions. An example of its application is Lagrange analysis in process design to find economical options in manufacturing engines, especially internal combustion engines, that meet specifications from a kinetic point of view with lower losses (Duarte et al., 2018).

Lagrange's equation is also used in the field of control systems to ensure stability and optimal response for various industrial applications. For example, Lagrange's equation for controlling vibrations of smart sensors in monitoring electricity networks (Cheng, 2022). The Lagrange equation helps in modeling the dynamics of the surrounding environment, such as applying the Lagrange method to optimize interactions in aquatic systems, which can increase the efficiency of clean water use and waste processing in recycled water supply systems (Толбатов et al., 2017).

Overall, the application of Lagrange's equation in technology shows how important this method is in solving complex challenges in various fields. Through deep understanding and proper application, this equation not only helps increase efficiency and accuracy but also drives innovation in the development of future technologies. Therefore, it is very important to carry out further research and find new applications of this equation.

## Materials and Methods

This study is a systematic literature review (SLR). While writing this article, SLR findings can be considered original work. The Cochrane Collaboration recommended that seven articles be written using this method. This Systematic Literature Review used the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) design, which consists of Selections, Reports, Items for Systemic Reviews, and Meta-Analyses (Page & Moher, 2017). First, asking the research question; second, finding the research; third, critical evaluation of the research; fourth, data collection; fifth, data analysis and reporting; sixth, interpretation of the research results; and fifth, refining and updating the review (Rother, 2007). Information sources were collected through Google Scholar and ScienceDirect. Keyword searches on Google Scholar and ScienceDirect were Lagrange applications from 2015-2022. In addition, the search for articles published in English within a certain period. The eligibility selection is adjusted to focus on the application or application of Lagrange's equation, and Scopus indexes the article, so the data selected is 30 articles.

The screening process will be conducted independently by two reviewers to ensure the reliability of the results. Articles that pass the initial stage will be evaluated based on predetermined inclusion criteria, such as topic relevance, publication type, and data availability. Articles that do not meet the criteria will be excluded. The collected data will then be analyzed qualitatively to identify key themes, categories, and patterns when applying the Lagrange equation. For example, quantitative analysis will also be conducted where possible to describe the distribution of articles by publication year or technology field.

**Table 1.** Inclusion and exclusion criteria

Inclusion Criteria	
Research Subject	The research subject should be related to applying Lagrange's equation in technology.
Population Target	The population you wish to study must be related to Lagrange technology or applications.
General Characteristics	Research subjects must meet general characteristics relevant to the research problem.
Continuity	The research subject must be continuous and relevant to differential equations.
Exclusion Criteria	
Non-relevant subjects	Subjects that are irrelevant to applying Lagrange's equation in technology should be excluded.
Logistical limitations	Consider logistical limitations such as subject availability, equipment, expertise, and cost.
Criteria that do not support research	Avoid criteria that do not support the research problem.

Next, standards for eligibility review selection will be presented. These review selection criteria help produce results that are more accurate, objective, and relevant to the research. These criteria reduce the risk of errors. Table 2 shows the results of the review selection based on the criteria.

At this stage, articles are collected according to inclusion and exclusion criteria. Based on these criteria, eleven articles were included. The following is a table of articles included in the inclusion

**Table 2.** The results of the review selection based on the criteria

Criteria	Inclusion	Exclusion	Rationale
Type of publication	Scholarly and ScienceDirect articles	Reports and any other sources	To ensure research is academically sourced

Criteria	Inclusion	Exclusion	Rationale
Open Access	Open access	Non open access	To ensure that research is easily accessible
Publication year	Articles published from 2015-2022	The article was published prior to 2015	To ensure content validation of the research review as, the last 4 years is the time to observe the latest trends.
Language	English	Any language other than English	English is the official language for international articles.

Source: <https://www.medrxiv.org/content/10.1101/2023.02.19.23286155v1>

**Table 3.** Article data in exclusion criteria

Source	Article	Reason
(Tran et al., 2023)	Lagrange Multiplier-Based Optimization for Hybrid Energy Management System with Renewable Energy Sources and Electric Vehicles	The research subject must be continuous and relevant to the concept of differential equations.
(Ariska et al., 2020b)	Dynamic Analysis of Tippe Top on Cylinder's Inner Surface With and Without Friction based on Routh Reduction	The research subject must be continuous and relevant to the concept of differential equations.
(Vadlamani et al., 2020)	Physics successfully implements Lagrange multiplier optimization	Research subjects must meet general characteristics relevant to the research problem.
(Musielak et al., 2020)	Special Functions of Mathematical Physics: A Unified Lagrangian Formalism	The research subject must be continuous and relevant to the concept of differential equations.
(Heck & Uylings, 2020)	A Lagrangian approach to bungee jumping	Research subjects must meet general characteristics relevant to the research problem.
(Ohara, 2024)	Finger flow modeling in snow porous media based on lagrangian mechanics	Research subjects must meet general characteristics relevant to the research problem
(Jia & Al Dmour, 2022)	The Optimal Application of Lagrangian Mathematical Equations in Computer Data Analysis	The research subject should be related to the application of Lagrange's equation in technology.
(Cheng, 2022)	Application of the Lagrange's equation for Intelligent Sensor Vibration Control for Power Network Monitoring	The research subject should be related to the application of Lagrange's equation in technology.
(Tran et al., 2023)	Lagrange Multiplier-Based Optimization for Hybrid Energy Management System with Renewable Energy Sources and Electric Vehicles	The research subject should be related to applying the Lagrange equation in technology.

Source	Article	Reason
(Roy & Kar, 2017)	Robust Control of Uncertain Euler-Lagrange Systems with Time-Varying Input Delay	The research subject should be related to applying the Lagrange equation in technology.
(Duarte et al., 2018)	Application of Lagrange Equations in the Analysis of Slider-Crank Mechanisms	Application of Lagrange Equations in the Analysis of Slider-Crank Mechanisms.

The table data illustrates the number of publications per year from 2008 to 2023, which likely relates to scholarly articles exploring or reviewing the applications of Lagrange's Equations in technology. By analyzing this data, several key trends and insights emerge that shed light on the trajectory of research interest and output in this field over the years. These patterns are useful for understanding the development of scholarly focus and the factors that might influence academic productivity in this specific area of technological application.

One of the first observations is the increase in publications in certain years, which suggests periods of heightened interest in the field. From 2008 to 2010, the number of publications remained steady, with just one publication per year. However, in 2014, this number increased to three, indicating a notable surge in research output related to Lagrange's Equations and their technological applications. The upward trend continued, peaking in 2018 with five publications. This peak year might reflect a culmination of research interest and advancements in the field, as new technological applications and challenges may have driven a surge in scholarly attention. The increase observed in these years hints at a broader trend in which certain milestones or technological needs might have prompted researchers to apply mathematical approaches like Lagrange's Equations.

The data also reveals fluctuations, with some years showing a significant publication drop. Notably, in 2012, 2013, and 2017, the number of publications fell to zero. This decrease could result from various factors, such as shifts in academic or technological interests, availability of research funding, or the emergence of alternative mathematical methods that temporarily overshadowed the application of Lagrange's Equations. For example, suppose other analytical techniques or computational tools have been developed or popularized in the past few years. In that case, researchers might have focused on these new approaches, leading to fewer publications on Lagrange's Equations. This variability highlights the influence of external academic trends and technological innovations on research priorities. It indicates that broader shifts within the scientific community affect interest in Lagrange's Equations.

Despite these fluctuations, an overall upward trend in publications is apparent, especially after 2013. This suggests that, over time, interest in applying Lagrange's Equations in technology has grown, paralleling advancements in technological fields that increasingly require sophisticated mathematical frameworks. As technology continues to evolve, so do the complexities of its challenges, necessitating advanced mathematical approaches to address them. The steady rise in publications after 2013 could indicate that Lagrange's Equations have become more relevant for tackling these complex problems as researchers seek robust methodologies to model and solve issues such as robotics, engineering, and artificial intelligence. This trend reflects the maturation of the field and the growing recognition of Lagrange's Equations as valuable tools for solving real-world technological problems.

Interestingly, there is a noticeable decline in specific years, particularly in 2022, when the number of publications again dropped to zero. This sharp decrease may be attributable to external factors, such as the COVID-19 pandemic, which significantly disrupted academic productivity and publication rates worldwide. The pandemic caused delays in research projects, restricted access to laboratories, and impacted collaborative efforts, all of which likely contributed to a temporary decline in scholarly output. Although there was a slight recovery in 2023, the effects of such global events serve as a reminder of how external pressures can influence research timelines and outputs, even in specialized fields like the application of Lagrange's Equations in technology. This drop underscores the sensitivity of academic productivity to external disruptions, and it may have lasting effects on research momentum in the coming years as the field works to regain its pre-pandemic pace.

In preliminary conclusion, this data suggests that research on applying Lagrange's Equations in technology shows a general upward trend with occasional fluctuations and specific periods of decline. The pattern reflects

the growing relevance and application potential of Lagrange's Equations in various technological domains. This relevance is likely driven by the increasing complexity of technical challenges that demand precise and efficient mathematical models for optimal solutions. Additionally, as technology advances, the need for mathematical frameworks like Lagrange's Equations may further increase, prompting more research interest and, potentially, a steady rise in publications in the future. Thus, despite periodic declines, the field appears poised for sustained growth as complex technological problems require advanced mathematical approaches like those provided by Lagrange's Equations. This trend indicates an exciting horizon for researchers, as technological advancements will likely continue to drive innovations and applications of mathematical techniques, fostering further academic exploration and expanding the impact of Lagrange's Equations in diverse technology areas.

Article analysis is carried out at this stage based on the questions identified in the first stage. A total of 11 articles were found that met the inclusion criteria. The article has been edited to suit the topic, especially applying Lagrange's equation. The PRISMA step diagram (Figure 1) represents the data extraction steps.

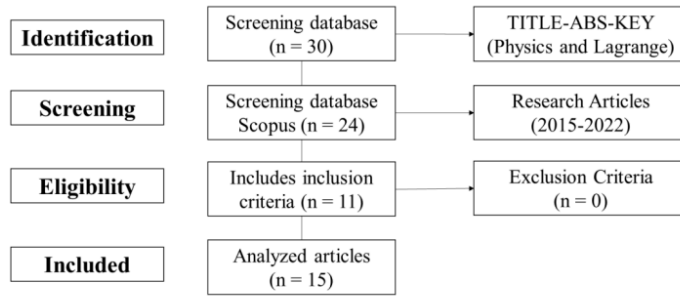


Figure 1. PRISMA stages

## Results

### How are Lagrange's equations applied in various technological fields?

Implements Lagrange multiplier optimization incorporated into the Lagrangian function, which is then used to find solutions through optimization techniques (Vadlamani et al., 2020). we introduce a Lagrange function  $L(x, \lambda)$  defined as follows:

$$L(x, \lambda) = f(x) - \sum_{i=1}^p \lambda_i g_i(x) \quad (1)$$

Which can be optimized by gradient descent or other methods to solve for  $x^*$  and  $\lambda^*$ . The theory of Lagrange multipliers and the popular "Augmented Lagrange Method of Multipliers" algorithm are used to solve for locally optimal  $(x^*, \lambda^*)$  (Musielak et al., 2020; Vadlamani et al., 2020).

Optimizing Energy Flow in Smart Grids With the rise of renewable energy sources and complex power grids, Lagrange's equations can optimize energy flow management, considering real-time demand, transmission losses, and capacity constraints (Ariska et al., 2020).

$$\left( \frac{\partial T}{\partial s^l} \right) - c_{li}^T(q) s^l \frac{\partial T}{\partial \sigma^l} = S_i \quad (2)$$

### How has using the Lagrange equation impacted the development and efficiency of various technologies?

Based on the article's analyses, the use of Lagrange's Equations has a positive developmental impact or progress in various fields of technology, including 1) Control System Optimization to design controllers based on the Lagrange equation, automatic control technology and robotics can be optimized to achieve better performance. This can improve the efficiency of the control system and the system's response to a given input. 2) Application to Electrical and Electronics Technology: In electrical and electronics engineering, using the



Lagrange equation can help analyze and design more efficient electrical and electronic circuits. This can increase the efficiency of electronic systems and improve the system's response to input signals. 3) Design Optimization using the Lagrange approximation, technology can be optimized to design more efficient and stable systems. For example, a better understanding of kinetic and potential energy can help design safer and more effective bungee cords in bungee jumping design. Then, the field of Machinery is the design of more efficient and energy-saving machines. Reduction of engine vibration and noise, thereby improving reliability and comfort. Developing new machines that are more powerful and environmentally friendly, such as intelligent sensor vibration control in power grid monitoring and optimization of interactions in aquatic systems, can improve the efficiency of clean water use and waste treatment in recycled water supply systems.

#### **What are the limitations of using Lagrange's equations in technological contexts? Are there emerging methods that could address these limitations?**

Based on the articles analyzed using the Lagrange equation has limitations in a technological context, including:

1. Differentiation requirements
2. Local optimization that provides a minimum solution for the system
3. Complex systems

New methods that can overcome limitations include:

1. Development of numerical and computational methods
2. Collaboration of physics and computer science
3. The Finite Element Method (FEM) is a method for numerically solving differential equations arising in Engineering and mathematical modeling, covering the fields of fluid flow, mass transport, and electromagnetics. FEM is a general numerical method that solves partial differential problems in two or three space variables.

#### **How can research efforts leverage the strengths of Lagrange's equations to address emerging technological challenges?**

Lagrange's equations are fantastic for finding extremum values (minimum or maximum) when dealing with constraints (Vadlamani et al., 2020). Many technological challenges involve optimizing a desired outcome-maximizing efficiency, minimizing energy consumption, or achieving a specific design goal while respecting limitations imposed by materials, cost, or physical laws (Tran et al., 2023). By translating the challenge into a mathematical model with these constraints as Lagrange multipliers, researchers can leverage the equations to identify the optimal solution. For instance, designing a lightweight yet strong bridge involves maximizing strength while minimizing weight (material usage). Lagrange's equations can help find the optimal bridge structure that fulfills these conflicting goals.

Lagrange's equations are adept at describing how forces influence the motion of a system over time. This is crucial for developing control systems that ensure desired behavior in dynamic environments (Ariska et al., 2020). By incorporating these equations into control algorithms, researchers can design systems that react and adapt to changing conditions (Ohara, 2024). For example, controlling a self-balancing scooter involves constantly adjusting motor forces based on the scooter's tilt and movement. Lagrange's equations can help design a control system that analyzes these dynamics and applies the right forces for optimal balance (Heck & Uylings, 2020).

These equations excel at describing the motion and interaction of interconnected components. This is highly relevant for developing complex technological systems like robots, autonomous vehicles, or spacecraft (Tran et al., 2023). Researchers can analyze the overall behavior and optimize control strategies by modeling each component and its interaction with others as a system with constraints (Duarte et al., 2018). Imagine designing a robotic arm. Lagrange's equations can help determine the optimal movement of each joint to achieve a specific task while considering limitations like joint angles and motor power (Cheng, 2022).

#### **Discussion**

Smart grids are complex systems that integrate renewable energy sources, traditional power plants, and distributed energy resources (like rooftop solar panels) with the goal of efficient and reliable energy delivery. Lagrange multipliers offer a powerful tool to optimize energy flow within these grids while considering various constraints. The Lagrange multipliers come into play here. For each constraint, a Lagrange multiplier is

introduced. These multipliers are then used to combine the objective function with the constraints in a single equation, the Lagrangian function. This function essentially captures the trade-off between minimizing losses or maximizing renewables while respecting the grid's limitations. By taking partial derivatives of the Lagrangian function with respect to all variables (energy flow between points, power generation at different plants, and the Lagrange multipliers), we obtain a system of equations. Solving this system leads to the optimal values for energy flow within the grid that minimizes losses or maximizes renewables while adhering to the constraints. While the optimal energy flow values dictate how power should be routed, the Lagrange multipliers themselves hold valuable information. Their signs can indicate if a particular constraint is active (influencing the solution) or inactive (not a limiting factor at that moment). This helps grid operators understand which constraints are most critical and how they might need to adjust generation or consumption patterns if those constraints become active.

Lagrange's Equations are a powerful mathematical tool, particularly well-suited for optimizing outcomes in the presence of constraints, a feature emphasized by Vadlamani et al.(2020). This optimization capability is essential for tackling various technological challenges where the goal is to maximize efficiency, minimize energy consumption, or achieve a specific design objective, all while adhering to limitations such as material properties, costs, or physical laws (Tran et al., 2023). Optimization with Constraints by Lagrange's equations is excellent for finding extremum values (minimum or maximum) in the presence of constraints, as highlighted by (Vadlamani et al., 2020). Many technological problems require optimization under such constraints. For instance, designing a bridge in structural engineering necessitates balancing conflicting goals: maximizing strength while minimizing weight. By employing Lagrange multipliers, researchers can translate these engineering challenges into mathematical models, allowing for the identification of optimal solutions. This method ensures that the designed structure is lightweight and strong, effectively balancing material usage and structural integrity. Lagrange's Equations also excel in describing how forces influence system motion over time, which is crucial for developing robust control systems. These systems need to ensure desired behavior in dynamic environments, as noted by Ariska et al. (2020). Incorporating Lagrange's equations into control algorithms enables systems to react and adapt to changing conditions effectively. For example, a self-balancing scooter requires constant adjustments to motor forces based on its tilt and movement. Using Lagrange's equations, a control system can be designed to analyze these dynamics accurately and apply the necessary forces to maintain optimal balance (Heck & Uylings, 2020). This approach can significantly enhance the stability and performance of such devices in real-time applications.

The strength of Lagrange's equations in describing the motion and interaction of interconnected components makes them indispensable in the development of complex technological systems. This is particularly relevant for robots, autonomous vehicles, and spacecraft, where each component must work harmoniously with others. Researchers can analyze overall behavior and optimize control strategies by modeling these components and their interactions as a system with constraints (Tran et al., 2023).

## Conclusion

In conclusion, Lagrange multipliers provide a powerful tool for optimizing energy flow within complex smart grids. These grids integrate various energy sources while aiming for efficient and reliable delivery. Lagrange multipliers help navigate the trade-offs between minimizing energy losses or maximizing renewable energy usage, all while respecting the grid's limitations. The benefits extend beyond smart grids. Lagrange's equations are a powerful mathematical tool applicable to various engineering challenges that involve finding optimal solutions under constraints. Examples include designing strong yet lightweight bridges and developing control systems for dynamic environments like self-balancing scooters. Their ability to describe the interaction of interconnected components makes them valuable for optimizing complex technological systems like robots and spacecraft.

## Acknowledgment

All contributions of the third parties can be acknowledged in this section

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