USING NUMBER LINE TO SCAFFOLD STUDENTS IN LEARNING ADDITION MIXED NUMBERS

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Abstract

Mixed numbers or fraction in general have been a problem for students. Students have difficulty not only understanding the concept of the numbers since its differences to natural number, but also they also have a difficulty in understanding the operation of it because students see the operation as the algorithm. That is why it is not surprising when they have more difficulty in learning mixed numbers operation such as addition. This research will address students' problem in learning addition mixed numbers and fractions. Furthermore, the purpose of his research is to develop a theory and materials related to helping students to understand the notion of mixed numbers and their addition. In order to develop the theory, this research will use design research as the method. The data in this research is collected by generating the video recording of the classroom's event, student's written work, students' result from pretest and posttest, the result of interview, and also the fields' note from the observation. This data gathered throughout the teaching experiment in a group of four of fourth grade students in SD Pusri, Palembang. From the teaching experiment, it can be seen that in line with learning addition of fractions, students also need the understanding in equivalence fractions as the base of learning addition mixed numbers. Moreover, the experiment also shows that students also easily relate the addition of mixed numbers to addition of fractions when learning addition mixed numbers.

Keywords: mixed numbers, number line, addition of mixed numbers, design research, realistic mathematics education

INTRODUCTION

Despite all the problems that arise when learning fractions, fractions still play an important role in mathematics. Fractions are especially important in algebraic reasoning, for example when learning algebraic equation (Norton & Wilkins: 2013). Students need to understand fractions in order to be able to manipulate the variables in the algebraic expression. Furthermore, regardless of their limitation in daily life practice, fractions can be seen in many aspect of students' lives, such as in measurement (2 and a quarter meters), or the notion of part of something (three quarters of a loaf). As such, this context can be chosen as the context to help students gain a better understanding in fractions.

According to dictionary of English, mixed number (n, d) is a number that consist of an integer and a fraction. It means that mixed numbers have a tight relation to fractions in terms of its definitions and its use also. In Indonesia, according to Kemendiknas (*ministry of Educations)* mixed numbers are taught in elementary school as part of the learning sequence of fractions in the curriculum. However, the study about mixed numbers is very few. It is very hard to find the study about it.

Thus, The aim of this research is to develop a theory and materials related to helping students to understand the notion of mixed numbers and their addition. The question that is formulated for this research is

"How can we support students' understanding of the addition of mixed numbers?"

THEORETICAL FRAMEWORK

Difficulty in Learning Mixed Numbers

Mixed numbers and Fractions are closely related since in mixed number there is fractional part. So, it is not surprising that students' difficulty in fractions may also occur in mixed numbers.

Fractions have been closed in human life, but fractions in mathematics become a very frightening lesson for students (Hasemann; 1981, Streefland; 1991, Keijzer: 2003). There are three main reasons why it seems so. First, students are unable to relate the fraction to their daily life practice (Keijzer, 2003). Also, students have difficulty in understanding the meaning behind fraction symbols (Mix et al., 1999; Hasemann, 1981). Lastly, there are many meaningless procedural rules regarding arithmetic operations on fractions that are taught to students (Hasemann, 1981; Pitkethly & Hunting, 1996).

Adition of Mixed Numbers

There are two methods of addition mixed numbers (Dept Mathematics, 2012). First, by changing the mixed numbers into improper fractions. Second, by adding each part of mixed numbers separately. However, in both ways the addition of mixed fractions are related to addition of fractions.

In Addition to fractions, there are concepts that students need as the prior knowledge. According to Petit et.al those concepts are equivalence of fractions, and also part-whole and quantity construct of fractions (cited in Bruce et.al, 2013) .

PMRI

One reason why *Pendidikan Matematika Realistik Indonesia* (PMRI) was because it is developed not only for implementing an alternative way of teaching and learning mathematics, but also to achieve social transformation within Indonesia (Sembiring et al., 2008). PMRI in Indonesia is a movement to change the paradigm of teaching and learning

mathematics. According to Sembiring et al. (2008), there are five approaches to reform education. Those approaches are changing (1) top down learning to bottom up implementation, (2) teachers' passivity in designing lessons and developing material to active involvement, (3) material and frameworks based on and developed behind the desk to classroom research, (4) students as listeners to students as active thinkers using day by day implementation strategies, and (5) context and materials that are developed separate from the school environment and interests of students to contexts and materials that are closely related both.

METHODS

Research Approach

The research approach of this study is design research. Design research is chosen because it has the similar aim to this study. The aim of this study is to develop a Local Instructional Theory (LIT) in addition of mixed numbers, while the aim of the design research is to generating new theories (Edelson, 2002). Additionally, LIT meant in this study is a theory not only in how the students' learning process but also in how to support students in the learning process (Gravemeijer & van Eerde, 2009).

One of the crucial features of design research is its cyclic character (van Eerde, 2013). Cyclic, here, has a meaning as repetitive process. In general, there are three activities in each cycle. First, thought experiment where researcher will use the current knowledge that he know to design problems and activities. Afterwards, the teaching experiment will be conduct using the problems and activities design beforehand. Then, the researcher will reflect on the experiment. This will resulted in new knowledge (van Eerde, 2013). This new knowledge will be used in the new cycle that starts from thought experiment and so on. Furthermore, since this paper is part of the bigger study, this paper will only focused in the first cycle.

As it mentioned above, there will be design in problems and activities. In this study the problems and activities are designed in a learning sequence of addition mixed numbers. There will be five meeting in the sequence. The general illustration can be seen in the diagram below (see figure 1).

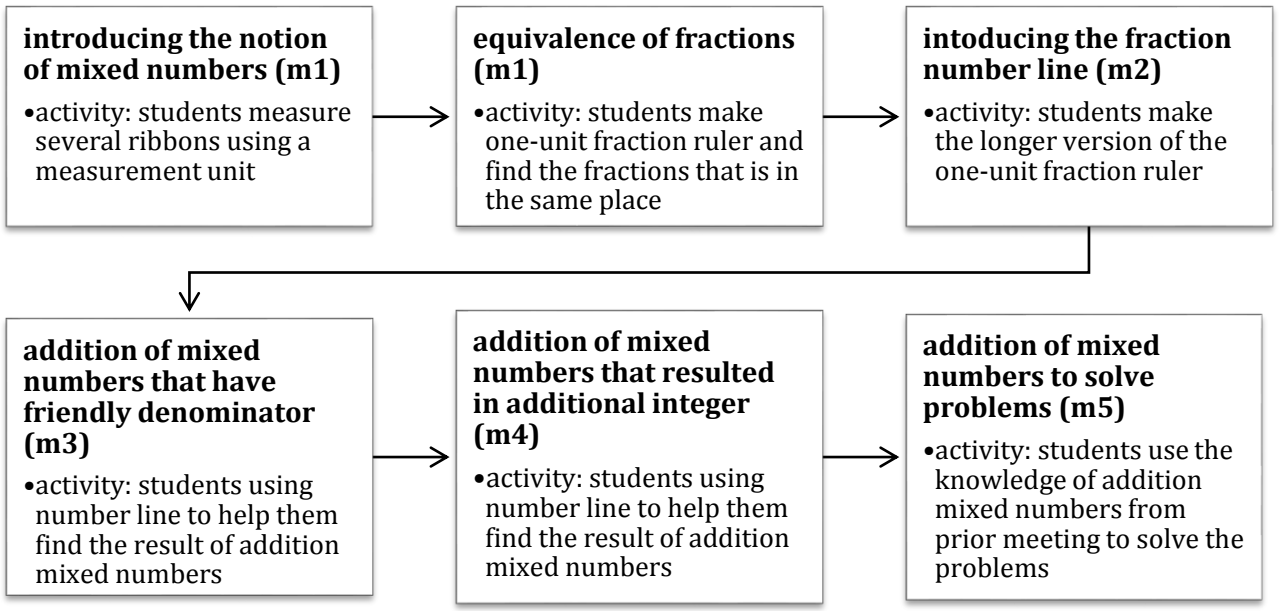

Figure 1 general illustration of this study's learning sequence.
(M1 means meeting 1, m2 mean meeting 2 and soon)

There is one thing to be noted from this sequence. Even if The general goal of the learning process is that students able to do addition mixed numbers, the sequence is designed for students that have not learnt about addition of fractions that have different denominator. Thus, the mixed numbers that is used in the problems will be the relative easy, such as halves, quarters, eights, thirds, and sixths. Furthermore, the students do not need to know the addition of fractions that have different denominator before learning addition of mixed numbers.

Collecting data

Generally, there is three phases in design research. Those phases are preparation, teaching experiments, and reciprocal analysis. Data collection will be tied to the first two phases. In the preparations, the data are collected from students' written work in pretest and video recording of students' interview. This data is utilized to know students' prelimainary knowledge of mixed numbers and additon of mixed numbers. In the teaching experiments, the data taken throughout the lessons are all of students' written work, video and audio recording of the lesson, and also field note. All this data will be analyzed in the last phase.

There is four elementary students that involved in this study. They are fourth grader from SD Pusri Palembang. Those students are recommended by their teacher as the middle achiever, but quite communicative. In addition, the researcher will pose as the teacher in this first cycle.

Analyzing data

The analysis of the data will be conducted by comparing what really happen in the classroom and the conjecture that is made beforehand in the hyphotetical learning trajectory (HLT). Thus, HLT is used as the guidelines for researcher in determining the focus of the analysis (Bakker & van Eerde, in press). To be precise, HLT is consist of not only the activities and the goal of it, but also the conjecture/ predictions of students thinking and understanding evolved in the lesson (Simon, 1995). This prediction will be used in the comparison.

Result and Discussion

In this section, not all of the activities will be presented. There will only three parts of the big picture that will be presented here due to its importance in the sequence. those three parts are students' preliminary knowledge, third meeting (students learning addition mixed numbers), and also one mini lesson where students attempt to solve bigger mixed numbers.

Students' preliminary knowledge

According to students' pretest answer and their interview about the pretest, there are several things that can be concluded. First, students did not fair sharing (equipartition) concept in finding the fractional parts. Second, students did not really familiar with mixed numbers. This second finding is related to the third poblem in the pretest where it asks students to draw $2\frac{3}{4}\ yards$ from a given 1 yards (see figure 2). From both interview and the pretest, most of the students did not understand the meaning of mixed number' symbols. Hence, they cannot comprehend the meaning of mixed numbers.

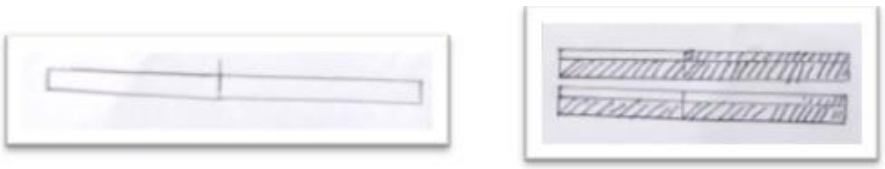

Figure 2 students' written work in solving third problem in pretest

In regards to addition mixed numbers most of the students still did not know how to solve the problems (see figure 3). It is clear that the students are not familiar with the addition of mixed numbers yet.

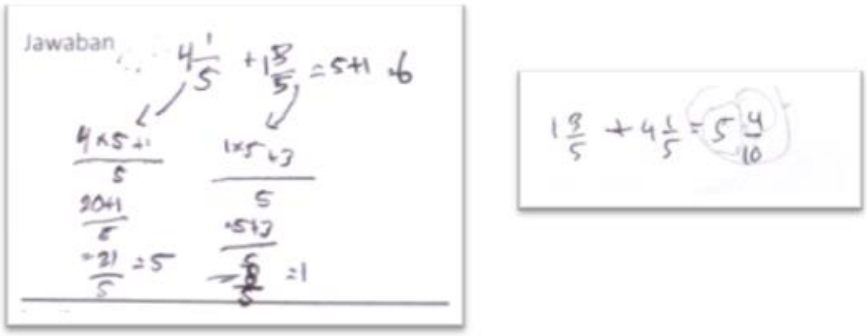

Figure 3 students' written answer in fifth problem in pretest

From the figure above it can be seen that the first students (left figure) was attemping to change mixed numbers into improper fractions. However, it can be seen that there is a misunderstanding in what she did (refer to $\frac{21}{5} = 5$ *and* $\frac{8}{5} = 1$). The second students (right figure), he already able to add the integer, but did not give regards to the fraction part. It can be seen from the way he did the calculation, he only add all the numbers in the fraction part.

Addition mixed numbers that have a friendly denominator

There is only one activity in this lesson that is finding the total length of the ribbons. The main goal of this activity is that the students could reason about the addition of mixed numbers. Thus, the classroom discussion in the end of the lesson is important.

This activity still have the similar context to the previous meeting in which students made the fraction ruler. Here, students will be provided fraction number lines (fractions ruler) and paper strips to help them find the result of the addition mixed numbers. In this activity the students worked in pair and both groups are able to find the results of the addition mixed numbers. Both groups use the given properties but in different way (see figure 4).

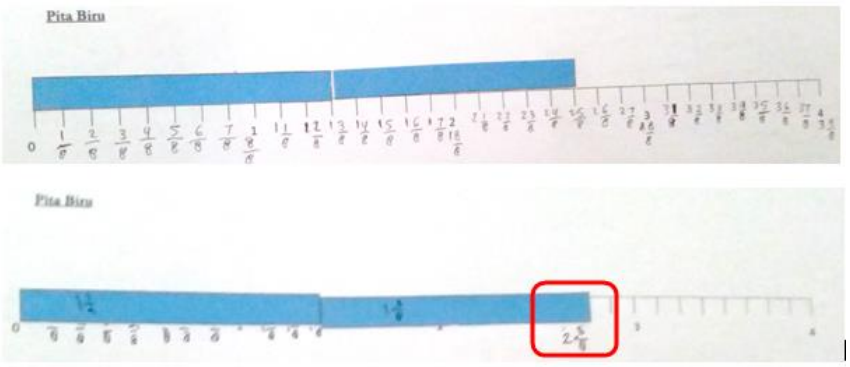

Figure 4 students' written work in solving the last problems in the worksheet. (above: girl group's, below: boy group's)

The important part of the meeting is the discussion of this problems. In the class discussion, teacher asks students to reason what happen to the addition mixed numbers result. The problem is $1\frac{1}{4} + 1\frac{3}{8} = 2\frac{5}{8}$. At first students are still confused of what the reason the number are doing that way. The struggle and the finding are shown in fragment 1 below

Fragment 1: finding the reason of additional mixed numbers

1. Researcher: can anybody give an explanation?
2. Students 1 : I can(enthusiasticly and suddenly raise her hand), but how can the result is 5? (refer to the nominator of the result).
3. Students 2 : it should be 3 plus 1. (refer to the nominator of fractions in the problem)
4. Students 1 : yeah.. (students 1 nodding her head). how come?
5. Researcher : let see the denominator
6. Students 1 : oh the denominator is 8 plus eh ... (scratching her head)
7. (*students keep thinking and struggling, after a while one of the students are able find the reason of the addition mixed numbers and what happen to the numbers*)
8. Researcher : is there anyone else that can explain why this can happen?
9. Students 3 : (stand up) oh yo 1/4 is changed into ... hmm
10. Students 1 : 5/8?
11. Students 3 : no..it became 2/8.
12. Researcher : 1/4 is changed into 2/8. so, is there any questions?
13. (*Researcher write down the changing of the fractions*)
14. Researcher : so, 1 2/8 + 1 3/8, what is the result?
15. Students 3 : ooh... the result is 2 5/8. (confidently)
16. Students 1 : (after a while and thinking what is written in the explanations) oooh that's right

From the fragment 1, it can be seen that students at first did not make the relation between the addition and the equivalence fractions that they have learnt before. It can be seen that at the first struggle, students got confused with the numbers (refer to line 1-3) they cannot grasp why 1 (refer to the numerator of $\frac{1}{4}$) plus 3 (refer to the numerator of $\frac{3}{8}$) can resulted in 5 (refer to the numerator of $\frac{5}{8}$, the result f the addition). it is clear that students did not pay attention to the denominator of the fraction part of the mixed numbers. However, after the researcher asked he students the denominator, one of the students are able to find the idea that both addition and equivalence of fractions are related.

In this study the equivalence of fractions have a different term that is an alias. Moreover, the students already quite flexible to find the alias of several fractions such as half, quarter, a third, a eight. In other words, students need the deeper understanding of

equivalence number in order to be able to understanding the addition of fraction part of mixed number.

Aside from this finding, a big questions appeared, what students did with the integer part of the mixed numbers. Thus, the researcher gave additional problems in which forced the students to use their reason and not the strips and number line that provide before. Figure 5 shows of addition of mixed numbers but with bigger integer number than the previous problems.

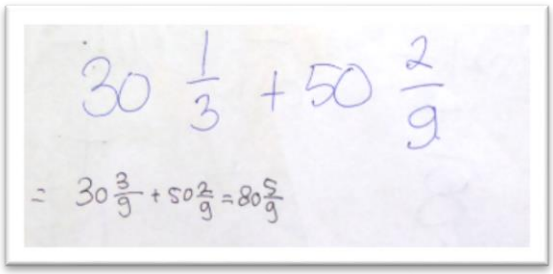

Figure 5

In the problem above one of the students are able to solve the additional of mixed numbers. From figure 5, it clearly seen that the students are able to find the alias of $\frac{1}{3}$ as $\frac{3}{9}$ before doing any addition. When the researcher asks the students how she solve the problems, she explain that first she change $\frac{1}{3}$ into $\frac{3}{9}$ and she directly added the integer part without any difficulties and added the fraction parts afterwards. From here, it can be seen that the students do not have any difficulties in adding the integer part, the challenging parts is where the students find the alias of certain fractions.

Conclusion and Limitation of the study

From the analysis above, there are two things can be concluded. First, the understanding of the equivalence fractions are crucial in learning addition of fractions. Second, when adding mixed numbers, students tend to Then, to answer the research questions how to support students in learning addition mixed number is that the teacher have to make the students understand the equivalence fractions.

However, since the scope of the fraction part of the denominator is quite limited, it might hinder the understanding of the other addition of mixed numbers. In addition to that, the method of solving addition mixed numbers that is used here, is not the nly methods. Thus, the question for further research is how we can support students to be able to use both methods through assessing the situation beforehand.

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REFERENCES

Bruce, C., Chang, D., & Flynn, T. (2013). Foundations to Learning and Teaching Fractions : Addition and Subtraction Literature Review.

Departemet of Mathematics. (2012). *Prealgebra Textbook.* California: College of Redwoods.

Edelson. (2002). Design Research : what we learn when we engage in Design. *Journal of Learning Science, 11(1)*, 105-121.

Fraction. *Encyclopedia of Mathematics.* URL: http://www.encyclopediaofmath.org/index.php?title=Fraction&oldid=30892.

Gravemeijer, K., & van Eerde, D. (2009). Design Research as a Means for Building a Knowledge Base for Teachers and Teacing in Mathematic Education. *The Elementary School Journal, 109*, 510-524.

Hasemann, K. (1981). On difficulties with fractions. *Educational Studies in Mathematics*, *12*, 71-87.

Keijzer, R. (2003). *Teaching Formal Mathematics in Primary Education.*Amsterdam: Universiteit Amsterdam.

Kemendiknas (2012). *Kompetensi Dasar Kurikulum 2013 untuk Sekolah Dasar (SD) dan Madrasah Ibtidaiyah (MI)* [Basic Competence Curriculum 2013 for Primary School and Madrasah Ibtidaiyah]. Jakarta: Kemendikbud

Mix, K. S., Levine, S. C., & Huttenlocher, J. (1999). Early fraction calculation ability. *Developmental Psychology*, 35, 164-174.

Mixed numbers (n.d.). *The American Heritage® Dictionary of the English Language, Fourth Edition*. (2003). Retrieved November 5 2014 from http://www.thefreedictionary.com/mixed+number.

Norton, A., & Wilkins, J. L. M. (2013). Supporting Students' Constructions of the Splitting Operation. *Cognition and Instruction*, *31*(1), 2–28. http://doi.org/10.1080/07370008.2012.742085

Saxe, G., Taylor, E., McIntosh, C., & Gearhart, M. (2005). Representing fractions with standard notation: A developmental analysis. *Journal for Research in Mathematics Educations*, *36*(2), 137–157. Retrieved from http://www.jstor.org/stable/30034828

Sembiring, R., Hadi, S., Dolk, M. (2008). Reforming Mathematics Learning in Indonesian Classroom through RME. *ZDM- International Journal on Mathematics Educations, 40*, 927-939.

Streefland, L. (1991). *Fractions in Realistic Mathematics Education*. Dordrecht: Kluwer Academics Publishers.

Van Eerde, D. (2012). Design Research : Looking into The Heart of Mathematics Education. In Zulkardi (Ed.), *1st SEA-DR* (pp. 1–11). Palembang. Retrieved from http://eprints.unsri.ac.id/2537/