DEVELOPING SEVENTH GRADERS' ALGEBRAIC THINKING ON LINEAR EQUATION IN ONE VARIABLE TOPIC

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Abstract

This paper discusses how to develop seventh graders' algebraic thinking on linear equation in one variable. For many students in the first year of middle school, algebra is given as formal and abstract topic. Most of them know how to solve the problems of linear equation in one variable related to the arithmetical thinking directly, but they have difficulties in understanding algebra topic of variable in equations deal with real-world algebra problems. They have misunderstanding of equal sign for solving linear equation correctly as well. We designed classroom activities to develop students' algebraic thinking in two components which can be identified for seventh grade students, that is, concept of equality and generalization. These components will be underpinned by Realistic Mathematics Education (RME). This research uses a design research methodology which consists of three phases. The first phase is thought experiment that means designing classroom activities and making conjectures of students' learning. The second phase is teaching experiment that was conducted in one class of grade 7 at SMPN 44 Jakarta. The last phase is retrospective analysis, we analysed the data of students' learning process based on video recording, students' work, interviews, and field notes, then compared them with the hypothetical learning trajectory. The result shows that RME-based activities, starting with inequality problems and pictorial equation solving, have supported students to understand the equal sign as an indication of equality and to understand the relationship between the situations using linear generalizing problems.

Keywords: *algebraic thinking, linear equation in one variable, pictorial equation solving, RME.*

INTRODUCTION

Algebra is one of topics in Indonesian mathematics curriculum and originally be introduced for middle school. It is used as a mathematical method for solving problems related to the problems in real life. Yee (2007) revealed that algebra has been called a language of mathematics. It is conceived as the branch of mathematics that deals with symbolizing general numerical relationship and structures to construct students' algebraic thinking.

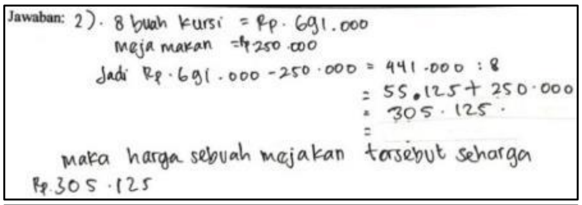
Linear equation in one variable gives the relationship between algebra and its application which need students' algebraic thinking. Ontario Ministry of Education (2013) described the importance of algebraic thinking that would focus on generalizing, expressing relationship, and exploring the concepts. This would push mathematics understanding of the students beyond the result of specific calculations and the procedural application of formulas.

Based on the statements above, in fact, there are problems in teaching and learning of that topic in the class. The teacher just started it procedurally, emphasized the skilled use of algebraic procedures, and did not construct students' understanding about the concept so that the students just knew the procedural ways rather than operations and processes. The following contextual problems support students to solve the problems related to algebraic thinking for the topic of linear equation in one variable.

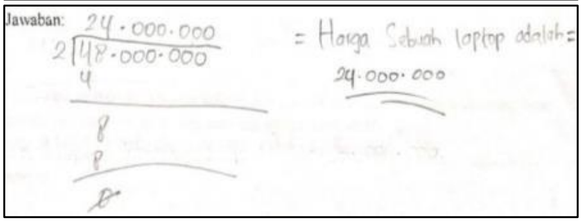
Mr.Henri bought 1 dining table and 8 chairs Rp691.000,00. If the price of one dining table was Rp250.000,00 more expensive than each chair, what was the price of one dining table?
The price of laptop is 5 times as much money as printers. If the price of 5 printers and 2 laptops is Rp48.000.000,- then how much does a laptop cost?

Figure 1: Problems of Equality Concept in Trial Test

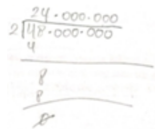
Figure 1 supports the students to identify the concept. It is one of the instructions to develop students' algebraic thinking about equality concept. In fact, there were only three students that knew about the given information, however they still gave wrong meaning or had difficulties in understanding the concept. Besides, they did not try to translate the problems into mathematical symbols. Here are some students' mistakes:


Jawaban: 2). 8 buah kursi = Rp. 691.000
Meja makan = Rp 250.000
Jadi Rp. 691.000 - 250.000 = 441.000 : 8
= 55.125 + 250.000
= 305.125
=
maka harga sebuah mejakan tersebut seharga
Rp. 305.125

Answer: 8 chairs = Rp691.000,-
dining table = Rp 250.000,-
So, 691.000 – 250.000 = 441.000 : 8
= 55.125 + 250.000
= 305.125
Hence, the price of dining table is Rp305.125,-


Jawaban: 24.000.000
2 | 48.000.000
4
8
8
0
= Harga Sebuah laptop adalah = 24.000.000

Answer:



The price of laptop is Rp24.000.000,-
Therefore, the price of dining table is Rp305.125,-

Figure 2: Students' Mistakes of Solving the First and Second Problem

Based on the trial test, we found that they still had difficulties to develop algebraic thinking on linear equation in one variable. They did not understand the operations and inequality form. They did not translate the problems into the models which could assist them to solve the problems.

The other fact was revealed by Yee (2006) that students are not familiar in word problems of algebra. They have difficulties to translate verbal language into algebraic expressions or equations and vice-versa as a translation activity. Moreover, they find algebra too abstract because it is dominated by symbols. These difficulties could be seen in their answer which all of the problems using verbal language. Knuth, Stephens, McNeil, and Alibali (2006) also told that understanding the equals sign has previously been shown to be crucial for algebraic problem solution and students' difficulty with the concept has been relatively well documented. Those conditions show that students' successful to learn linear equation in one variable need to be supported by contextual problems. Those problems will facilitate the students to develop algebraic thinking, so they can solve their problems in their life. This is confirmed by Treffers and Goffree in Wijaya (2012), realistic problems are going to be meaningful as the initial step because context has important function and role such as applicability and practice specific abilities in a situation applied. Thus, teaching and learning of linear equation in one variable using contextual problems enables students to develop their algebraic thinking.

The application of Realistic Mathematics Education (RME) can be an approach which suitable with developing students' algebraic thinking on teaching and learning of linear equation in one variable in the class. We provided activities about equality concept and generalization using contextual problems. Those problems could support them to get the strategy such as pictorial equation solving. Therefore we posed the following question to be answered through design research: "How RME can develop seventh graders' algebraic thinking on linear equation in one variable?"

THEORETICAL FRAMEWORK

Realistic Mathematics Education (RME)

Pendidikan Matematika Realistik Indonesia (PMRI) is adapted from Realistic Mathematics Education (RME) which is applied for the context in Indonesia or called the Indonesia Realistic Mathematics Education. RME is designed by Freudenthal and developed in Netherland. Freudenthal in Wijaya (2012) stated that mathematics need to be connected to realistic problem and should be expressed as a human activity. It means mathematics must be connected to the real situation in students' life and must be taught as human activity in every student's learning. The term 'realistic' stresses that problem situations should be meaningful and implies to something real which can be understood and imagined by students so that they can apply mathematical concept based on teaching and learning of mathematics in the class when solving problems in the real life.

Gravemeijer in Bakker (2004) revealed RME principles, namely guided reinvention through progressive mathematization, didactical phenomenology, and self-developed or emergent models. The first one is process of students for rediscovering mathematical concepts by learning environment and teacher guidance. In the second one, teaching and learning of mathematics requires to the use of reasoning in the way of finding problem situations that could provide methods for development of mathematical concepts. The last one is connected to helping students to make progress formal mathematical activity from informal knowledge.

Treffers in Wijaya (2012) has also defined five tenets for RME. They are phenomenological exploration, using models and symbols for progressive mathematization, using students' own constructions and productions, interactivity, and intertwinement. Phenomenological exploration makes students to do exploration of problems actively which for finding the final answers and developing strategies of problem-solving. The second one is using models and symbols for progressive mathematization. Model is developed by students based on informal knowledge and pre knowledge. Contextual problems bring out 'model of' from the situation, afterward show 'model for' to formal mathematics when the students have aimed the model for solving problems mathematically. In the third one, it is assumed that when the students can construct their knowledge, they can get meaningful teaching and learning of mathematics for them. Hence, students' construction and production with the guidance is essential part of instruction. The fourth one is interactivity, that is, the characteristic which is needed in RME. It is important because it can support communication skill and has contribution for developing students' knowledge. Learning process will become meaningful when the students can communicate their ideas to the others in the classroom. The last one is intertwinement and it is the important thing for integrating students' knowledge. Mathematics education should lead to useful integrated knowledge. It means teaching and learning of mathematics consider an instructional sequence in its relation to other domains and it is applied for solving problems.

Linear Equation in One Variable

Blitzer (2003) stated about equation which consists of two algebraic expressions (the left-hand side and the right-hand side) and be separated by an equal sign. Billstein, Libeskin, and Lott (2007) revealed that the equal sign indicates equality of the value for both sides. The concept is usually illustrated such a balance-scale model for solving equations and inequalities.

Variables are usually associated with equations. Van de Walle and Folk (2005) described a variable as "a symbol that can stand for any one of a set of numbers or other objects." It has different uses depending on the context, then it is as a tool to better think about and understand mathematical idea. Billstein, Libeskin, and Lott (2010) emphasized a major aspect of algebraic thinking from the concept of a variable, "one of the big ideas of algebraic thinking is the concept of variable" so that understanding about it is a fundamental thing.

Blitzer (2003) indicated a linear equation in one variable as an equation which has the power of 1 for the exponent on the variable. It is called a first-degree equation since the greatest power on the variable is one. If we solve an equation with finding the set of numbers that makes the equation a true statement, then that number is a solution of the equation. We just have to change the variable to make it a true statement. Substituting a solution to an equation for the variable makes the right-hand side equal to the left-hand side. We may use the addition, subtraction, multiplication, and division properties of equality. These properties state that adding, subtracting, multiplying, or dividing both sides by the same thing will make an equivalent equation, but we may not multiply or divide it by 0.

Algebraic Thinking

Van de Walle, Karp, and Bay-Williams (2011) described that "algebraic thinking or reasoning involves forming generalizations from experiences with number and computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts of pattern and functions." Based on their opinion, students will build their algebraic thinking with the experiences on daily activity, then use algebraic form to solve their problems, so they learn algebra not only the procedures, but also the meaning of each symbols. Russels in Kamol and Har (2010) also revealed about algebraic thinking as a tool for understanding abstraction. It is necessary for improving students' learning in mathematics such as cultivating mathematical conceptualization, especially for algebra topic.

Biggs and Collis in Kamol and Har (2010) identified that algebraic thinking has four levels of thinking: prestructural (level 1), unistructural (level 2), multistructural (level 3), and relational (level 4). It is based on SOLO (Structure of the Observing Learning Outcome) model. The algebraic thinking framework developed can be a guidance for the teachers to observe development of students' algebraic thinking.

Students, on level 1 thinking, were hypothesized that they were confused or unable to understand the tasks. They would focus on irrelevant data when answering the questions or even avoided answering them. Students exhibiting level 2 when they would engage the task in a relevant way but were not able to proceed further or just pursue only one aspect of it. They might try to represent the ideas through quantitative, but sometimes they would regress to prestructural thinking because they would be in transition between level 1 and level 2 thinking. Students on level 3 when they demonstrated an ability to complete the tasks and began to focus on more than one aspect of the task, but they would not always integrate their thinking to the original problem. Furthermore, students on level 4 were able to see relationship between the given data – making connection among the various relevant features. They also used all features and provided logical perspectives and explanations of data situations.

One of the ideas that may help the students to develop algebraic ways of thinking is using pictorial equation solving, Cai and Moyer (2008). It uses rectangles to solve algebra word problems involving whole numbers. The models constructed will represent quantities and relationships between and among quantities and unknowns. In addition, the models make them relatively easy for the representations to be partitioned into smaller units so that it will make the students to express their creativity when solving the problems.

METHOD

The research used a design research methodology which consists of three phases as Gravemeijer and Cobb (2006) explained. The first phase is thought experiment that is designing hypothetical learning trajectory (HLT) for classroom activities. The second phase is teaching experiment using one cycle. It focused on a seventh grade class that was consisted of 34 students at SMPN 44 Jakarta. The teacher would manage the students to do the activities prepared and used HLT as a guideline. The last phase is retrospective analysis that means analyzing the data of students' learning process based on video recording, photos, students' works on worksheets and their oral explanation, interviews, and field notes in every meetings, then compared them with the HLT. This phase had been started in the teaching experiment as an evaluation which enabled for changing the next activities. Hence, the result would be used to answer research question which had been formulated and could contribute to the HLT of further research.

The internal validity of this study is guaranteed by employing data collections and the soundness of the reasoning that has led to the conclusions as Bakker (2004) described. The teacher and observers discussed the process of students' learning based on the data collected. External validity is mostly interpreted as generalization of the results and used by analyzing students' participation in their classroom activity. There are also two kinds of reliability. Internal reliability was used by managing the data collected, then discussing them with the observers during design research phases. External reliability is about replicability. Bakker (2004) stated that it means all of the results of research must be reported both of failures and successes, procedures followed, the conceptual framework used, and the reasons for making certain choices.

RESULT AND DISCUSSION

We had three activities in our teaching experiment that used "Party Time Preparation" as the contextual problems on linear equation in one variable. Every activities had to be worked in group. Those activities enabled students to develop algebraic thinking through pictorial equation solving as a strategy which made them understand about variables. Hence, they not only knew the procedural ways of solving problems, but also knew the meaning of symbols built.

Activity 1: Understanding equality using inequality problems

Here we directed students to construct the equality concept. We posed this activity and developed the problems based on Yee (2007) which described how to teach about equality by demonstrating inequality (imbalance) first. The problems directed students to think flexibly about quantities and to learn how to compare the objects to another.

Lisa's mother was preparing cupcakes for Lisa's birthday. She was preparing them into two plates. Firstly, she has put them with the same numbers into those plates. Afterwards, she add the cupcakes and we could know that the second plate has 6 more cupcakes than the first one now. If she add 12 more cupcakes on the first plate, then:

a. *which plate does have more cupcakes now?*
b. *if she move 2 cupcakes from the first one to the second one, which plate does have more cupcakes now?*
c. *what should she do to get same numbers of cupcakes on both of plates?*

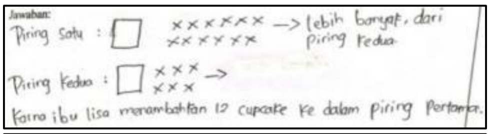
Figure 3: Inequality Problem for Achieving Equality

Figure 3 shows the contextual problem of inequality which could direct the students to learn about the equality concept. Here the students had to compare the quantities in imbalanced situation until they knew the process of achieving equality and the meaning of how the situations called well-balanced. This allowed them to build the strategy in various ways. They answered it using pictures, the box as a missing element and numbers, or comparation way. The conversation of fragment 1 shows about student's understanding to build strategy:

Fragment 1: Student's Understanding to Build Strategy

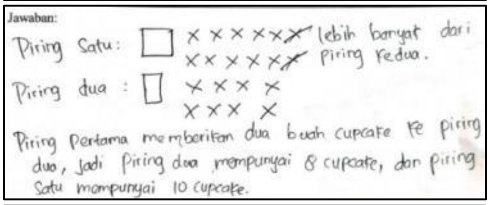
1	Teacher	: How to solve this problem?
2	Student 1	: We don't know the numbers of cupcakes on the plates before, but we know that the first plate has more cupcakes now.
3	Teacher	: Why? Can you explain it?
4	Student 1	: The second plate has 6 more pieces than the first one, but Lisa's mother add 12 cupcakes to the first one. Although we don't know the numbers of cupcakes on those plates before, but we can know that the first plate has more because 12 is more than 6.
5	Teacher	: So, how do you show it on this paper (answer's column)?
6	Student 1	: Hmm..we're confused how to show it, maybe we want to explain it by words.
7	Teacher	: How about drawing something to represent your idea?
8	Student 1	: Oh well, maybe **we can draw some shapes to represent the cupcakes and plates**. Let's try it, guys!

Based on fragment 1, the student is on level 2 of algebraic thinking. She knows what the given information are and what must be found. She knows that there is something unknown from the number of cupcakes at first and knows about how to compare information. On the other side, she can not devise a plan to translate the problem into a representation. It is showed when she does not know how to represent word problem into pictorials or even variables. The teacher guided them to represent the information into model involving equations. Here is the students' strategy of solving the problem:



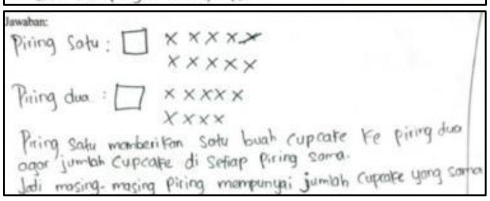
First plate → has more cupcakes than the second plate.

It's because Lisa's mother added 12 cupcakes to the first plate.



First plate: has more cupcakes than the second plate.

Because we know that Lisa's mother moved 2 cupcakes from the first one to the second one, so the second plate has 8 cupcakes while the first plate has 10 cupcakes.



Lisa's mother should move one cupcake from the first plate to thesecondone in order to get the same numbers of cupcakes.
So, each plates has the same number of cupcakes now.

Figure 4: The Group 3's Strategy for Solving the First Problem

Based on figure 4 above, the students tried to describe the information using pictures as the strategy. They represented the pictures for inequality problem until it was built on balanced situation. It showed us that they were able to understand about comparing quantities. They also developed algebraic thinking when they built the model of situation in which squares to represent unknown numbers of cupcakes on the plates before and cross to represent additional cupcakes. Those models directed them to understand about unknown value which always came up in algebraic problems.

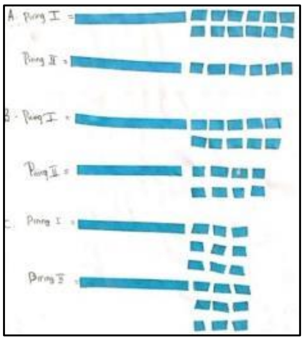


Figure 5: Students Used Pictorial Equation Solving

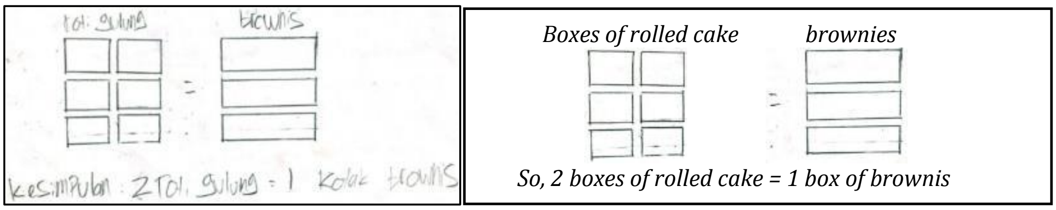
The teacher also guided various students' strategies to create pictorial equation solving more common using rectangles. Rectangles represent something which ease them to make partition and compare things easily. Figure 5 shows final representation using rectangles which were more common and ease them to solve in another situations. Although the squares created still showed different sizes, they could understand what the squares meant as additional cupcakes.

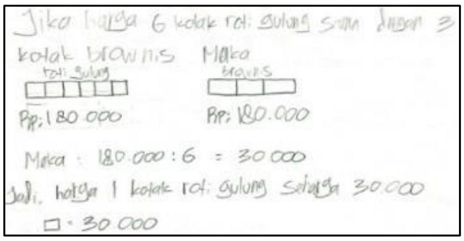
Activity 2: Understanding the equal sign using pictorial equation solving

These activities would direct the students to use pictorial equation solving for solving the problems. Even though they did not realize what the problems about, they actually could use pictorial equations well. Figure 6 is the first problem:

Lisa's mother wants to add the other cakes. She go to bakery and buy two kinds of cake: rolled cake and brownies. If she buys 6 boxes of rolled cake, then the price is same with 3 boxes of brownies. How do you show the illustration above? If Lisa's mother wants to buy 6 boxes of rolled cakes and 3 boxes of brownies, then she has to pay Rp360.000,00. What's the price of rolled cake, say, for a box?

Figure 6: The First Problem of Equal Sign





If the price of 6 boxes of rolled cake equals the price of 3 boxes of brownies,
Boxes of rolled cake brownies
Then 180.000 : 6 = 30.000.
So, the price of a box of rolled cake is Rp30.000,00.

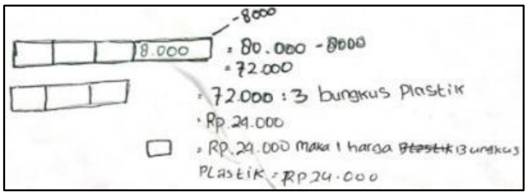
Figure 7: Students of Group 6 Used Pictorial Equation Solving

Figure 7 shows that the students can understand what the squares or rectangles exactly meant. They use those models of situation for representing the boxes of cakes. They can show illustrations and understand the equality concept using equality sign and pictorial equation solving. Besides, they can know the meaning of sign and demonstrate the relationship between the numbers on each side by analysing information given. It shows that they are able to reach level 4 of algebraic thinking with equal sign problem. Hence, the teacher guide them to build variable concept. The next problem on figure 8 will direct them to apply how to translate pictorial equation into algebraic expressions.

After buying some cakes, she went to a shop to buy the other things. When she arrived home, actually she still needed some packs of plastics. So, she asked Lisa's younger brother for buying them. She just gave him Rp80.000,00 and she had already known that he will get Rp8000,00 as change. How does he know about the price of one pack of plastics?

Figure 8: The Second Problem of Equal Sign

Figure 8 not only use variable, but also constants. Consequently, the students have to be able to find representation of their pictorial equation used. Figure 9 is the students' answer of that problem.



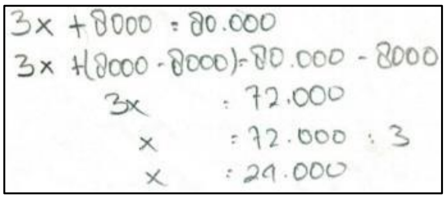


Figure 9: Students' Answer of Group 2 for Solving the Second Problem

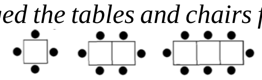
In this case, there is development of model from model of situation to model for solving problem simply which used a variable and constants. Figure 9 shows that the students can represent pictorial equation into algebraic expression as formal level of model for finding the price of one pack of plastics. They also had known that the activities done was about linear equation in one variable. Therefore, the goal was accomplished which related to Van de Walle, Karp, and Bay-Williams (2011) about algebraic thinking that they understand not only not only procedural ways to solve but also the meaning of those ways.

Activity 3: Expressing relationship between the situations using linear generalizing problem

Here the students were directed to understand relationship between variables. It was supposed so that they could generalize the properties, see the relationship between arithmetic and algebra, and justify why they work for any numbers as Watanabe (2008) described. It showed them that algebra is no longer meaningless symbol. Figure 10 is the problem.

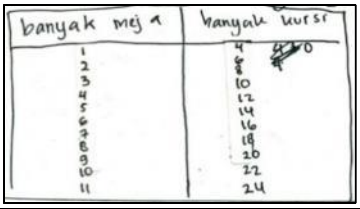
Based on figure 11, it shows that the students make a table, realize the pattern, and justify how they work. They are able to develop their algebraic thinking about analyzing relationship: how something changes anothers. It indicates them on level 3 of algebraic thinking with linear generalizing problem. They can find the pattern using inductive reasoning and analyze the function just for the condition of problem. They can not clearly explain how the numbers come up to the formula.

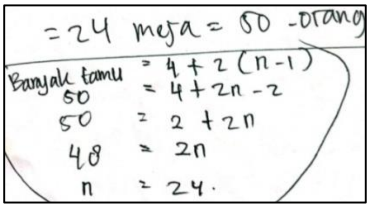
Lisa's birthday is begun. She has arranged the tables and chairs for her guests as the figure shown below:

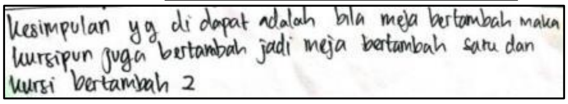


How many tables are needed for50 guests? What can you analyze from the relationship between tables and chairs?

Figure 10: Linear Generalizing Problem







We conlude that if the tables increase 1, then the chairs do too, that is, increase 2.

Figure 11: Students' Strategy of Group 4 for Solving the Problems

CONCLUSION

The result of this research shows that RME-based activities can develop students' algebraic thinking on linear equation in one variable. The development can be seen when they start to solve inequality and equality problems using pictorial equation solving, then develop the models into formal model using algebraic expression. Furthermore, they get the relationship between variables using linear generalizing problems. This skill is also supported by giving contextual problems which can be imagined and interpreted to the real situation. Hence, they understand not only procedural ways for this topic, but also the meaning of those ways.

This research actually has not been able to represent all of the students' answers because it only used 6 students as research subject. However, this study might give new information about teaching and learning of linear equation in one variable using RME approach for developing students' algebraic thinking.

REFERENCES

Bakker, A. (2004). *Design research in statistics education: On symbolizing and computer tools* (Doctoral dissertation, Utrecht University, Netherland). Retrieved from http://igitur-archive.library.uu.nl/dissertations/2004-0513-153943/full.pdf

Billstein, R., Libeskind, S., Lott, J.W. (2007). *A problem solving approach to mathematics for elementary school teachers – ninth edition.* Boston: Pearson Education.

Billstein, R., Libeskind, S., Lott, J.W. (2010). *A problem solving approach to mathematics for elementary school teachers – tenth edition.* Boston: Pearson Education.

Blitzer, R. (2003). *Thinking mathematically.* New Jersey: Prentice-Hall.

Cai, J., Moyer, J. (2008). Developing algebraic thinking in earlier grades: some insights from international comparative studies. In C. Greenes (Ed.), *Algebra and Algebraic Thinking in School Mathematics: Seventieth Yearbook* (pp. 169-179). Reston, VA: National Council of Teachers of Mathematics.

Ontario Ministry of Education. (2013). *Paying attention to algebraic reasoning support document for paying attention to mathematics education.* Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/PayingAttentiontoAlgebra.pdf

Gravemeijer, K. & Cobb, P. (2006). *Design research from a learning design perspective. educational design research.* London, New York: Routledge.

Kamol, N., Har, Y.B. (2010, July). *Upper primary school students' algebraic thinking*. Paper presented at the meeting of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia, Fremantle, Western Australia.

Knuth, E.J., Stephens, A.C., McNeil, N.M., Alibali, M.W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37*(4), 297-312. doi: 10.2307/30034852

Van De Walle, J.A. & Folk, S. (2005). *Elementary and middle school mathematics: Teaching developmentally.* Ontario: Pearson Education Canada.

Van De Walle, J.A., Karp, K., Bay-Williams, J. (2011). *Elementary and middle school mathematics: Teaching developmentally.* Boston, MA: Allyn & Bacon.

Watanabe, T. (2008). Algebra in elementary school: A japanese perspectives. In C. Greenes (Ed.), *Algebra and Algebraic Thinking in School Mathematics: Seventieth Yearbook* (pp. 183-193). Reston, VA: National Council of Teachers of Mathematics.

Wijaya, A. (2012). *Pendidikan matematika realistik suatu alternatif pendekatan pembelajaran matematika* [Mathematics realistic education as an alternative way of mathematics learning approach]. Yogyakarta: Graha Ilmu.

Yee, L.P. (2006). *Teaching secondary school mathematics: A resource book.* Singapore: McGrawHill Education.

Yee, L.P. (2007). *Teaching primary school mathematics: A resource book.* Singapore: McGrawHill Education.