SUPPORTING STUDENTS' SPATIAL ABILITY IN UNDERSTANDING THREE-DIMENSIONAL REPRESENTATIONS

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Abstract

Spatial ability is known as a key to develop students' 3-dimensional (Abbreviated as 3D) geometry thinking. One of the types of 3D geometry thinking is understanding 3D representations which anchors the other types of geometry thinking. Therefore, to develop students' understanding of 3D representations, we must develop their spatial ability. The present study aims to design a sequence of activities to develop students' spatial ability in understanding 3D representations. To develop such activities, we combined spatial visualization tasks and spatial orientation tasks with the aspect of understanding 3D representations. These activities are identifying pictures, drawing and constructing objects of building blocks. The activities are also designed based on the characteristics of Realistic Mathematics Education (RME) and students' learning style. In addition, this study also aims to contribute to local instruction theory of developing students' spatial ability such as how the design works and how students' learning goes. Consequently, design research is chosen as the research approach in order to develop both the learning materials and the instructional theory. The study involved six 3rd grade students of SD Laboratorium UNESA, Surabaya. The result showed that the activities support the development of students' spatial ability in understanding 3D representations. During the implementation, these activities gave students chance and guided them to explore the views of 3D objects and their representations.

Keywords: *Spatial ability, spatial visualization, spatial orientation, understanding 3D representation*

INTRODUCTION

Spatial ability has been widely known to be an important skill for students. Many studies have proved that it is positively related to students' achievement in mathematics (Hegarty & Waller, 2005; Newcombe, 2010; Pittalis & Christou, 2010; Cheng & Mix, 2014). However, it is not yet specifically known which topics of mathematics are supported by spatial ability. In 2010, a study revealed that spatial ability support the development of students' 3D geometry thinking which consists of four types of reasoning: 1) understanding 3D representations, 2) spatial structuring, 3) conceptualization of mathematical properties, and 4) measurement (Pittalis & Christou, 2010). The study concluded that the development of students' spatial ability will promote the development of students' 3D geometry thinking. Although spatial ability has an important role in the development of students' 3D geometry thinking, it does not yet receive much attention in the Indonesian mathematics classroom (Revina, Zulkardi, Darmowijoyo, & van Galen, 2011; Risma, Putri, & Hartono, 2013).

In primary education, there are some activities related to students' spatial ability such as constructing the net of cubes, determining directions, finding the volume of space figures and map reading. However, these activities do not supporting students' 3D geometry thinking, especially understanding 3D representations. According to Duval (1999), understanding 3D representations play a crucial role to support the other types of 3D geometry thinking. It becomes the foundation of students' 3D geometry thinking. Lack of understanding 3D representations can impede students' learning, especially in the field of geometry. Therefore, we need activities to support students' spatial ability in understanding 3D representations.

The present study aims to design a sequence of activities to support students' spatial ability in understanding 3D representations. At the same time, this study also aims to contribute to the local instruction theory of how to develop students' spatial ability. This study will be guided by the following research question: *How can we support the development of students' spatial ability in understanding 3D representations?*

THEORETICAL FRAMEWORK

Spatial Ability in Understanding 3D representations

There are many definitions of spatial ability. McGee (1979) defined spatial ability as the ability to formulate mental images and manipulate them mentally. Supporting McGee's definition, Tartre (1990) added that spatial ability is a mental skill concerned with understanding, manipulating, reorganizing, or interpreting relationships visually. In short, spatial ability is the ability concerned with the use of space (Olkun, 2003). Summarizing all the definitions of spatial ability, we defined it as the ability to mentally interpret, manipulate, reorganize, and process information visually related to the use of space. There are many factors that build students' spatial ability. Lohman (1988) stated that there are three major factors that determine students' spatial ability. They are 1) spatial visualization, 2) spatial orientation, and 3) spatial relation.

Spatial visualization is the ability to handle imaginary movement in three dimensional space such as moving, rotating, and manipulating (Lohman, 1988). Clements (2003) added that spatial visualization is understanding and performing mental movements of two or three dimensional object. The key word of spatial visualization is "mental movement". Meanwhile, students' spatial orientation is the ability of understanding and operating the relationship between information of different positions in space with respect to the information in students' position (McGee, 1979). It suggests the students to do not confused if the perceptual perspective of the students in space is changed (Lohman, 1988). Unlike spatial visualization, spatial orientation does not require to mentally move the object. The last major factor of spatial ability is spatial relation. It is the ability to do mental rotations of an object as a unit quickly (Lohman, 1988). Some scientists only consider two major factors that determine students' spatial ability (McGee 1979; Clements, 2003). These two factors are spatial orientation and spatial visualization.

According to the study of Pittalis and Christou (2010), spatial ability supports the development of students' 3D geometry thinking which consists of 4 types. The types are 1) understanding 3D representations, 2) spatial structuring, 3) conceptualization of mathematical properties, and 4) measurement. These four types of geometry thinking are in line with Duval's (1998) theory. Duval explained that understanding 3D representations becomes the foundation of the other types of 3D geometry thinking. The following figure 1 illustrates the relation between the four types of 3D geometry thinking.

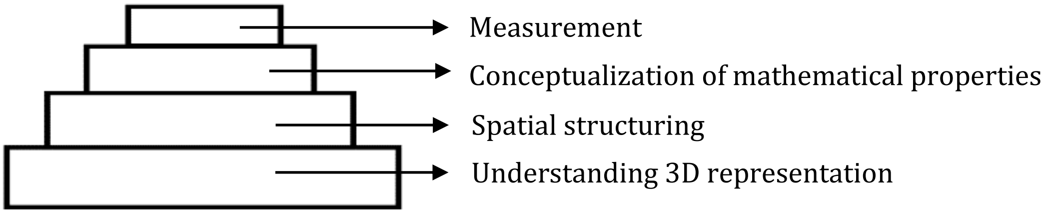

Figure 1: Four types of 3D geometry thinking

Understanding 3D representation itself is indicated by two aspects: 1) recognizing, and 2) manipulating 3D representations. Based on the study of Pittalis and Christou (2010), understanding 3D representations is supported by only two main factors of spatial ability. They are spatial visualization and orientation. Therefore, to improve students' ability in understanding 3D representation, we must support students' spatial visualization and orientation.

The Present Study

This study aims to develop students' spatial ability in understanding 3D representations. To achieve this goal, we design a sequence of lessons consisting of spatial visualization tasks and spatial orientation tasks combined with the aspect of understanding 3D representations. The designing process is based on the five principles of Realistic Mathematics Education: 1) the use of context, 2) the use of model, 3) the use of students' constructions and productions, 3) interactivity, and 4) intertwinement. Furthermore, we also consider students' learning style in designing the activities. According to Park (2000), most of the elementary students have a kinesthetic and visual learning style. This means they are easier to learn by experiencing, touching, and observing things visually. Therefore, the tasks involve the use of real objects such as building blocks and model of buildings. We design the activities such that students can experience directly spatial orientation and spatial visualization of an object. Later, these objects are gradually replaced by distant representations such as pictures and photographs. The following table 1 describes the level of representations employed in the tasks based on Parzysz's level of representations (1988).

Table 1: Levels of 3D representations

	Geometry Objects	
Levels	2D	3D

	Level 0	Shape of Object	Object
Close representation	Level 1	Drawing	Model
Distant representation	Level 2		Drawing

METHOD

This study uses design research as the research approach because it enables us not only to develop activities and learning materials, but also to develop a local instruction theory on how the activities support students' spatial ability (Gravemeijer & Cobb, 2006). Design research consists of 3 phases: 1) preparation and designing, 2) teaching experiment, and 3) retrospective analysis (Cobb, Confrey, Lehrer, & Schauble, 2003). We designed the activities in the preparation and designing phase. In addition, we also developed a Hypothetical Learning Trajectory (HLT) for the activities. In the teaching experiment phase, we did two cycles: 1) a preliminary teaching experiment (cycle 1) and 2) a teaching experiment (cycle 2). The preliminary teaching experiment involves 6 students of grade 3B of SD Laboratorium UNESA. On the other hand, the cycle 2 involves 26 students of grade 3C of SD Laboratorium UNESA. The result of cycle 1 would be analyzed to revise and improve the activities and the HLT. The current article reports only on the first cycle of the study in which 6 students were involved to test the activities and the HLT before we implemented the design in the class.

During the preliminary teaching experiment, we conducted a classroom observation registered by videos and field notes. All the students' written works were collected to be analyzed in the retrospective analysis phase. Before and after the activities, students did a pretest and a posttest about their knowledge regarding the topic. After the posttest, we conducted an interview with the students to further dig up findings during the learning process.

RESULTS AND DISCUSSION

There are five lessons in this study. The first lesson is named "playing with the camera part 1". It aims to let students experience the orientation of an object. Lesson 1 also became a preliminary activity for the students before solving problems related to spatial orientation tasks. The second lesson is "playing with the camera part 2". In this lesson, students get spatial orientation tasks to develop their spatial orientation ability. In the next lesson, students have drawing activities in the context of reporting on newly built temples. The aims is to develop students' spatial visualization and to bridge students thinking from using the model into using the distant representations of the object. Lesson 4 enables students further to explore the properties of the views of an object. At the end of the sequence, students will have "building temple" activities. In this lesson, students have to build an object based on its photo. During the lessons, students worked in two small groups of 3 students. The details of each lesson and how students' learning developed will be discussed below.

Lesson 1: Playing with the camera part 1

In this lesson, students became photographers with the task to find the best position to capture the buildings in a city. During the discussion, each group was given a set of miniatures of the city, camera model, and digital camera. Camera model is made from paper formed like a cuboid without the upper and down side. This camera model will be used to help students looking the object. The miniatures of the city can be seen in figure 2. The investigation in this lesson consisted of three problems. In the first problem, students had to determine the buildings they see from each position. During the investigation, students moved around the table while seeing through the camera model to directly investigate the building they will see from each position. Group 1 preferred to use the digital camera since they could directly see the photo. However, group 2 argued that using the camera model was more efficient and easier to use. Both of group successfully completed the tasks correctly.

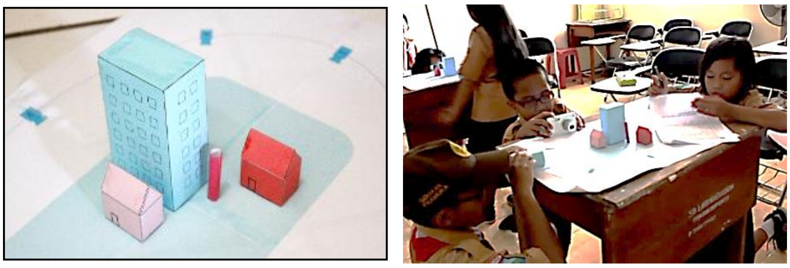

Figure 2: Miniature of the city and students' investigations

In the second problem, students had to describe the position of buildings in the photo from different stand points by using terms like right, left, front or back. In describing the position, some students used the orientation of their body to determine the position while some others used the orientation of the apartment. In this case, the teacher brought the students to an agreement of using the orientation of the apartment. Furthermore, students also used the term "back", "front", and even "front-left side" (*sebelah kiri bagian depan*) to describe the position. None of them used cardinal directions to describe the position of the building.

The third problem was finding the position that enables them to capture two, three, or all the buildings. To find the position to capture some or all the buildings, students again moved around the object while seeing the objects to find the desired position. None of them used their previous investigation in problem one to determine the position. In the end, both group suggested positions where they could see all the building. Based on our observation, students' activities and strategies during the lesson indicates that they had directly experienced the orientation of the objects.

Lesson 2: Playing with the camera part 2

In lesson 2, students were asked to investigate 12 photographs and to find the position where the photos were taken. However, among these 12 photos, there were 4 photos that were not of the objects. Therefore, students have to analyze the photo carefully in order to pick the right photos and find its position. The teacher provided each group the objects in the photo, the camera model and the digital camera. However, when investigating the position of the photos, students were not allowed to use the digital camera because it would be used later in the discussion to confirm their answer. To find the position, students moved around the object while holding the photo. They tried to find a match between what they saw on the photo and the view of the real object. At the beginning, students used the camera model to see the object but then after several photos they stopped using the camera model. This indicates that students had been able to visualize the view of the object just by looking from the bird eye angle. Hence, students had develop basic spatial visualization by manipulating the image of the object in their mind and visualize its view.

To determine the wrong photo, students firstly tried to find the position by moving around the object. If they did not find a match, they claimed the photo was not of the object. Some students determined the wrong photos by identifying the difference between the real object and the object on the photo from a certain stand point. Figure 3 shows one of students' answers on this activity. In the figure, students circled parts of the object on the photo and wrote "*kebalik*" or "reversed". This shows how students claimed that the photo is not from the object. At the end of the activity, both groups took the correct photos and put them in the right position. To confirm their answers, each group used their camera digital to capture the real object. Based on the description above, students were able to relate the information they had on the photo with the information they saw in the real object. This means that students had developed their spatial orientation.

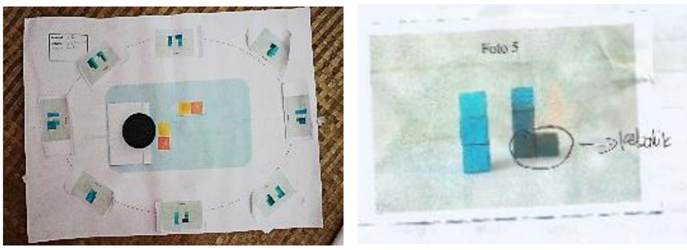

Figure 3: Students' answer for the activity in lesson 2

Lesson 3: Reporting new temples

The third lesson was about reporting on new temples by drawing its standard views. The first problem asked students to construct a temple by using exactly 6 cubes and then to draw its standard views. During the process, students did not have much difficulties to identify the shape of the standard views. Some used the camera to help them draw while other students did not need it. Group 2 showed an unexpected drawing since they shaded some cubes to indicate their position. A shaded cube means that the cube lies behind the unshaded cubes. The following figure is the answer of group 2 for the first problem of lesson 3.

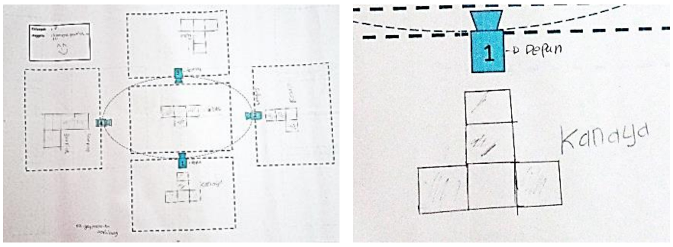

Figure 4: Second group's drawings for the first problem in lesson 3

In the next problem, students were given a bird eye photo of building blocks. They had to draw the standard view from the side of the object. At first, students struggled to imagine the shape since they did not have the object. Kanaya from group 2 suggested to look at the photo from its edge to help her imagine the shape. This indicates that Kanaya actually tried to apply her strategy in the first problem to solve the second problem. However, she did not have the object which forced her to imagine the movement. The following figure is the drawings of group 1 and 2 for the second problem of lesson 3. It can be seen that group 2 also apply the shading strategy in this drawing.

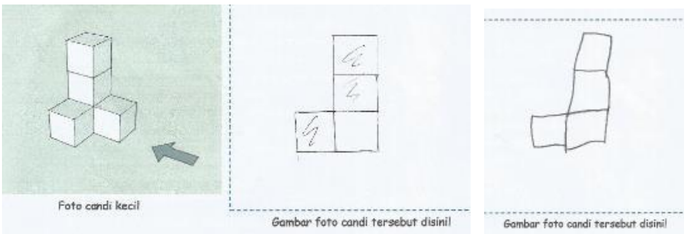

Figure 5: Students' drawings for the second problem in lesson 3

Based on the students' learning, we conclude that students were able to determine and draw the standard views of the object. The activities helped students moved working with the concrete model to the use of distant representation (picture). The first activity gave students preliminary experiences to do mental movements in the second problem.

Lesson 4: Fixing the reports

In this lesson, students were given a problem of arranging the photos of standard views of an object. There were 5 standard views given to the students: top view, left view, right view, front view and back view and they had to arrange the photos in the layout. The aim is to support students to discover the properties of the standard views. This properties are 1) the back and the front view of an object are "similar" 2) the left and the right view are also "similar". "Similar" means they had the same shape but they are reversed. During the discussion, we noticed that students struggled to discover this properties directly from the activity. The teacher had to give direct assistance to help students finding the idea of arranging the photos. Although students accepted the properties of the standard views, they seemed confused and did not understand it. This lack of sense of the properties was proved when the students worked on the second problem. In this problem, students had to draw the lost standard views in three reports and determine which report can be repaired by drawing all its standard views. Students repaired the first report correctly since the lost photos were the left view and the front view. They drew the left and the front view by using its opposite view (right and back view) as the reference since the opposite photo in the layout had a "similar" shape. However, in the second report where the front and the back photo were lost, students drew the shape randomly by assuming the view from the top view. We noticed that students tried to imagine the object in the middle of the layout. However, they could not imagine the object because they did not yet experienced this activity. In this lesson, we found a jump from lesson 3 to lesson 4 that requires students to imagine building the object. Although students could discover the properties, they did not fully understand it. Therefore, we conclude that the activities in this lesson should be done after the lesson 5. Also a preliminary investigation is required to guide students to discover the properties of standard views of an object.

Lesson 5: Building new temples

Lesson 5 was about building an object based on its bird eye picture and its standard views. There were two problems of building an object. The first one was building an object based on its bird eye photograph (Figure 6a). The activity aims to help students noticing the blind spot of distant representation. In the process, both groups constructed the object in the same way. They assumed the back part of the object (Figure 6b). As a response to students' answers, the teacher then proposed another construction by removing two cubes in the back (Figure 6c). By doing this, students realized that the object was still similar to the photo but it had differences in the back from the previous construction. After discussing it with the teacher, students concluded that they cannot build the object unless they had more information such as another photo of the object from the back.

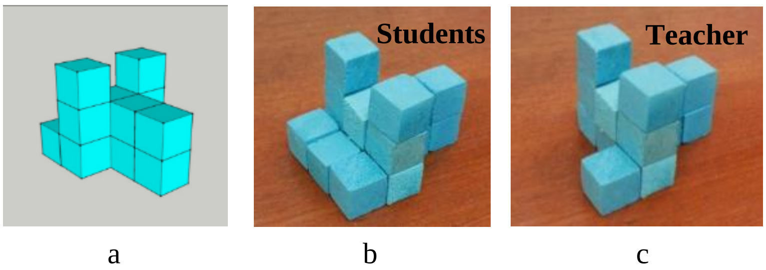

Figure 6: Students' construction and teacher's proposed model

The following fragment illustrates the discussion between the teacher and the students regarding the blind spot of the distant representation.

Fragment 1: Students discussed the blind spot of the representation

1 Teacher : So, the temple of group 1 and group 2 are the same. Has the temple been similar to the photo?
2 Both group : Yes
3 Teacher : What if I take these two cubes and this cube. Is the temple still similar to the photo?
4 Aydin : Yes, it is
5 Hafid : No
6 Aura and Aydin: Yes, it is still similar to the photo, Fid!

7 Kanaya : If you look from this point, yes it is still similar
8 Teacher : So, which is the correct one? Does it have these cubes (removed cubes) or not?
9 Kanaya : Yes, it has
10 Teacher : How did you know it Kanaya?
11 Kanaya and Aydin: hehehe
12 Aura : guess it sir.
13 Teacher : Oh by guessing, if you are not allowed to guess it, then how?
14 Kanaya : then you could not figure it out
15 Aura : we have to ask Reza first or the government
16 Teacher : what do you want to ask?
17 Aura : Sir, is there another cubes in the back of the temple?
18 Teacher : okay, so if you just have this photo, can you figure out what the back part looks like?
19 Students : No, we cannot
20 Teacher : So, can you build the miniature of the temple?
21 Ratu : No, you cannot
22 Kanaya and Aura: You can build it, but the back part is incomplete

From the discussion, we notice that students firstly assumed they could know what the back part of the object looks like without realizing that actually it is impossible. After the teacher proposed another constructions, they realized that they had assumed the back part of the temple. Afterwards, the teacher gave the standard views of the temple from the back, the right, and the top. Students then had to finish their construction based on all the given photos. To adjust the construction, students worked from the back view and then the side view. Finally, they got the same temple after adjusting the object with its top view. This activity indicated two thing. First, students were able to interpret and read the distant representation of an object (photos) which suggests they had develop an understanding of 3D representations. Second, they were able after the support of the teacher to combine their spatial visualization and orientation ability to build the object. Building the object based on its bird eye view involves both spatial orientation and spatial visualization.

In the second problem of lesson 5, students built an object only based on its standard views (Figure 7). As we had predicted, students began the construction from the top view and then adjusted the temple based on its side views. However, the two groups came up with two different constructions. After they checked each other work, they concluded that both temples were correct and that there were two possible answers. At that point, the teacher gave additional information that the temple only had 9 cubes. This additional condition leads both groups to have the same construction in figure 7.

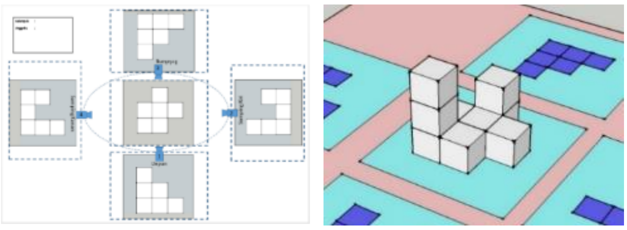

Figure 7: The second problem of lesson 5 and students' answer

These activities show that students were able to connect the information they had in each standard view of the temple to construct and build the temple by using the cubes. This indicates a development of students' spatial ability in interpreting, recognizing, and manipulating representation of 3D objects.

CONCLUSION

Based on the discussion and analysis of the students' learning above, we conclude several points regarding the activities and recommendations for the next cycle.

1. In the lesson 1 and the lesson 2, the activities helped students experiencing and developing spatial orientation. This is indicated by how students develop the strategies to solve the problems in the activity. They made connections between information in the photo with the real situation visually. Furthermore, students also could differentiate the photos of the object and the photos that are not of the objects.
2. Drawing activities in lesson 3 supported the development of students' spatial visualization. In this activity, students' strategies to identify the shape slowly shifted to work without the building block. This indicated that they performed mental movement on the distant representation of an object. They also could identify and drew the standard views of an object based on both its model and its photo.
3. There is a gap between lesson 4 and lesson 3 since students had not yet experienced building object based on its standard views. In addition, in this lesson, students did not use building blocks which made the activity even more difficult to imagine for them. Therefore, lesson 4 should be moved and conducted after the lesson 5.
4. Lesson 5 helped students to identify and predict the blind spot of representation with the support of the teacher. Furthermore, the activities supported students' spatial ability in understanding 3D representations. They could interpret, recognize, and connect the standard views of 3D objects to build and construct it by using cubes.

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