

Optimization of Internet Pricing Under Multiple QoS Networks

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Abstract

As internet is becoming critical in economics life, Internet Service Providers (ISPs) now deal with high demand to promote good quality information. However, the knowledge to develop new pricing plans that serve both customers and supplier is known, but only a few pricing plans involve multiple networks. This research will analyze the dynamical situation in network where new proposed pricing plans are offered with multiple networks involved. Preliminary findings show that we can begin solve the simple QoS networks, generalize the model into multiple QoS networks and also compare two multiple QoS models to get best rofit maximization.

Key words: charging scheme, multiple QoS networks, profit maximization

1 Introduction

The Internet has, in a short space of time, become fundamental to the global economy. It helps experimental research network and creates economic activities from all areas of life [1]. It is a big job for Internet service provider (ISP) to promote good service in achieving high quality of information and obtain valuable profit from available resources. Providing better and different Quality of Service (QoS) is the best way to improve revenue and third parties who use these schemes will be able to develop new ICT based as a result of products. Currently, a proper pricing mechanism for network service provider is lacking to provide technical solution that is logical and persuasive for customers, and in addition the service cost of network service is poor in a very competitive

environment in the market resulting no revenue for the companies creating those products [2, 3].

Pricing product or service is critical business decisions or core activity we want to focus on this research. We have to offer the products for a price that our target market is willing to pay. The product is that produces a profit to our company. There are many approaches to pricing involving scientific method or otherwise [4].

Pricing has become a very interesting topic in network business. In supporting this business, internet has to provide the best QoS meaning that the mechanism that allows differentiation of network services based on their unique service requirements [2, 5, 6]. Their papers are basically one of the few studies about pricing which focuses on economic point of view. Byun & Chatterjee [2] discussed about designing pricing models for internet services at various levels of quality which focus on usage based pricing scheme since that scheme reflects congestion level in details. The parameter involved is basically based on bandwidth and by creating suitable formula, these parameters are to be set up to obtain pricing formula that can be used to develop research on pricing model. The model was tested on OPNET simulation program and the results show that by designing proper pricing scheme with quality index is in pricing formula yields simpler formula but of course it is also dynamic. The possible changes in service pricing and revenue changes can also be made. The disadvantage of their result is actually only can be applied in theoretical situations since they only consider single route from the source application where in real situation, we deal with multiple routes from source to reach destinations.

Wu et al [7] described the optimal pricing schemes both in consumer's and supplier's perspectives. They view three pricing schemes such as flat fee, pure usage-based and two-part tariff scheme. Basically, in their paper, they analyze the situation where providers can gain better profit if they choose to one pricing scheme and how much it can charge. When we mean that optimal or maximal profit obtained by providers, it does not mean that the providers get the highest profit but rather they can optimize their resources well, so it functions properly. The analysis of pricing strategy is divided into two parts; by considering the homogenous and heterogeneous customers. In homogenous case, all customers have the same utility on consumption level per day while in heterogeneous case, customers have two segments according to their willingness to pay and level of usage.

Sain [8] also considered four pricing schemes namely flat rate, usage-based, transaction-based, and version-based pricing. He analyzes those pricing strategy by grouping into two parts: the one component and two-component strategy and shows that usage based pricing is more efficient than one component pricing strategy by giving examples of well known communication network companies in the world.

Recent work on multiple service network are due to [9, 10, 11]. The papers describe the pricing scheme based on auction to allocate QoS and maximize ISP's revenue. The auction pricing scheme is scalability, efficiency and fairness in sharing resources. The solution of the optimization problem goes from single

bottleneck link in the network and then she generalizes into multiple bottleneck link using heuristic method. In this paper, she uses only single QoS parameter-bandwidth, while in networks, there are many parameters affect QoS that can be considered.

Although QoS mechanisms are available in some researches, there are few practical QoS networks. Even recently a work in this QoS network [2], it only applies simple network involving one single route from source to destination.

1.1 Research Statement

Due to scarcity research on multiple networks applied to pricing schemes to obtain optimal pricing strategy, where actually this issue remains critical especially as we then advance into the second generation internet. Telecommunication companies (telcos) face challenging problems due to user's preferences on flat rate pricing. Telcos develop multiple QoS networks to give customers choice to choose the service. However, telcos are having difficulties in coming out with the right pricing schemes with this multiple QoS networks.

Li et al [12] contended that the price could also be possible to be constraint in QoS networks due to the facts that resources is available in the network. It needs more observation to know the details about integration between multiple networks and pricing schemes.

Therefore, this research seeks to study and analyze the interaction between multiple networks and pricing schemes and develop new pricing plans that can dynamically work under multiple QoS networks.

1.2 Objectives of the Study

The objectives of this research will be to

1. Formulate the new optimal pricing schemes under multiple QoS networks.
2. Analyze the optimal pricing schemes under multiple QoS networks.
3. Determine whether in those pricing schemes which schemes offer better pricing that can enhance the customers and make advantages to telcos

1.3 Significance of the Study

Pricing schemes are critical issues in this current internet networks. Nowadays, telcos face a great challenge in managing appropriate pricing plans in these dynamical networks. With precise pricing plans, telcos are able to control congestion, maintain resources such as bandwidth, delay, etc optimally while also satisfy customer demands and gain optimal profit [13].

1.4 Scope of The Study

In this research, the pricing schemes proposed are based on telcos concerns on wired networks and the advantages of new pricing schemes are under telcos' perspective to maximize their profits.

2 Literature Review

The pricing schemes of the past were mainly responsive pricing that is only charging extra when network congestion indicated that the users had QoS degradation, with size of changes related to degree of congestion by comparing three different schemes for allocating a simple network's resources. Firstly is the pricing scheme that use no feedback and user adaptation to the network state. Secondly is the use of a closed-loop form of feedback and adaptation and lastly is a closed loop variation or tight loop as it shortens the delay in the control loop [14]. Other scheme are congestion avoidance algorithm proposed by [15] and also scheme that combined congestion avoidance algorithm and one type of responsive pricing scheme that was smart market mechanism by Network Protocol proposed by [16, 17].

One important thing why we wanted to create pricing mechanism was due to reducing congestion. What happened if we could not avoid congestion? Karp [18] explained problems related to congestion and how to control it. If, for instance, there was single flow which was sending packets from source to destination, if it transmitted at certain rate, it got dropped packet, but if it chose to send other rate, it could reach destination. It got acknowledgment from destination about the received packet. But how did we know how much flow can go through? The problem could be formulated as follows. How can the source A, for instance, knew and managed its flow over continuing certain time, meaning that time was divided into duration length of time like explained in [19, 20]. Others dealing with analysis of pricing strategy were to optimize profits, did not raise profits by guiding us to efficient pricing strategy which could control the congestion.

Tuffin [21], Ros & Tuffin [22] and Odlyzko [23] also proposed Paris metro pricing scheme for charging the network. In this case, the different service class would have different price. The user had choice to choose channels to travel and price to pay. The scheme basically made use of user to partition into classes and move to other class it found same service from other class with lower unit price. But still, they only considered with the case of single network which is not suitable with current internet.

Meanwhile, Altmann & Chu [24] offered new pricing plan that gave benefit to ISP and users. This plan was combination of flat rate and usage based pricing. In this plan, user would get benefit from unlimited access by choosing higher QoS and at the same time ISP was able to reduce its peak load. The drawback was still due to lack of information how that plans could be adopted into multiple route networks.

For the next generation internet, the availability of fast transportation of data is required. The multicast communication can decrease due to limitation of bandwidth. So we need QoS specification and compute optimal routes to a multi-constrained problem, by using greedy algorithm such as meta-heuristics algorithm , like suggested in [25].

3 Methodology

The research will give a valuable understanding of pricing schemes explained. Since the research involves multiple networks, we will formulate the algorithm to solve those networks and also simulate the result, We will make use of software applications such as LINGO to solve the optimization problem.

Basically, the steps of methodology involved are

1. Derive pricing schemes for multiple networks
2. Derive optimization problem for pricing schemes
3. Create flow chart/algorithm to solve optimization problem
4. Run the optimization problem of pricing schemes
5. Test the program of that optimization problem The result of simulation is for making decision whether we can adopt pricing scheme
6. Analyze the testing of optimization problem of pricing schemes and making conclusion about the result
7. Finally, write up the pricing schemes for multiple networks

4 Findings

In this preliminary findings, we would like to modify the mathematical formulation of [2, 9, 10, 11] since it could also combine into simpler formulation by taking into consideration the utility function, base price, quality premium, index performance, capacity and also bandwidth required. Then, we consider the problem of internet charging scheme as Mixed Binary Integer Nonlinear Programming (MBINLP) that can be solved using LINGO version 12.0 [26] to obtain optimal solution (see in [27]). In this part, we also would like to compare two models in which whether we fix decision variable of user admission to the class or not (see in [28]).

Assume that there is only one single network from source to destination since we concentrate on service pricing scheme. Assume that the routing schemes are already set up by the ISP. As [10] pointed out, we have 2 parts of utility function namely, base cost which does not depend on resource consumption and cost which depends on resource consumption. The utility function has

characteristics as marginal profit as function of bandwidth diminishing with increasing bandwidth.

The Objective of ISP is to obtain maximized revenue subject to constraints based on system' available resources.

We have parameters and decision variables, respectively

- α_j : base price for class j
- β_j : quality premium of class j that has I_q^j service performance
- Q : total bandwidth
- V_i : minimum bandwidth required by user i
- X_j : bandwidth for class j
- M : a very large positive number

- $Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$
- X_{ij} : final bandwidth obtained by user i for class j
- L_{mj} : minimum bandwidth for class j
- W_j : price for class j
- I_q^j : quality index of class j

The mathematical model will be

$$\max P_{ij}.U_{ij} = \sum_{j=1} \sum_i (\alpha_j + \beta_j * I_q^j) W_j * \log \frac{X_{ij}}{L_{mj}} * Z_{ij} \quad (1)$$

subject to

$$\sum_j \sum_i X_{ij} \leq Q \quad (2)$$

$$0 \leq I_q^j \leq 1 \quad (3)$$

$$X_{ij} \geq L_{mj} - (1 - Z_{ij}) * M \quad (4)$$

$$W_j \leq W_{ij} + (1 - Z_{ij}) * M \quad (5)$$

$$X_{ij} \geq V_i - (1 - Z_{ij}) * M \quad (6)$$

$$X_{ij} \geq X_j - (1 - Z_{ij}) * M \quad (7)$$

$$X_{ij} \geq 0 + Z_{ij} * M \quad (8)$$

$$X_{ij} \geq 0; L_{mj} \geq 0; W_j \geq 0 \quad (9)$$

$$X_{ij} \leq X_j \quad (10)$$

$$Z_{ij} = 0 \text{ or } 1 \quad (11)$$

We also have the second mathematical model that is

$$Max P_{ij} \cdot U_{ij} = \sum_j \sum_i (\alpha_j * Z_{ij} + \beta_j * I_q^j) * W_j \log \left(\frac{X_{ij}}{L_{m_j}} \right) \quad (12)$$

subject to (2)-(11)

Objective function (1) basically states that ISP wants to maximize its revenue from total sum of price and its utility function. Eq(2) tells as that total final bandwidth of all users cannot exceed the total bandwidth available. Quality index is the average of service quality that has value between 0 (meaning at base quality) or 1 (meaning that has best quality) as Eq(3) showed. Eq(4) states that bandwidth for user i has greater than the negative of minimum bandwidth for class j if user i is admitted to class j or otherwise. Eq(5) tells us about price for class j should be less than the price of user i willing to pay in class j if the user i will admit to class j . Next, Eq (6) basically shows that final bandwidth obtained by user i for class j will exceed negative of minimum bandwidth required by i if user i is admitted to class j or otherwise. Eq(7) states that final bandwidth obtained by user i at class j should be exceed bandwidth for class j if user i is admitted to class j or otherwise. Eq(8) tells us that final bandwidth obtained by user i should be greater than a very large positive number if user i is admitted to class j or not, if otherwise. Eq(9) state about the nonnegative requirements of the variables, Eq(10) shows that final bandwidth of user i to class j should not exceed the bandwidth of class j and lastly, Eq(11) tells us about decision if user i is admitted to class j or not.

Figure 1. Case when $\mathbf{Q}=\mathbf{M}=\mathbf{X}$

	$V_1 < V_2, W_1 < W_2$	$V_1 > V_2, W_1 > W_2$	$V_1 = V_2, W_1 = W_2$
V_1, V_2	5, 6	6, 5	5, 5
W_1, W_2	7, 8	8, 7	8, 8
GMU, ER	23, 0	23, 0	23, 0
OV, OB	82.89, 82.89	82.89, 82.89	82.89, 82.89
ESS, TSI	3, 229	3, 212	3, 239
I_q, W, L_m	1, 8, 0	1, 8, 0	1, 8, 0
X_1, X_2	0, 50	50, 0	0, 50
Z_1, Z_2	0, 1	1, 0	0, 1

First, for single class network, we can see that from Fig. 1, when $V_1 < V_2$ then $Z_1 = 0, Z_2 = 1$. It means that User 2 is admitted to the class since minimum bandwidth required by user 2 is larger than 1's. So between two users within one class, user that has larger minimum bandwidth required will be admitted to the class with price for that class is $W = \max\{W_1, W_2\}$.

Figure 2. Case when $Q > X, Q > M$ and $X = M$

	$V_1 < V_2, W_1 < W_2$	$V_1 > V_2, W_1 > W_2$	$V_1 = V_2, W_1 = W_2$
V_1, V_2	5, 6	6, 5	5, 5
W_1, W_2	7, 8	8, 7	8, 8
GMU, ER	23, 0	23, 0	23, 0
OV, OB	145.06, 145.06	145.06, 145.06	145.06, 145.06
ESS, TSI	3, 550	3, 524	3, 554
I_q, W, L_m	1, 7, 0	1, 7, 0	1, 7, 0
X_1, X_2	50, 50	50, 50	50, 50
Z_1, Z_2	1, 1	1, 1	1, 1

Fig. 2 explains different things. Since $Q > X, Q > M$ and $X = M$ then the value of $X_1 = X_2 = X$ and all users are admitted to that class with price for that class $W = \min\{W_1, W_2\}$. $I_q = 1$ means that $\alpha + \beta$ is the upper bound price for perfect service [2].

Figure 3. $Q > M, X_1 = M, X_2 > M$

	$V_1 < V_2, X_1 < X_2$ $W_{11} < W_{21}, W_{12} < W_{22}$	$V_1 > V_2, X_1 > X_2$ $W_{11} > W_{21}, W_{12} > W_{22}$
V_1, V_2, X_1, X_2	5, 6, 50, 60	6, 5, 60, 50
$W_{11}, W_{21}, W_{12}, W_{22}$	7, 8, 7, 8	8, 7, 8, 7
GMU, ER	29, 2	29, 2
OV, OB	1378.1, 1378.1	1402.27, 1402.27
I_q^1, I_q^2	1, 1	0, 48, 1
W_1, W_2	37.2, 57	0.53, 58
$X_{11}, X_{21}, X_{12}, X_{22}$	19.8, 20.2, 0, 60	0, 0, 6.07, 50
L_{m1}, L_{m2}	0, 0	0.17, 0
$Z_{11}, Z_{21}, Z_{12}, Z_{22}$	0, 0, 0, 1	0, 0, 0, 1

For case 1, only one user is admitted to only one class j . In this case, User 2 is admitted to Class 2 ($Z_{22} = 1$) having $I_q^2 = 1$ and minimum bandwidth for Class 2 is 0. Final bandwidth obtained by user i for class j who is admitted to class j , $X_{ij} = \min\{X_j\}$.

In Fig. 3, we can see the computation in multiple QoS networks. For case 1, only one user is admitted to only one class j . In this case, User 2 is admitted to class 2 ($Z_{22} = 1$) having $I_q^2 = 1$ and minimum bandwidth for class 2 is 0. Final bandwidth obtained by user i for class j who is admitted to class j , $X_{ij} = \min\{X_j\}$. For case 2, if we put quantities on parameters that are $Q = 100$ bps, $X_1 = X_2 = M = 50$ bps, $V_1 = V_2 = 5, W_{11} = 8, W_{21} = 7, W_{12} = 8, W_{22} = 7$ then we have the same results discussed in Fig. 3 ($V_1 > V_2, X_1 > X_2, W_{11} > W_{21}, W_{12} > W_{22}$). But if we see the in QoS networks, each class must have different bandwidth. so it is not possible to have $X_1 = X_2$, it should be $X_1 > X_2$ or $X_1 < X_2$.

Figure 4. Conditions of the Model

Case 1	Case 2	Case 3
$Q > M$	$Q > M$	$Q > M$
$V_1 < V_2$	$V_1 > V_2$	$V_1 < V_2$
$X_1 < X_2$	$X_1 > X_2$	$X_1 < X_2$
$W_{11} < W_{21}$	$W_{11} > W_{21}$	$W_{11} < W_{21}$
$X_1 = M$	$X_1 > M$	$X_1 = M$
$X_2 > M$	$X_2 = M$	$X_2 > M$

Next for comparing two models (1) and (12), we have conditions of model and parameter quantities in Fig.4 and Fig.5 respectively.

Figure 5. Parameter Quantities

	Case 1	Case 2	Case 3
Q, M	100, 50	100, 50	100, 60
V_1, V_2	5, 6	6, 5	5, 5
X_1, X_2	50, 60	60, 50	60, 60
W_{11}, W_{21}	7, 8	8, 7	8, 7
W_{12}, W_{22}	7, 8	8, 7	8, 7
α, β	0.1, 0.05	0.1, 0.05	0.1, 0.05

Then the solution of optimization problems is shown in Fig.6 below.

Figure 5. Results for 2 models and 3 cases

	case 1		case 2		case 3	
	1 st model	2 nd model	1 st mode	2 nd model	1 st model	2 nd model
GMU(K)	28	28	28	28	28	28
ER(sec)	0	0	1	0	1	0
OV(\$)	1378.097	2362.452	1402.274	2396.991	1644.046	461.6
Iter	117	434	113	21	7	113
Z_{11}, Z_{21}	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
Z_{12}, Z_{22}	0, 1	0, 1	0, 1	0, 1	0, 1	0, 0
I_q^1, I_q^2	0.53, 1	1, 1	0.53, 1	1, 1	0.45, 1	1, 1
W_1, W_2 (bps)	0.77, 57	57, 57	0.78, 58	57, 58	0.29, 68	66.8, 0
X_{11}, X_{21} (bps)	0, 0	0, 0	0, 0	0, 0	0, 0	0.16, 0.0067
X_{12}, X_{22} (bps)	40, 60	40, 60	50, 50	50, 50	0, 60	0, 60
L_{m1}, L_{m2} (bps)	0.03, 0	0, 0	0.03, 0	0, 0	0, 0	0, 0

In Model (1), only User 2 is admitted to Class 2 ($Z_{22}=1$) for each case with final bandwidth obtained by User 2 for Class 2 (X_{22}) is 60 bps (for Case 1 and Case 3) and 50 bps (for Case 2). Minimum bandwidth for Class 2 (L_{m2}) in Model 1 is 0 bps with price for Class 2 (W_2) is \$57 (for Case 1), \$58 (for Case 2) and \$68(Case 3). Quality index of Class 2 (I_q^2) has the highest index of 1 for each case. Generated memory used (GMU) for each case is 28K with elapsed runtime (ER) is 0 sec (Case 1) and 1 sec (Case 2).

Also, in Model (2), User 2 is only user that is admitted to Class 2 (for Case 1 and 2 only) with $X_{22}=60$ bps for Case 1 and $X_{22} = 50$ bps for Case 2. $L_{m2} = 0$

bps in each case with $W_2=57$ bps (in Case 1) and $W_2=58$ bps (Case 2). Quality index of Class 2 is 1. Meanwhile in Case 3, there are no user is allowed to each class ($Z_{ij}=0$).

If we compare with conditions and result, we can see some changes in parameter values. In Case 1, the change of value of $W_{22} = 57$ bps does not violate condition of $W_{12} < W_{22}$. In both Case 2 and 3, the value change of W_{22} violates the condition of $W_{12} > W_{22}$. Case 1 gives better result rather than Case 2 and Case 3 although in getting profit maximization the result in Case 1 is lower than in other cases (OV= 1378.097 for Model 1 and OV= 362.452 for Model 2).

4.1 Concluding Remark

The model represented shows the connection between bandwidth required, bandwidth obtained and QoS by giving the assumptions and data;we can find the optimal solution with profit maximization. However, due to assumptions, we have limited the model into static optimal solution and cannot be dynamic solution where we should have various demands for capacity (peak and off-peak).

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