Viona Adelia

The development of web-based learning environment to enhance elementary students' numeracy and reasoning

Faculty of Teacher Training and Education
Universitas Sriwijaya

The Development of Web-Based Learning Environment to Enhance Elementary Students' Numeracy and Reasoning

This research was funded by a grant from the Ministry of Higher Education, Science, and Technology of the Republic of Indonesia under *Pendidikan Magister menuju Doktor untuk Sarjana Unggul* (PMDSU) grant.

This research was carried out in the context of the Elementary School Students in Palembang, Indonesia.

Adelia, Viona

The Development of Web-Based Learning Environment to Enhance Elementary Students' Numeracy and Reasoning / V. Adelia – Palembang: Faculty of Teacher Training and Education, Universitas Sriwijaya / Publisher, 2025

Dissertation Universitas Sriwijaya. With references. Ringkasan dalam Bahasa Indonesia

ISBN: XXX-XXX-XXXX-XX-X

Key words: Design Research / Development Studies / Fraction / Learning Environment / Numeracy

Cover design: Viona Adelia

Printed by: Publisher, Palembang

©2025 Viona Adelia, Palembang, Indonesia

THE DEVELOPMENT OF WEB-BASED LEARNING ENVIRONMENT TO ENHANCE ELEMENTARY STUDENTS' NUMERACY AND REASONING

PENGEMBANGAN LINGKUNGAN BELAJAR BERBASIS WEBSITE UNTUK MENGUATKAN KOMPETENSI NUMERASI DAN BERNALAR SISWA SD

(dengan ringkasan dalam bahasa Indonesia)

Disertasi

untuk memperoleh gelar Doktor di Universitas Sriwijaya atas wewenang Rektor, Prof. Dr. Taufiq Marwa, S.E., M.Si., sesuai dengan keputusan Dewan Doktor yang akan dipertahankan di depan umum pada hari Jumat, 13 Juni 2025, pukul 14.00 WIB

oleh
Viona Adelia

lahir pada tanggal 26 Februari 1999
di Lahat, Indonesia

Promotor	:	Prof. Dr. Ratu Ilma Indra Putri, M.Si.
Co-Promotor	:	Prof. Dr. Zulkardi, M.I.Kom., M.Sc.
	:	Assoc. Prof. Dr. Michiel Doorman

LEGALIZATION PAGE

Dissertation	:	The Development of Web-Based Learning Environment to Enhance Elementary Students' Numeracy and Reasoning
Name of Student	:	Viona Adelia
Student ID	:	06013682227002
Program of Study	:	Doctoral Program in Mathematics Education
Faculty	:	Teacher Training and Education

Approved by,

Promotor

Prof. Dr. Ratu Ilma Indra Putri, M.Si.
NIP 196908141993022001

Co-Promotor I

Prof. Dr. Zulkardi, M.I.Komp., M.Sc.
NIP. 196104201986031002

Co-Promotor II

Assoc. Prof. Dr. L. M. (Michiel) Doorman
NIP. -

Acknowledge by,

Dean of Faculty of Teacher Training and Education

Dr. Hartono, M.A.
NIP. 196710171993011001

Coordinator of Study Program

Prof. Dr. Ratu Ilma Indra Putri, M.Si.
NIP. 196908141993022001

Dipindai dengan CamScanner

LEGALIZATION PAGE

Dissertation	:	The Development of Web-Based Learning Environment to Enhance Elementary Students' Numeracy and Reasoning
Name of Student	:	Viona Adelia
Student ID	:	06013682227002
Program of Study	:	Doctoral Program in Mathematics Education
Faculty	:	Teacher Training and Education

Approved by,

Promotor

Prof. Dr. Ratu Ilma Indra Putri, M.Si.
NIP 196908141993022001

Co-Promotor I

Prof. Dr. Zulkardi, M.I.Komp., M.Sc.
NIP. 196104201986031002

Co-Promotor II

Assoc. Prof. Dr. L. M. (Michiel) Doorman
NIP. -

Acknowledge by,

Dean of Faculty of Teacher Training and Education

Dr. Hartono, M.A.
NIP. 196710171993011001

Coordinator of Study Program

Prof. Dr. Ratu Ilma Indra Putri, M.Si.
NIP. 196908141993022001

Dipindai dengan CamScanner

Pernyataan Integritas

Saya yang bertanda tangan di bawah ini:

Nama : Viona Adelia

NIM : 06013682227002

Program Studi : Doktor Pendidikan Matematika

Menyatakan dengan sesungguhnya bahwa Disertasi yang berjudul " ***The Development of Web-Based Learning Environment to Enhance Elementary Students' Numeracy and Reasoning***" ini beserta seluruh isinya adalah benar-benar karya saya sendiri, dan saya tidak melakukan penjiplakan atau pengutipan dengan cara yang tidak sesuai dengan etika keilmuan yang berlaku sesuai dengan Peraturan Menteri Pendidikan Nasional Republik Indonesia nomor 17 Tahun 2010 tentang pencegahan dan penanggulangan Plagiat di Perguruan Tinggi. Apabila di kemudian hari, ada pelanggaran yang ditemukan dalam disertasi ini dan/atau ada pengaduan dari pihak lain terhadap keaslian karya ini, saya bersedia menanggung sanksi yang dijatuhkan kepada saya.

Demikian pernyataan ini dibuat dengan sesungguhnya tanpa paksaan dari pihak manapun.

Palembang, 16 Juli 2025

Yang Membuat Pernyataan

Viona Adelia

NIM. 06013682227002

Preface

First and foremost, I would like to express my sincere gratitude to the Ministry of Higher Education, Science, and Technology of the Republic of Indonesia for the support provided through the program of *Pendidikan Magister menuju Doktor untuk Sarjana Unggul* (PMDSU) and the program of *Peningkatan Kualitas Publikasi Internasional* (PKPI).

I am deeply grateful to my supervisors. To Prof. Dr. Ratu Ilma Indra Putri, M.Si, thank you for believing in me and entrusting me with this long and meaningful research journey. To Prof. Zulkardi, M.IKomp, M.Sc, thank you for the critical and constructive feedback, and for every opportunity given to me. My heartfelt thanks also go to my Dutch supervisors. To Assoc. Prof. Dr. Michiel Doorman, thank you for the opportunity to carry out a research visit at the Freudenthal Institute, an eye-opening experience that allowed me to learn and grow within a world-class research environment. And to Dr. Michiel Veldhuis, thank you for your kind supervision, for always answering my questions with patience, critical insights, and constructive feedback, also for the meeting with Dr. Ronald Keijzer.

To my dissertation committee, Prof. Dr. Yusuf Hartono and Dr. Hapizah, S.Pd, M.T, thank you for your time, insightful suggestions and feedbacks, which have enriched this dissertation.

To the teachers, students, and schools who participated in this study, I am deeply grateful for your openness and contribution, without which this dissertation would not have been possible.

Last, but not least, I deeply appreciate the love and prayers from my family, mama, papa, and my brother

Warm Regards,

Viona Adelia

Abstract

This study addresses the urgent need to improve Indonesian students' numeracy, following declining mathematics performance in PISA 2022 despite curriculum reforms. It aims to provide insights into effective teaching practices by developing an empirically based, web-based learning environment focused on fractions. Guided by two main questions: (1) how to integrate numeracy and fraction learning in elementary classrooms and (2) what characteristics make a learning environment valid, practical, and impactful. The study combines literature review, learning trajectory design, and iterative development. The resulting local instructional theory emphasizes real-life applicability of part-whole relationships, equivalence, addition with unlike denominators, and division of fractions. The developed web-based environment aligns with theoretical foundations (validity), is feasible and usable for students (practicality), and shows potential effects by promoting students' reasoning and engagement. Evaluation through Guskey's five levels revealed positive student responses, active participation, and improved test performance, supported by strong organizational commitment. Overall, this dissertation demonstrates how carefully designed learning environments can foster numeracy competence and offers implications for future research and practice.

Keywords: Design Research, Fraction, Numeracy, Reasoning, Web-Based Learning Environment

Abstract

Penelitian ini menanggapi rendahnya hasil literasi matematika siswa Indonesia pada PISA 2022 meskipun telah dilakukan reformasi kurikulum yang menekankan pada numerasi. Tujuan utamanya adalah memberikan wawasan tentang praktik pembelajaran melalui pengembangan lingkungan belajar berbasis web yang berfokus pada materi pecahan. Penelitian ini menjawab dua pertanyaan utama: (1) bagaimana mengintegrasikan pembelajaran numerasi dan pecahan di sekolah dasar, serta (2) bagaimana karakteristik lingkungan belajar yang valid, praktis, dan berdampak. Melalui tinjauan pustaka, perancangan lintasan belajar, dan proses pengembangan bertahap, dihasilkan teori instruksional lokal yang menekankan penerapan pecahan dalam konteks nyata, meliputi relasi bagian-keseluruhan, pecahan senilai, penjumlahan pecahan berbeda penyebut, dan pembagian pecahan. Lingkungan belajar yang dikembangkan terbukti valid (sesuai dasar teoritis dan empiris), praktis (mudah digunakan siswa), serta memiliki potensi dampak positif. Evaluasi menggunakan lima level Guskey menunjukkan respons positif siswa, keterlibatan aktif, peningkatan pemahaman pecahan, serta dukungan organisasi sekolah yang kuat. Secara keseluruhan, disertasi ini menunjukkan bagaimana desain pembelajaran yang terstruktur dapat memperkuat kompetensi numerasi siswa, serta memberikan implikasi bagi penelitian dan praktik selanjutnya.

Keywords: *Design Research*, Lingkungan Belajar Berbasis Website, Numerasi, Pecahan, Penalaran

Table of Contents

To Mama and Papa

Chapter 1
Introduction

Introduction

1.1 Why numeracy matters

Mathematics education aims to develop students' ability to solve real-life quantitative problems (Hoogland, 2016; Niss & Jablonka, 2014; O'Donoghue, 2002). This means numeracy is the intended outcome of mathematics education. According to Goos et al. (2018), numeracy is a competency that enables individuals to solve real-life situations involving mathematical elements. There are several terms used to describe this definition, for example quantitative literacy and mathematical literacy (Goos et al., 2018; Niss & Jablonka, 2014; Tout & Gal, 2015). This study will use the term numeracy.

Goos et al. (2018) further elaborate on common situations where numeracy is needed: at home (for cooking, doing groceries, keeping scores of a favorite basketball team), at work (as nowadays, every job requires numeracy, even low-skilled job (Liu, 2020), and in community and civic life, as numeracy allows individual to be active and thoughtful citizens (Crowe, 2010; Geiger et al., 2015).

Because of its importance, many countries participate in international assessment to evaluate their citizens' level of numeracy. Several international assessments measure numeracy, such as the Programme for International Student Assessment (PISA) and The Programme for the International Assessment of Adult Competencies (PIAAC). These assessments provide evidence for governments and stakeholders regarding the numeracy levels of their citizens, reflecting the educational conditions of each country (Li et al., 2025). This highlights the significance of numeracy for a country's population.

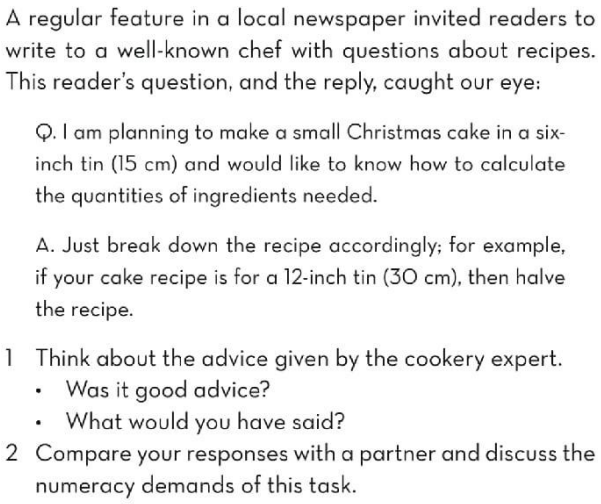
Numeracy is considered an essential outcome of compulsory schooling (Gal et al., 2020). Therefore, it is crucial that the education students receive in compulsory school helps them to become numerate (Bennison, 2015), an individual who possesses strong numeracy skills. For example, Singapore designs its mathematics curriculum to focus on using and applying mathematics in practical tasks, real-life problems, and within mathematics itself (Toh et al., 2019). Through this curriculum, Singapore effectively supports their students in becoming numerate. As a results, in PISA 2022, Singaporean students ranked significantly above the OECD average and have consistently maintained this high ranking since their first participation in 2009 (OECD, 2023b, 2023a). This demonstrates Singapore's successful curriculum implementation.

In contrast, in 2019, Indonesia reformed its curriculum to place greater emphasis on numeracy and literacy (Pusat Asesmen dan Pembelajaran, 2020). One of the driving factors was the Indonesian Ministry of Education's goal for students to apply their mathematical knowledge in various fields (Direktorat Jenderal Pendidikan Anak Usia Dini Pendidikan Dasar dan Pendidikan Menengah, 2021). In other words, the reform aimed to support students in becoming numerate. However, this reformation has not yet had a significant impact on Indonesia's PISA 2022 result. Indonesia's average performace in 2022 declined compared to 2018, particularly in mathematics (OECD, 2023b, 2023a). This suggests that factors beyond curriculum design, such as curriculum implementation, affect students' numeracy levels. In fact, based on our preliminary study, mathematics teachers in Indonesia, particularly in Palembang, struggle to provide supportive and appropriate learning experiences that help their students become numerate (Adelia et al., 2024a).

1.2 Students' initial numeracy competence

In order to develop a meaningful instructional design, it is essential to consider students' starting points (Gravemeijer & Cobb, 2006). This section provides an overview of elementary students' initial numeracy competence based on their responses to selected numeracy tasks.

One of the problems used (see Figure 1), adapted from Goos et al. (2018), required students to evaluate advice given by a chef about adjusting recipe quantities when changing the size of a cake tin. Solving this problem demands reasoning with measurement, geometry, and proportional reasoning. To support students' visualization of the tins' dimensions, an additional prompt (see Figure 2) was given, asking them to identify which tins had the same shape.



A regular feature in a local newspaper invited readers to write to a well-known chef with questions about recipes. This reader's question, and the reply, caught our eye:

Q. I am planning to make a small Christmas cake in a six-inch tin (15 cm) and would like to know how to calculate the quantities of ingredients needed.

A. Just break down the recipe accordingly; for example, if your cake recipe is for a 12-inch tin (30 cm), then halve the recipe.

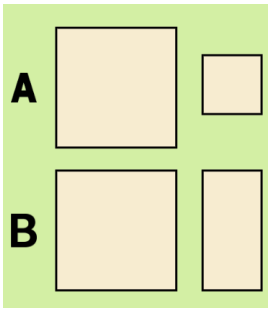
1 Think about the advice given by the cookery expert.
- Was it good advice?
- What would you have said?

2 Compare your responses with a partner and discuss the numeracy demands of this task.

Figure 1. Students' initial numeracy task

Although most students correctly identified option A as the pair of tins with the same shape (noticing that both have square bases), many still agreed that halving the recipe was a good strategy when reducing from a 30 cm to a 15 cm tin. This response pattern suggests that students struggled to integrate different mathematical domains, geometry, measurement, and proportional reasoning, in a real-world context (Goos et al., 2018). Their justification was often limited to surface features (15 is half of 30) without attending to the deeper mathematical relationship involving area.

In analyzing students' responses, we also paid attention to their critical orientation, that is, their tendency to question, justify, and reflect on their reasoning (Goos et al., 2018). However, most students accepted the surface-level advice without challenging it or offering alternative reasoning, revealing a lack of critical engagement with the mathematical context. For instance, students could have raised several assumptions worth examining: whether the depth of the tins is the same, whether the tins are square-based or round-based, and whether the shape affects how the recipe should be adjusted. The absence of such dimension indicates that students were not yet accustomed to questioning underlying assumptions. To sum up, we see that students' numeracy capabilities remains limited. Therefore, we thought that it is important to provide learning experiences that help students develop the numeracy capabilities.



With a 10 x 10 cm cake pan, which pair of the picture that is correct?

Figure 2. Additional aid for numeracy task

1.3 Infusing numeracy into learning

Prior studies have outlined several solutions to support students' numeracy learning. Australia implemented organizational and whole-school management approaches to ensure that all students have access to numeracy learning (Colbert, 2011). Another solution is building partnerships between home, school and community to enhance children's numeracy education (Goos & Jolly, 2004). From the teachers' perspective, it has been found that to effectively embed numeracy into their teaching, teachers need to possess appropriate knowledge of mathematics, pedagogy, and curriculum (Bennison, 2015). These studies highlight that infusing numeracy into students' learning can be achieved in various ways.

This study offers another solution by providing an environment that contains empirical-based mathematics materials to support students' numeracy learning. Moreover, numeracy supports students' critical thinking by providing a foundation for reasoning and problem-solving (Sellars, 2017). In other words, numeracy serves as the basis for reasoning, and strengthening numeracy skills directly contributes to the development of students' reasoning abilities. Therefore, the initial objective of this study is to provide insights into teaching practices within a numeracy learning environment and the effects it has on students' reasoning.

1.4 Fraction and numeracy in action

To provide effective numeracy learning, it is essential to embed the implication of mathematics in real-life contexts (Adelia et al., 2024b). Therefore, the choice of mathematical domains must be carefully considered. Fractions are one such domain, as they originate from human activities (Streefland, 1991). Thus, fraction and numeracy are naturally interconnected.

This study designs fraction learning based on its original roots, human activities, which also serves as the study's aspiration. To achieve this, activities such as fair sharing and measuring are employed to introduce and develop the concept of fractions. In this way, the learning developed in this study not only addresses the philosophical origins of fractions but also fulfils the broader aim of promoting numeracy.

1.5 Web-based learning environment to support students' numeracy

In response to the problem of Indonesian students' numeracy levels, this study proposes a web-based learning environment that provides empirically grounded mathematics materials designed to support students' numeracy. A web-based learning environment serves as an educational tool offering diverse learning experiences for both students and educators (Nam & Smith-Jackson, 2007). These environments have been widely utilized across educational levels, from elementary schools to higher education, either to complement face-to-face teaching with targeted instructional

resources or to deliver unique and engaging learning experiences that may not be feasible in conventional classroom settings (Woo & Reeves, 2019).

The increasing need for web-based learning environments is driven by several factors. First, the growing integration of technology in education has been shown to benefit students' overall learning outcomes (Hillmayr et al., 2020). This also aligns with PISA findings, which indicate that the use of digital tools for learning positively supports students' creative thinking (OECD, 2023a, 2023b). Second, there is a demand for accessible and flexible learning resources for teachers in supporting students' numeracy. This is reflected in the findings of our preliminary study (Adelia et al., 2024a), which will be elaborated further in Chapter 2. Furthermore, digital learning environments enable the creation of realistic situations and virtual training scenarios (Cirneanu & Moldoveanu, 2024), which are essential for fostering students' numeracy competence through connections to real-life applications.

In this study, the web-based learning environment is utilized as a direct solution for both students and teachers. It is designed to create meaningful numeracy learning experiences by embedding mathematical concepts within realistic contexts, fostering students' reasoning abilities, and providing teachers with structured resources to guide instruction. Therefore, this study aims to document the development process of the web-based learning environment and to identify the characteristics of this environment that support students' numeracy and reasoning.

1.6 Description of development process

The development process of the web-based learning environment is described to address the study's aim of documenting how such an environment is developed. This process follows three main phases—preliminary, prototyping, and assessment (Zulkardi, 2002), with each phase comprising distinct activities (see Figure 3).

The preliminary phase focuses on gathering essential information to inform the development of the web-based learning environment. It begins with a needs analysis to identify the requirements and contextual needs for developing a web-based learning environment. Subsequently, a literature review is conducted to explore existing research and best practices in designing learning that support students' numeracy.

The prototyping phase involves iterative design research studies aimed at formulating design principles for fraction learning that enhance students' numeracy. These principles guide the development of the web-based learning environment's components, namely instructional video. Another main component is numeracy test item. These components form the first prototype, which is then subjected to limited formative evaluation to assess the validity and practicality.

Building on these evaluations, the web platform is established as the final prototype. The assessment phase seeks to address goal of this study by identifying the characteristics of the developed web-based learning environment that support students' numeracy and reasoning. This phase evaluates the practicality and effectiveness of the web-based learning environment through small group testing and field testing, providing empirical evidence of its potential impact on students' reasoning.

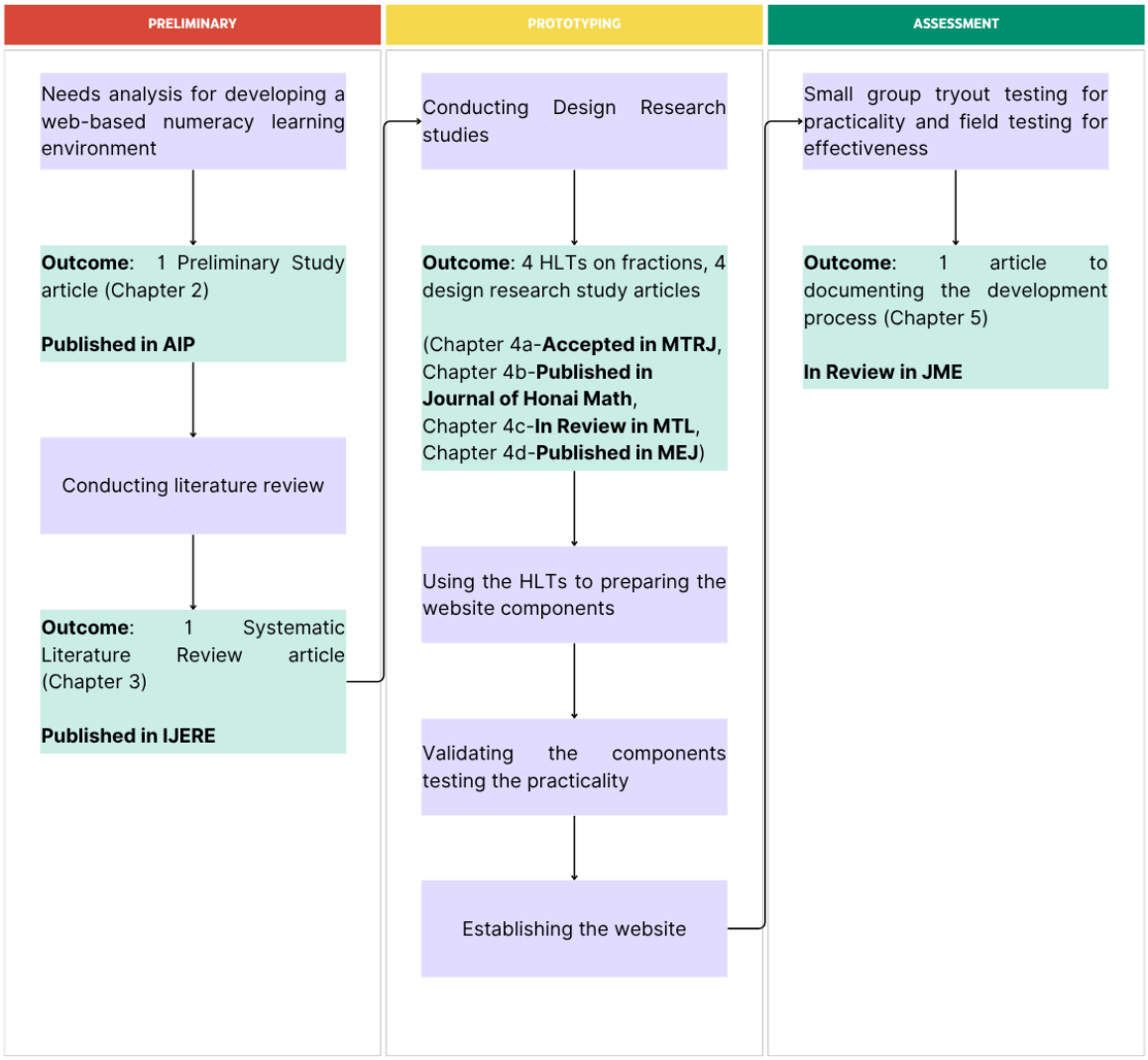


Figure 3. Development process

1.7 Focus of this dissertation

As stated earlier, this study aims to provide insights into teaching practice within a web-based learning environment, with a particular focus on fraction learning. Additionally, it explores the potential of the web-based learning environment in supporting students' mathematical reasoning, as numeracy is reflected across various mathematical processes (Díez-Palomar et al., 2023). Therefore, since this study seeks to provide a numeracy web-based learning environment, it is necessary to inform the development process in detail. Hence, this study is guided by two main research questions:

1. How to integrate numeracy and fraction learning into mathematics classroom for elementary school students?
2. What are the characteristics of the developed web-based learning environment that strengthens elementary students' numeracy competence for reasoning in a way that is valid, practical, and has potential effects?

Several interrelated chapters are provided to answer the main research questions. The initial research question is answered through Chapter 3 and 4, while Chapter 2 serves as the foundational background for the study. The second research question will be answered in Chapter 5.

1.8 Structure of this dissertation

This dissertation comprises a series of journal articles formatted as chapters, each focussing on a different aspect of the study on developing a web-based learning environment. Table 1 illustrates the structure of this dissertation, outlining the topics addressed and the sub-research questions for each chapter.

Table 1
Structure of this Dissertation

	Topic	Research question	Sub-research question
Chapter 1	Introduction		
Chapter 2	Mathematics teachers' perspective of numeracy	How is the numeracy learning practice in Palembang?	
Chapter 3	Approaches to implementing numeracy learning	How to integrate numeracy and fraction learning into mathematics	How to support students in becoming numerate in the classroom?
Chapter 4	Designing fraction learning	classroom for elementary school students?	How do students understand and reason about the key components of the part-whole subconstruct in fractions? How to design a learning trajectory of fraction equivalence by providing students measurement activity? How does students thinking about fraction addition with unlike denominators develop in a sharing activity? How do students utilize their understanding of fractions as parts of a whole to comprehend the concept of partition division?
Chapter 5	Developing the web-based learning environment	What are the characteristics of the developed web-based learning environment that strengthens elementary students' numeracy competence for reasoning in a way that is valid, practical, and has potential effects?	
Chapter 6	Conclusion		

Chapter 2 presents a preliminary study conducted to gather essential information that informs the development of the web-based learning environment. By examining the current practices of numeracy learning in Palembang, a city in southern Sumatera, this chapter establishes the foundational groundwork for the study. The findings highlight existing gaps and identify specific areas that requiring further support, providing valuable insights to guide the subsequent development process.

Chapter 3 reports the results of a systematic literature review, emphasizing the importance of appropriate numeracy learning in supporting students to become numerate. While the significance of numeracy learning is well recognized, there remains a lack of clarity on how to effectively design such learning. To address this gap, the systematic literature review gathers insights and best practices for designing numeracy-focused instruction.

Chapter 4 provides insights into designing a learning trajectory for fractions through the Design Research methodology. These learning trajectories address numeracy tenets, mathematics that are applicable in real-life settings, and cover four features of fractions: part-whole relationship, fraction equivalence, fraction addition with unlike denominators, and fraction division. These learning trajectories are them synthesized to produce a local instructional theory (LIT) for learning fractions through human activities.

Chapter 5 highlights the development process of the web-based learning environment. The development is grounded in the LIT, which serves as the theoretical foundation for designing the key components of the web-based learning environment. This environment is developed through iterative cycles following the Design Research methodology, aiming to address the second main research question regarding the characteristics of an effective web-based learning environment.

Finally, *Chapter 6* summarizes the findings from all studies and connects them to answer the main research questions. The usefulness and limitations of the developed web-based learning environment are discussed, along with implication and suggestions for further research.

References

Adelia, V., Putri, R. I. I., & Zulkardi, Z. (2024a). A systematic literature review: how do we support students to become numerate? International Journal of Evaluation and Research in Education (IJERE), 13(3), 1816. https://doi.org/10.11591/ijere.v13i3.26849

Adelia, V., Putri, R. I. I., & Zulkardi. (2024b). Mathematics teachers in Palembang teaching practice: Numeracy perspective. THE 5TH SRIWIJAYA UNIVERSITY LEARNING AND EDUCATION (SULE) INTERNATIONAL CONFERENCE 2023: Reimagining Learning toward Equitable Education, 020069. https://doi.org/10.1063/5.0201098

Adelia, V., Putri, R. I. I., & Zulkardi. (2025). From Parts to Wholes: Investigating Fraction Division through Partitioning Strategies. Jurnal Pendidikan Matematika, 19(2), 241–254.

Adelia, V., Putri, R. I. I., Zulkardi, Z., & Mulyono, B. (2022). Learning trajectory for equivalent fraction learning: An insight. Journal of Honai Math, 5(1), 47–60. https://doi.org/10.30862/jhm.v5i1.233

Adu-Gyamfi, K., Schwartz, C. S., Sinicrope, R., & Bossé, M. J. (2019). Making sense of fraction division: domain and representation knowledge of preservice elementary teachers on a fraction division task. Mathematics Education Research Journal, 31, 507-528. https://doi.org/10.1007/s13394-019-00265-2

Aguilar, M. S., & Castaneda, A. (2021). What mathematical competencies does a citizen need to interpret Mexico's official information about the COVID-19 pandemic? Educational Studies in Mathematics, 108(1–2), 227–248. https://doi.org/10.1007/s10649-021-10082-9

Aguilar, M. S., & Zavaleta, J. G. M. (2012). On the links between mathematics education and democracy: A literature review. Pythagoras, 33(2), 1-15. https://doi.org/10.4102/pythagoras.v33i2.164

Alvarez, I., Guasch, T., & Espasa, A. (2009). University teacher roles and competencies in online learning environments: a theoretical analysis of teaching and learning practices. European Journal of Teacher Education, 32(3), 321–336. https://doi.org/10.1080/02619760802624104

Amuah, E., & Davis, E. K. (2023). Strategies and procedural and conceptual knowledge of addition of unlike denominator fractions: the case of grade 8 children in two districts of the central region of ghana. African Journal of Research in Mathematics, Science and Technology Education, 27(2), 123-136. https://doi.org/10.1080/18117295.2023.2226546

Astuti, E. P., Wijaya, A., & Hanum, F. (2024). Characteristics of junior high school teachers beliefs in developing students numeracy skills through ethnomathematics-based numeracy learning. Journal of Pedagogical Research. https://doi.org/10.33902/JPR.202423405

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. Journal of Experimental Child Psychology, 113(3), 447-455. https://doi.org/10.1016/j.jecp.2012.06.004

Bakker, A. (2018). What is design research in education? In Design research in education: A practical guide for early career researchers (pp. 3-22). Routledge. https://doi.org/10.4324/9780203701010

Bakker, A., & Smit, J. (2018). Using hypothetical learning trajectories in design research. In Design research in education: A practical guide for early career researchers (pp. 255-271). Routledge. https://doi.org/10.4324/9780203701010

Bakker, A., Wijers, M., Jonker, V., & Akkerman, S. (2011). The use, nature and purposes of measurement in intermediate-level occupations. ZDM, 43, 737-746. https://doi.org/10.1007/s11858-011-0328-3

Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2020). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. Journal of Educational Psychology, 112(3), 628. https://psycnet.apa.org/doi/10.1037/edu0000384

Barlovits, S., Caldeira, A., Fesakis, G., Jablonski, S., Koutsomanoli Filippaki, D., Lázaro, C., Ludwig, M., Mammana, M. F., Moura, A., Oehler, D.-X. K., Recio, T., Taranto, E., & Volika, S. (2022). Adaptive, Synchronous, and Mobile Online Education: Developing the ASYMPTOTE Learning Environment. Mathematics, 10(10), 1628. https://doi.org/10.3390/math10101628

Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). Research on Teaching and Learning Probability (pp. 1–33). https://doi.org/10.1007/978-3-319-31625-3_1

Bennison, A. (2015). Supporting teachers to embed numeracy across the curriculum: a sociocultural approach. ZDM, 47(4), 561–573. https://doi.org/10.1007/s11858-015-0706-3

Bennison, A. (2016). A sociocultural approach to understanding identity as an embedder-of-numeracy: A case of numeracy and history. European Educational Research Journal, 15(4), 491–502. https://doi.org/10.1177/1474904116643327

Braithwaite, D. W., Tian, J., & Siegler, R. S. (2018). Do children understand fraction addition?. Developmental Science, 21(4). https://doi.org/10.1111/desc.12601

Brousseau, G., Brousseau, N., & Warfield, V. (2004). Rationals and decimals as required in the school curriculum: Part 1: Rationals as measurement. The Journal of Mathematical Behavior, 23(1), 1-20. https://doi.org/10.1016/j.jmathb.2003.12.001

Bruce, C. D., Flynn, T., Yearley, S., & Hawes, Z. (2023). Leveraging number lines and unit fractions to build student understanding: insights from a mixed methods study. Canadian Journal of Science, Mathematics and Technology Education, 23(2), 322-339. https://doi.org/10.1007/s42330-023-00278-x

Bulgar, S. (2003). Children's sense-making of division of fractions. The Journal of Mathematical Behavior, 22(3), 319-334. https://doi.org/10.1016/S0732-3123(03)00024-5

Bush, J. B. (2021). Software-based intervention with digital manipulatives to support student conceptual understandings of fractions. British Journal of Educational Technology, 52(6), 2299-2318. https://doi.org/10.1111/bjet.13139

Čadež, T. H, & Kolar, V. M. (2018). How fifth-grade pupils reason about fractions: a reliance on part-whole subconstructs. Educational Studies in Mathematics, 99, 335-357. https://doi.org/10.1007/s10649-018-9838-z

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. Educational studies in mathematics, 64, 293-316. https://doi.org/10.1007/s10649-006-9036-2

Chetty, R., Friedman, J. N., & Rockoff, J. E. (2014). Measuring the impacts of teachers II: Teacher value-added and student outcomes in adulthood. American economic review, 104(9), 2633-2679. https://doi.org/10.1257/aer.104.9.2633

Cirneanu, A.-L., & Moldoveanu, C.-E. (2024). Use of Digital Technology in Integrated Mathematics Education. Applied System Innovation, 7(4), 66. https://doi.org/10.3390/asi7040066

Clements, D. H., & Sarama, J. (2012). Learning trajectories in mathematics education. In D. H. Clements & J. Sarama (Eds.), Hypothetical learning trajectories (pp. 81-90). Routledge.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. Educational Researcher, 32(1), 9-13. https://doi.org/10.3102/0013189X032001009

Cohen, P., Cuban, L., Ellis, W. Jr., Ewell, P., Hallett, D., & Kennedy, D. (2001). Mathematics and Democracy (L. A. Steen, Ed.). The Woodrow Wilson National Fellowship Foundation.

Colbert, P. (2011). Essential Provisions for Quality Learning Support: Connecting Literacy, Numeracy and Learning Needs. In Multiple Perspectives on Difficulties in Learning Literacy and Numeracy (pp. 133–158). Springer Netherlands. https://doi.org/10.1007/978-1-4020-8864-3_6

Confrey, J., & Maloney, A. (2015). A design research study of a curriculum and diagnostic assessment system for a learning trajectory on equipartitioning. ZDM, 47, 919-932. https://doi.org/10.1007/s11858-015-0699-y

Copur-Gencturk, Y. (2021). Teachers' conceptual understanding of fraction operations: results from a national sample of elementary school teachers. Educational Studies in Mathematics, 107(3), 525-545. https://doi.org/10.1007/s10649-021-10033-4

Cramer, K., & Wyberg, T. (2009). Efficacy of different concrete models for teaching the part-whole construct for fractions. Mathematical thinking and learning, 11(4), 226-257. https://doi.org/10.1080/10986060903246479

Cramer, K., Monson, D., Whitney, S., Leavitt, S., & Wyberg, T. (2010). Dividing fractions and problem solving. Mathematics Teaching in the Middle School, 15(6), 338-346. https://doi.org/10.5951/MTMS.15.6.0338

Crowe, A. R. (2010). "What's Math Got to Do With It?": Numeracy and Social Studies Education. The Social Studies, 101(3), 105–110. https://doi.org/10.1080/00377990903493846

de Lange, J. (2003). Mathematics for Literacy. Quantitative Literacy: Why Numeracy Matters for Schools and Colleges, 75–89. http://www.maa.org/ql/pgs75_89.pdf.

de Oliveira Júnior, A. P., Zamora, P. R., Azevedo de Oliveira, L., & Costa de Souza, T. (2018). Student's Attitudes Towards Probability and Statistics and Academic Achievement on Higher Education. Acta Didactica Napocensia, 11(2), 43–56. https://doi.org/10.24193/adn.11.2.4

Díez-Palomar, J., Ramis-Salas, M., Močnik, I., Simonič, M., & Hoogland, K. (2023). Challenges for numeracy awareness in the 21st century: making visible the invisible. Frontiers in Education, 8. https://doi.org/10.3389/feduc.2023.1295781

Direktorat Jenderal Pendidikan Anak Usia Dini Pendidikan Dasar dan Pendidikan Menengah. (2021). Direktorat Sekolah Menengah Pertama Inspirasi Pembelajaran yang Menguatkan Numerasi.

Doorman, L. M. (2005). Modeling motion: From trace graphs to instantaneous change. Wilco Press.

Draganov, A. (2022). Mathematical Tools for Real-World Applications A Gentle Introduction for Students and Practitioners. The MIT Press.

Drijvers, P. (2015). Digital Technology in Mathematics Education: Why It Works (Or Doesn't). In Selected Regular Lectures from the 12th International Congress on Mathematical Education (pp. 135–151). Springer International Publishing. https://doi.org/10.1007/978-3-319-17187-6_8

Engel, J. (2019). Statistical literacy and society: What is civic statistics? (M. Gea, M. López-Martín, & E. Molina-Portillo, Eds.; pp. 1–17). the third international virtual congress on statistics education. https://digibug.ugr.es/bitstream/handle/10481/55027/engel.pdf

Ennis, R. P., & Losinski, M. (2019). SRSD fractions: Helping students at risk for disabilities add/subtract fractions with unlike denominators. Journal of Learning Disabilities, 52(5), 399-412. https://doi.org/10.1177/0022219419859509

Fazio, L., & Siegler, R. S. (2011). Teaching fractions (Vol. 22). North Coburg: International Academy of Education.

Fosnot, C. T., & Dolk, M. (2002). Young Mathematicians at Work: Constructing Fractions, Decimals, and Percents In V. Merecki & L. Peake (Eds.). HEINEMANN • Portsmouth, NH.

Frankenstein, M. (2009). Developing a criticalmathematical numeracy through real real-life word problems. In Words and worlds (pp. 111-130). Brill. https://doi.org/10.1163/9789087909383_008

Fraser, B. J. (1998). The Birth of a New Journal: Editor's Introduction. Learning Environments Research, 1(1), 1–5. https://doi.org/10.1023/A:1009994030661

Freudenthal, H. (1968). Why to teach mathematics so as to be useful. Educational Studies in Mathematics, 1(1–2), 3–8. https://doi.org/10.1007/BF00426224

Freudenthal, H. (1973). Mathematics as an Educational Task. Reidel Publishing Company.

Freudenthal, H. (1991). Revisiting mathematics education. Kluwer Academic Publisher

Gaggi, O., Ciraulo, F., & Casagrande, M. (2018). Eating Pizza to learn fractions. Proceedings of the 4th EAI International Conference on Smart Objects and Technologies for Social Good, 220–225. https://doi.org/10.1145/3284869.3284921

Gal, I., Grotlüschen, A., Tout, D., & Kaiser, G. (2020). Numeracy, adult education, and vulnerable adults: a critical view of a neglected field. ZDM, 52(3), 377–394. https://doi.org/10.1007/s11858-020-01155-9

Gal, I., van Groenestijn, M., Manly, M., Schmitt, M. J., & Tout, D. (2005). Adult numeracy and its assessment in the ALL survey: A conceptual framework and pilot results. International Adult Literacy Survey: Measuring Adult Literacy and Life Skills: New Frameworks for Assessment, 137–184. http://www.nald.ca/library/research/measlit/measlit.pdf

Geiger, V. (2019). Using mathematics as evidence supporting critical reasoning and enquiry in primary science classrooms. ZDM, 51(6), 929–940. https://doi.org/10.1007/s11858-019-01068-2

Geiger, V., Goos, M., & Dole, S. (2015). The Role of Digital Technologies in Numeracy Teaching and Learning. International Journal of Science and Mathematics Education, 13(5), 1115–1137. https://doi.org/10.1007/s10763-014-9530-4

Geiger, V., Goos, M., & Forgasz, H. (2015). A rich interpretation of numeracy for the 21st century: a survey of the state of the field. ZDM, 47(4), 531–548. https://doi.org/10.1007/s11858-015-0708-1

Goos, M., & Jolly, L. (2004). Building partnerships with families and communities to support children's numeracy learning. In I. Putt, R. Faragher, & M. McLean (Eds.), Mathematics Education for the Third Millennium: Towards 2010 (pp. 279–286). MERGA, Inc.

Goos, M., Dole, S., & Geiger, V. (2011). Improving numeracy education in rural schools: a professional development approach. Mathematics Education Research Journal, 23(2), 129–148. https://doi.org/10.1007/s13394-011-0008-1

Goos, M., Geiger, V., & Dole, S. (2012). Auditing the numeracy demands of the middle years curriculum. PNA. Revista de Investigación En Didáctica de La Matemática, 6(4), 147–158. https://doi.org/10.30827/pna.v6i4.6138

Goos, M., Geiger, V., & Dole, S. (2014). Transforming professional practice in numeracy teaching. Transforming mathematics instruction: Multiple approaches and practices, 81-102. https://doi.org/10.1007/978-3-319-04993-9_6

Goos, M., Geiger, V., Dole, S., Forgasz, H., & Bennison, A. (2018). Numeracy across the curriculum: Research-based strategies for enhancing teaching and learning. Routledge.

Goos, M., Geiger, V., Dole, S., Forgasz, H., & Bennison, A. (2018). Numeracy across the curriculum: Research-based strategies for enhancing teaching and learning. Routledge.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. V. D. Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), Educational design research (pp. 29-63). Routledge. https://doi.org/10.4324/9780203088364

Grønmo, L. S. (2018). The Role of Algebra in School Mathematics (pp. 175–193). https://doi.org/10.1007/978-3-319-72170-5_11

Guskey, T. (2000). Evaluating professional development (Vol. 1). Thousand Oaks: Corwin Press.

Guskey, T. (2000). Evaluating professional development (Vol. 1). Thousand Oaks: Corwin Press.

Hacker, D. J., Kiuhara, S. A., & Levin, J. R. (2019). A metacognitive intervention for teaching fractions to students with or at-risk for learning disabilities in mathematics. ZDM, 51(4), 601–612. https://doi.org/10.1007/s11858-019-01040-0

Hains, D., Intindola, M., Lepisto, D., & Wagner, B. (2019). Scrimmage! Teaching quantitative literacy through a multidimensional simulation. The International Journal of Management Education, 17(1), 119-129. https://doi.org/10.1016/j.ijme.2019.01.002

Haris, D., & Putri, R. I. I. (2011). The role of context in third graders' learning of area measurement. Journal on Mathematics Education, 2(1), 55-66. https://doi.org/10.22342/jme.2.1.778.55-66

Heilmann, L. (2020). Health and numeracy: the role of numeracy skills in health satisfaction and health-related behaviour. ZDM, 52(3), 407–418. https://doi.org/10.1007/s11858-019-01106-z

Hillmayr, D., Ziernwald, L., Reinhold, F., Hofer, S. I., & Reiss, K. M. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: A context-specific meta-analysis. Computers & Education, 153, 103897. https://doi.org/10.1016/j.compedu.2020.103897

Hoogland, C. P. (2016). Images of numeracy: investigating the effects of visual representations of problem situations in contextual mathematical problem solving. https://pure.tue.nl/ws/portalfiles/portal/42888388/20161108_CO_Hoogland.pdf

Hooper, M., Mullis, I. V. S., Martin, M. O., & In. (2013). TIMSS 2015 Context Questionnaire Framework Martin. In TIMSS 2015 Assessment Frameworks. http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED545250&site=ehost-live

Indefenso, E. E., & Yazon, A. D. (2020). Numeracy Level, Mathematics Problem Skills, and Financial Literacy. Universal Journal of Educational Research, 8(10), 4393–4399. https://doi.org/10.13189/ujer.2020.081005

Ingram, J. (2022). Randomness and probability: exploring student teachers' conceptions. Mathematical Thinking and Learning, 1–19. https://doi.org/10.1080/10986065.2021.2016029

Ivars, P., Fernández, C. & Llinares, S. (2020). A learning trajectory as a scaffold for pre-service teachers' noticing of students' mathematical understanding. International Journal of Science and Mathematics Education, 18, 529–548. https://doi.org/10.1007/s10763-019-09973-4

Jablonka, E. (2015). The evolvement of numeracy and mathematical literacy curricula and the construction of hierarchies of numerate or mathematically literate subjects. ZDM, 47, 599-609. https://doi.org/10.1007/s11858-015-0691-6

Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. Educational Studies in Mathematics, 96(1), 1–16. https://doi.org/10.1007/s10649-017-9761-8

Jenderal Pendidikan Anak Usia Dini, D., Dasar, P., Pendidikan Menengah Kementerian Pendidikan, D., Teknologi Tahun, D., & Mata Pelajaran Matematika untuk Jenjang Sekolah Menengah Pertama, P. (2021). Direktorat Sekolah Menengah Pertama Inspirasi Pembelajaran yang Menguatkan Numerasi.

Jerrim, J., & Sims, S. (2021). When is high workload bad for teacher wellbeing? Accounting for the non-linear contribution of specific teaching tasks. Teaching and teacher education, 105, 103395. https://doi.org/10.1016/j.tate.2021.103395

Jiang, Z., Mok, I. A. C., & Li, J. (2021). Chinese students' hierarchical understanding of part-whole and measure subconstructs. International Journal of Science and Mathematics Education, 19, 1441-1461. https://doi.org/10.1007/s10763-020-10118-1

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. Journal of Experimental Child Psychology, 116(1), 45-58. https://doi.org/10.1016/j.jecp.2013.02.001

Kamid, K., Huda, N., Syafmen, W., Sufri, S., & Sofnidar, S. (2021). The relationship between students' mathematical disposition and their learning outcomes. Journal of Education and Learning (EduLearn), 15(3), 376–382. https://doi.org/10.11591/edulearn.v15i3.17604

Kara, M., Simon, M. A., & Placa, N. (2018). An empirically-based trajectory for fostering abstraction of equivalent-fraction concepts: A study of the Learning Through Activity research program. Journal of Mathematical Behavior, 52, 134–150. https://doi.org/10.1016/j.jmathb.2018.03.008

Kieran, C. (2014). Algebra Teaching and Learning. In Encyclopedia of Mathematics Education (pp. 27–32). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8_6

Kieren, T. E. (1976). On the mathematical, cognitive and instructional. In R. Lesh (Eds.) Number and measurement: Papers from a research workshop ERIC/SMEAC (pp. 101-144). Eric.

Kong, S. C., & Kwok, L. F. (2005). A cognitive tool for teaching the addition/subtraction of common fractions: A model of affordances. Computers & Education, 45(2), 245-265. https://doi.org/10.1016/j.compedu.2004.12.002

Koyama, M. (2010). Mathematics Curriculum in Japan. In Reforms and Issues in School Mathematics in East Asia (pp. 59–78). BRILL. https://doi.org/10.1163/9789460912283_005

Laidin, D. R., & Tengah, K. A. (2021). Applying butterflu method in the learning of addition and subtraction of fractions. Jurnal Pendidikan Matematika, 15(2), 161-174. https://doi.org/10.22342/jpm.15.2.13934.161-174

Lamon, S. J. (2012). Ratio and proportion: Children's cognitive and metacognitive processes. In T. P. Carpenter, E.Fennema, & T. A. Romberg (Eds.), Rational numbers: An integration of research (pp. 131–156). Routledge.

Lamon, S. J. (2020). Teaching Fractions and Ratios for Understanding (4th Edition). Routledge: New York. https://doi.org/10.4324/9781003008057

Laughlin, A. J. (2022). Adding fractions with unlike denominators, for grade 4 (8- and 9-year-old) students. In A. Takahashi, T. McDougal, S. Friedkin, & T. Watanabe (Eds.), Educators' learning from lesson study (pp. 170-180). Routledge. https://doi.org/10.4324/9781003230915

Laughlin, A. J. (2022). Lesson Study and the New Teacher: Adding Fractions with Unlike Denominators, for Grade 4 (8- and 9-year-old) Students. In A. Takahasi, T. McDougal, S. Friedkin, & T. Watanabe (Eds.), Educators' Learning from Lesson Study (1st ed.). Routledge. https://doi.org/10.4324/9781003230915

Lefa, B. (2014). the Piaget Theory of Cognitive Development : an Educational. September.

Lenz, K., Dreher, A., Holzäpfel, L., & Wittmann, G. (2020). Are conceptual knowledge and procedural knowledge empirically separable? The case of fractions. British Journal of Educational Psychology, 90(3), 809-829. https://doi.org/10.1111/bjep.12333

Levy, R., & Sinclair, N. (2017). Young Children Developing Literacy and Numeracy Skills with Technology. In Digital Technologies and Learning in the Early Years (pp. 69–86). SAGE Publications Ltd. https://doi.org/10.4135/9781526414502.n7

Lew, H. (2019). Current Mathematics Curriculum of South Korea and Its Embodiment into Textbooks (pp. 127–150). https://doi.org/10.1007/978-981-13-6312-2_7

Li, J., Xue, E., & Guo, S. (2025). The effects of PISA on global basic education reform: a systematic literature review. Humanities and Social Sciences Communications, 12(1), 106. https://doi.org/10.1057/s41599-025-04403-z

Li, Y. (2008). What do students need to learn about division of fractions?. Mathematics teaching in the middle school, 13(9), 546-552. https://doi.org/10.5951/MTMS.13.9.0546

Liljedahl, P. (2015). Numeracy task design: a case of changing mathematics teaching practice. ZDM, 47(4), 625–637. https://doi.org/10.1007/s11858-015-0703-6

Lithner, J. (2006). A framework for analysing creative and imitative mathematical reasoning.

Lithner, J. (2008). A research framework for creative and imitative reasoning. Educational Studies in Mathematics, 67(3), 255–276. https://doi.org/10.1007/s10649-007-9104-2

Liu, H. (2020). Low-numerate adults, motivational factors in learning, and their employment, education and training status in Germany, the US, and South Korea. ZDM, 52(3), 419–431. https://doi.org/10.1007/s11858-019-01108-x

Lo, J. J., & Luo, F. (2012). Prospective elementary teachers' knowledge of fraction division. Journal of Mathematics Teacher Education, 15, 481-500. https://doi.org/10.1007/s10857-012-9221-4

Low, J., Shahrill, M., & Zakir, N. (2020). Solving fractions by applying the bar model concept with the butterfly method. Jurnal Pendidikan Matematika, 14(2), 101-116. https://doi.org/10.22342/jpm.14.2.11261.101-116

Martin, T., Petrick Smith, C., Forsgren, N., Aghababyan, A., Janisiewicz, P., & Baker, S. (2015). Learning Fractions by Splitting: Using Learning Analytics to Illuminate the Development of Mathematical Understanding. Journal of the Learning Sciences, 24(4), 593–637. https://doi.org/10.1080/10508406.2015.1078244

McClure, R., & Sircar, S. (2008). Quantitative literacy for undergraduate business students in the 21st century. Journal of Education for Business, 83(6), 369-374. https://doi.org/10.3200/JOEB.83.6.369-374

Möhring, W., Newcombe, N. S., Levine, S. C., & Frick, A. (2016). Spatial proportional reasoning is associated with formal knowledge about fractions. Journal of Cognition and Development, 17(1), 67-84. https://doi.org/10.1080/15248372.2014.996289

Nam, C. S., & Smith-Jackson, T. L. (2007). Web-Based Learning Environment: A Theory-Based Design Process for Development and Evaluation. Journal of Information Technology Education: Research, 6, 023–043. https://doi.org/10.28945/200

Ni, Y., & Zhou, Y. D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. Educational Psychologist, 40(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3

Nieveen, N. (2010). Formative Evaluation in Educational Design Research. In T. Plomp & N. Nieveen (Eds.), An Introduction to Educational Design Research (pp. 89–101). SLO.

Nieveen, N. M., McKenney, S., & van den Akker, J. (2006). Educational design research: The value of variety. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), Educational design research: The value of variety (pp. 151-158). Routledge. https://research.utwente.nl/en/publications/educational-design-research-the-value-of-variety

Nieveen, N., & van den Akker, J. (1999). Exploring the potential of a computer tool for instructional developers. Educational Technology Research and Development, 47(3), 77–98. https://doi.org/10.1007/BF02299635

Nieveen, N., McKenney, S., & Van Den Akker, J. (2006). Educational Design Research: the Value of Variety. In J. Van Den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), Educational Design Research (pp. 151–158). Routledge.

Niss, M., & Jablonka, E. (2014). Mathematical Literacy. In Encyclopedia of Mathematics Education (pp. 391–396). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8_100

Noordin, N., Razak, F. A., Dollah, R., & Alias, R. (2012). Identifying equivalent pictorial representation of a fraction & a whole: A problem for 13-year olds in Malaysia. Procedia-Social and Behavioral Sciences, 42, 106-116. https://doi.org/10.1016/j.sbspro.2012.04.172

Nortvedt, G. A., & Wiese, E. (2020). Numeracy and migrant students: a case study of secondary level mathematics education in Norway. ZDM, 52(3), 527–539. https://doi.org/10.1007/s11858-020-01143-z

O'Donoghue, J. (2002). Numeracy and Mathematics. Irish Mathematical Society Bulletin, 0048, 47–56. https://doi.org/10.33232/BIMS.0048.47.56

OECD. (2021). The Assessment Frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies. OECD. https://doi.org/10.1787/4bc2342d-en

OECD. (2022). Mathematics Framework (Draft). Retrieved from PISA.

OECD. (2023a). PISA 2022 Results (Volume I). OECD. https://doi.org/10.1787/53f23881-en

OECD. (2023b). PISA 2022 Results (Volume II). OECD. https://doi.org/10.1787/a97db61c-en

Ott, J. M., Snook, D. L., & Gibson, D. L. (1991). Understanding partitive division of fractions. The Arithmetic Teacher, 39(2), 7-11. https://doi.org/10.5951/AT.39.2.0007

Pantziara, M., & Philippou, G. (2012). Levels of students'"conception" of fractions. Educational Studies in Mathematics, 79, 61-83. https://doi.org/10.1007/s10649-011-9338-x

Patkin, D., & Levenberg, I. (2012). Geometry from the World around Us. Learning and Teachning Mathematics, 13(September 2016), 14–18.

Pedersen, P. L., & Bjerre, M. (2021). Two conceptions of fraction equivalence. Educational Studies in Mathematics, 107(1), 135-157. https://doi.org/10.1007/s10649-021-10030-7

Plomp, T. (2010). Educational Design Research: an Introduction. In T. Plomp & N. Nieveen (Eds.), An Introduction to Educational Design Research (pp. 9–35). SLO. www.slo.nl

Pramudiani, P., Herman, T., Dolk, M., & Doorman, M. (2022). How does a missing part become important for primary school students in understanding fractions?. Journal on Mathematics Education, 13(4), 565-586. http://doi.org/10.22342/jme.v13i4.pp565-586

Prediger, S., Quabeck, K., & Erath, K. (2022). Conceptualizing micro-adaptive teaching practices in content-specific ways: Case study on fractions. Journal on Mathematics Education, 13(1), 1-30. https://doi.org/10.22342/jme.v13i1.pp1-30

Prince, R., & Frith, V. (2020). An investigation of the relationship between academic numeracy of university students in South Africa and their mathematical and language ability. ZDM, 52(3), 433–445. https://doi.org/10.1007/s11858-019-01063-7

Purnomo, Y. W., Arlini, R., Nuriadin, I., & Aziz, T. A. (2021). Learning trajectory based on fractional sub-constructs: Using fractions as quotients to introduce fractions. Mathematics Teaching Research Journal, 13(3), 183-207. https://files.eric.ed.gov/fulltext/EJ1382607.pdf

Purnomo, Y. W., Kowiyah, Alyani, F., & Assiti, S. S. (2014). Assessing number sense performance of Indonesian elementary school students. International Education Studies, 7(8), 74–84. https://doi.org/10.5539/ies.v7n8p74

Purnomo, Y. W., Widowati, C., & Ulfah, S. (2019). Incomprehension of the Indonesian elementary school students on fraction division problem. Infinity Journal, 8(1), 57-74. https://doi.org/10.22460/infinity.v8i1.p57-74

Pusat Asesmen dan Pembelajaran. (2020). AKM dan Implikasinya pada Pembelajaran.

Putri, R. I. I. (2011). Improving mathematics communication ability of student in grade 2 through PMRI approach. Presented at International Seminar and the Fourth National Conference on Mathematics Education, 21-23 July 2011, Universitas Negeri Yogyakarta, Yogyakarta.

Putri, R. I. I., & Zulkardi. (2017). Fraction in shot-put: A learning trajectory. AIP Conference Proceedings, 1868, 050005. https://doi.org/10.1063/1.4995132

Putri, R. I. I., Setyorini, N. P., Meitrilova, A., Permatasari, R., Saskiyah, S. A., & Nusantara, D. S. (2021). Designing a healthy menu project for indonesian junior high school students. Journal on Mathematics Education, 12(1), 133-146. https://doi.org/10.22342/jme.11.1.9786.135-144

Putri, R. I. I., Zulkardi, Setyorini, N. P., Meitrilova, A., Permatasari, R., Saskiyah, S. A., & Nusantara, D. S. (2021). Designing a healthy menu project for Indonesian junior high school students. Journal on Mathematics Education, 12(1), 133–146. https://doi.org/10.22342/jme.12.1.13239.133-146

Regian, J. W., Shebilske, W. L., & Monk, J. M. (1992). Virtual Reality: An Instructional Medium for Visual-Spatial Tasks. Journal of Communication, 42(4), 136–149. https://doi.org/10.1111/j.1460-2466.1992.tb00815.x

Roche, A., & Clarke, D. M. (2013). Primary teachers' representations of division: Assessing mathematical knowledge that has pedagogical potential. Mathematics Education Research Journal, 25, 257-278. http://dx.doi.org/10.1007/s13394-012-0060-5

Roesslein, R. I., & Codding, R. S. (2019). Fraction interventions for struggling elementary math learners: A review of the literature. Psychology in the Schools, 56(3), 413-432. https://doi.org/10.1002/pits.22196

Rule, A. C., & Hallagan, J. E. (Eds.). (2006). Preservice elementary teachers use drawings and make sets of materials to explain multiplication and division of fractions. Proceedings from PMET "06: The 2nd

Annual Preparing Mathematicians to Educate Teachers Conference. Oswego, NY. https://files.eric.ed.gov/fulltext/ED494956.pdf

Santos-Trigo, M., Reyes-Martínez, I., & Aguilar-Magallón, D. (2015). The Use of Digital Technology in Extending Mathematical Problem Solving Reasoning (pp. 298–309). https://doi.org/10.1007/978-3-319-22629-3_24

Sari, I. P., Suryadi, D., Herman, T., Dahlan, J. A., & Supriyadi, E. (2024). Learning obstacles on fractions: A scoping review. Infinity Journal, 13(2), 377-392. https://doi.org/10.22460/infinity.v13i2.p377-392

Sari, Y. P., Zulkardi, Z., & Putri, R. I. I. (2023). The development of numeracy problems using light rail transit context. Jurnal Elemen, 9(1), 227–245. https://doi.org/10.29408/jel.v9i1.6923

Sellars, M. (2017). Numeracy across the curriculum: a pathway to critical thinking. International Journal Innovation Creativity and Change, 3(2), 75–83.

Şengül, S., & Erdoğan, F. (2014). A Study on the Elementary Students' Perceptions of Algebra. Procedia - Social and Behavioral Sciences, 116, 3683–3687. https://doi.org/10.1016/j.sbspro.2014.01.823

Setambah, M. A. B., Jaafar, A. N., Saad, M. I. M., & Yaakob, M. F. M. (2021). Fraction chiper: A way to enhance student ability in addition and subtraction fraction. Infinity Journal, 10(1), 81-92. https://doi.org/10.22460/infinity.v10i1.p81-92

Sharp, J., & Adams, B. (2002). Children's constructions of knowledge for fraction division after solving realistic problems. The Journal of Educational Research, 95(6), 333-347. http://dx.doi.org/10.1080/00220670209596608

Shin, J., & Lee, S. J. (2018). The alignment of student fraction learning with textbooks in Korea and the United States. The Journal of Mathematical Behavior, 51, 129-149. https://doi.org/10.1016/j.jmathb.2017.11.005

Shin, M., & Bryant, D. P. (2015). Fraction interventions for students struggling to learn mathematics: A research synthesis. Remedial and Special Education, 36(6), 374-387. https://doi.org/10.1177/0741932515572910

Sidney, P. G., Thompson, C. A., & Rivera, F. D. (2019). Number lines, but not area models, support children's accuracy and conceptual models of fraction division. Contemporary Educational Psychology, 58, 288-298. https://doi.org/10.1016/j.cedpsych.2019.03.011

Siegler, R. S., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. Journal of Educational Psychology, 107(3), 909. https://psycnet.apa.org/doi/10.1037/edu0000025

Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. Developmental Psychology, 49(10), 1994. https://psycnet.apa.org/doi/10.1037/a0031200

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., ... & Chen, M. (2012). Early predictors of high school mathematics achievement. Psychological Science, 23(7), 691-697. https://doi.org/10.1177/095679761244010

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. Trends in Cognitive Sciences, 17(1), 13–19. https://doi.org/10.1016/j.tics.2012.11.004

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal for Research in Mathematics, 26(2), 114-145. https://doi.org/10.5951/jresematheduc.26.2.0114

Simon, M. A. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. Mathematical Thinking and Learning, 8(4), 359-371. https://doi.org/10.1207/s15327833mtl0804_1

Simon, M. A., Placa, N., Avitzur, A., & Kara, M. (2018). Promoting a concept of fraction-as-measure: A study of the Learning Through Activity research program. The Journal of Mathematical Behavior, 52, 122-133. https://doi.org/10.1016/j.jmathb.2018.03.004

Sinicrope, R., Mick, H. W., & Kolb, J. R. (2002). Fraction division interpretat. In B. Litwiller & G. Bright (Eds.), Making sense of fractions, ratios, and proportions, NCTM 2002 yearbook (pp. 153–161). Reston, VA: National Council of Teachers of Mathematics. https://eric.ed.gov/?id=ED474863

Smith, J. P., van den Heuvel-Panhuizen, M., & Teppo, A. R. (2011). Learning, teaching, and using measurement: introduction to the issue. ZDM, 43(5), 617. https://doi.org/10.1007/s11858-011-0369-7

Son, J. W., & Senk, S. L. (2010). How reform curricula in the USA and Korea present multiplication and division of fractions. Educational Studies in Mathematics, 74, 117-142. https://doi.org/10.1007/s10649-010-9229-6

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. Learning and Instruction, 14, 503-518. https://doi.org/10.1016/j.learninstruc.2004.06.015

Steffe, L. P. & Olive, J. (2010). Children's fractional knowledge. New York, NY: Springer.

Streefland, L. (1991). Fractions in Realistic Mathematics Education: A Paradigm of Developmental Research (Vol. 8). The Netherlands: Springer Science & Business Media. https://doi.org/10.1007/978-94-011-3168-1

Sumpter, L. (2018). Grade 9 Students' Reasoning About Division of Fractions: What Are their Arguments Anchored in? In Students' and Teachers' Values, Attitudes, Feelings and Beliefs in Mathematics Classrooms (pp. 135–143). Springer International Publishing. https://doi.org/10.1007/978-3-319-70244-5_13

Toh, T. L., Chan, C. M. E., Tay, E. G., Leong, Y. H., Quek, K. S., Toh, P. C., Ho, W. K., Dindyal, J., Ho, F. H., & Dong, F. (2019). Problem Solving in the Singapore School Mathematics Curriculum (pp. 141–164). https://doi.org/10.1007/978-981-13-3573-0_7

Torres-Peña, R. C., González, D. P., & Ariza, E. A. (2024). Enhancing fraction learning through problem-solving and historical context: A didactic unit approach. Journal on Mathematics Education, 15(3), 815-834. http://doi.org/10.22342/jme.v15i3.pp815-834

Tout, D. (2020). Evolution of adult numeracy from quantitative literacy to numeracy: Lessons learned from international assessments. International Review of Education, 66(2–3), 183–209. https://doi.org/10.1007/s11159-020-09831-4

Tout, D. (2020). ISSUES IN THE TEACHING OF MATHEMATICS Critical Connections Between Numeracy and Mathematics. https://research.acer.edu.au/learning_processes/29

Tout, D., & Gal, I. (2015). Perspectives on numeracy: reflections from international assessments. ZDM, 47(4), 691–706. https://doi.org/10.1007/s11858-015-0672-9

Tsai, T. L., & Li, H. C. (2017). Towards a framework for developing students' fraction proficiency. International Journal of Mathematical Education in Science and Technology, 48(2), 244-255. https://doi.org/10.1080/0020739X.2016.1238520

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2008). Elementary and Middle School Mathematics: Teaching Developmentally. Boston, MA: Allyn and Bacon. https://books.google.co.id/books/about/Elementary_and_Middle_School_Mathematics.html?id=nisCzwEACAAJ&redir_esc=y

Van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (Eds.). (2006). Educational design research. Routledge.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic Mathematics Education. In Encyclopedia of Mathematics Education (pp. 713–717). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_170

Van den Heuvel-Panhuizen, M., & Wijers, M. (2005). Mathematics standards and curricula in the Netherlands. Zentralblatt Für Didaktik Der Mathematik, 37(4), 287–307. https://doi.org/10.1007/BF02655816

van Hiele, P. (1999). Developing Geometric Thinking through Activities that Begin with Play. Teaching Children Mathematics, 6(February 1999), 10–16.

Vandecandelaere, M., Speybroeck, S., Vanlaar, G., De Fraine, B., & Van Damme, J. (2012). Learning environment and students' mathematics attitude. Studies in Educational Evaluation, 38(3–4), 107–120. https://doi.org/10.1016/j.stueduc.2012.09.001

Wahyu, K., Kuzu, T. E., Subarinah, S., Ratnasari, D., & Mahfudy, S. (2020). Partitive fraction division: Revealing and promoting primary students' understanding. Journal on Mathematics Education, 11(2), 237-258. https://doi.org/10.22342/jme.11.2.11062.237-258

Wang, X. (2015). The Literature Review of Algebra Learning: Focusing on the Contributions to Students' Difficulties. Creative Education, 06(02), 144–153. https://doi.org/10.4236/ce.2015.62013

Watanabe, T. (2006). The teaching and learning of fractions A Japanese perspective. Teaching Children Mathematics, 12(7), 368–374.

Wijaya, A., Elmaini, & Doorman, M. (2021). A learning trajectory for probability: A case of game-based learning. Journal on Mathematics Education, 12(1), 1–16. https://doi.org/10.22342/jme.12.1.12836.1-16.

Wilkins, J. L. M., & Norton, A. (2018). Learning progression toward a measurement concept of fractions. International Journal of STEM Education, 5(1), 1-11. https://doi.org/10.1186/s40594-018-0119-2

Wong, M. (2010). Equivalent fractions: Developing a pathway of students' acquisition of knowledge and understanding. Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia (pp. 673-680). Fremantle: MERGA

Woo, Y., & Reeves, T. C. (2019). Interaction in asynchronous web- based learning environments: Strategies supported by educational research. Online Learning, 12(3–4). https://doi.org/10.24059/olj.v12i3-4.1691

Xiao, F., Barnard-Brak, L., Lan, W., & Burley, H. (2019). Examining problem-solving skills in technology-rich environments as related to numeracy and literacy. International Journal of Lifelong Education, 38(3), 327–338. https://doi.org/10.1080/02601370.2019.1598507

Xiao, Y., & Watson, M. (2019). Guidance on Conducting a Systematic Literature Review. Journal of Planning Education and Research, 39(1), 93–112. https://doi.org/10.1177/0739456X17723971

Yeo, S. (2019). Investigating children's informal knowledge and strategies: The case of fraction division. Research in Mathematical Education, 22(4), 283-304. http://doi.org/10.7468/jksmed.2019.22.4.283

Yoshida, H., & Sawano, K. (2002). Overcoming cognitive obstacles in learning fractions: Equal-partitioning and equal-whole. Japanese Psychological Research, 44(4), 183-195. https://doi.org/10.1111/1468-5884.00021

Zaleta, C. K. (2006). Invented strategies for division of fractions. In Alatorre, S., Cortina, J.L., Saiz, M., and Mendez, A (Eds.). Proceeding of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Merida, Mexico: Universidad Pedagogica Nacional. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=77136e0514893005b78c081f0df074677a40f3a6#page=509

Zhang, L., Shang, J., Pelton, T., & Pelton, L. F. (2020). Supporting primary students' learning of fraction conceptual knowledge through digital games. Journal of Computer Assisted Learning, 36(4), 540–548. https://doi.org/10.1111/jcal.12422

Zulkardi, & Putri, R. I. I. (2010). Pengembangan blog support untuk membantu siswa dan guru matematika Indonesia belajar Pendidikan Matematika Realistik Indonesia (PMRI) [Developing a support blog to help Indonesian mathematics students and teachers learn Indonesian Realistic Mathematics Education (IRME)]. Jurnal Inovasi Perekayasa Pendidikan (JIPP), 2(1), 1-24.

Zulkardi, Putri R. I. I., & Wijaya A. (2020). Two decades of realistic mathematics education in Indonesia. In van den Heuvel-Panhuizen M. (eds), International Reflections on the Netherlands Didactics of Mathematics. ICME-13 Monographs (pp. 325-340). Springer

Zulkardi. (2002). Developing A Learning Environment On Realistic Mathematics Education For Indonesian Student Teacher (Doctoral disertation, University of Twente, Enschede). 1– Bakker, A. (2004). Design research in statistics education: On symbolizing and computer tools. CD- β Press.