

PROMOTING STUDENTS' UNDERSTANDING OF THE ADDITION OF FRACTIONS

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Abstract

Many researchers have documented that students consider fractions as a difficult topic because there are many rules in operating fractions. In Indonesia, many teachers place more emphasize on the algorithms instead of students' understanding of the concept. Thus, students tend to memorize the algorithms without understanding the reasoning behinds it. Consequently, students often made mistakes in applying the algorithms, such as doing the procedure 'top+top and bottom+bottom' in solving the addition of fractions problems. Therefore, there is a need to develop instructional activities that support students' understanding of the addition of fractions. This study used design research approach and applied the idea of Realistic Mathematics Education (RME) which suggests that students have to construct their understanding actively by exploring contexts and models. The models used in this study were paper strips and bar model. This study was conducted in the third grade of elementary school in three cycles, which each cycle consisted of three meetings. The result of the first cycle showed that the students began to develop their fractions sense after producing measuring strips by themselves by using paper strips. After exploring the measuring strips, they were able to grasp the idea of the equivalent fractions. Afterwards, students could translate their insight of equivalence of fractions in the measuring strips to find the common denominator in adding fractions with either the same or the different denominators in the bars. In conclusion, the result of the first cycle provides evidence that the use of paper strips and bar model can help students in understanding the addition of fractions.

Keywords: Addition of fractions, RME, design research, fraction strips, bar model

INTRODUCTION

Background

The topic of fractions is well-known as a complicated mathematics topic for students (Charalambous & Pitta-Pantazi, 2007; Hasemann, 1981; Streefland, 1991; etc.). Students regard fractions as difficult since there are many complicated rules in the operation of fractions, which are different from the rules in the operation of whole numbers. Moreover, many teachers still use conventional teaching in which they emphasize on the algorithms and computation instead of students' understanding of the concept of fractions. As the result, the concept of the operation of fractions is not strongly embedded in students' mind. Consequently, students often apply the rules of whole numbers in the operation of fractions. In the case of addition of fractions, students often make a common mistake in adding fractions, in which they do 'top+top over

bottom+bottom' strategy. For instance, students argue that the result of $\frac{1}{2} + \frac{1}{3}$ is $\frac{2}{5}$. They apply such algorithm because they consider a fraction as two whole numbers, thus they add the numerators and add the denominators.

Freudenthal (1991) suggested that mathematics should be regarded as human activity. In the learning process, students should actively experience and construct their understanding of a concept. It is in line with the theory of RME, that in teaching and learning, teachers should relate mathematics topics to reality or contexts that are real or imaginable for students (Gravemeijer, 2004). In addition, according to Cramer et al. (2008), the use of models also can support students in understanding a concept since it can help students to build mental representations of a concept. They also argued that it can promote students' argument and reasoning. Van Galen et al. (2008) suggested that paper strips and bar model can be conceptual models that can help students to construct the concept and support their reasoning of fractions.

However, in Indonesia, many teachers still employ a conventional approach in teaching fractions (Ullyia et al., 2010; Sembiring et al., 2008). Conventional teaching, according to Lamon (2001), does not support students to have a meaningful learning. In this teaching approach, the teacher focuses more on how the students can apply algorithms instead of how the students understand how and why the algorithms work. Consequently, since students need to memorize many algorithms, they often misuse the algorithm, which they apply a wrong algorithm in a problem.

To deal with this issue, this study attempted to promote a meaningful teaching and learning in the addition of fractions. As suggested by van Galen et al. (2008), the lessons in this study integrated paper strips and bar model to support students' reasoning and to help them construct their mental representation of fractions.

Research Aim and Questions

Based on the background above, it is a necessity to reform the teaching and learning of fractions in Indonesia. Inspired by the theory of RME, the integration of contexts and models in the teaching and learning of fractions is considered fruitful to promote students meaningful learning. Therefore, the aim of this study is to investigate how we can support students' understanding of the addition of fractions. To reach this goal, we formulate a research question as follows:

How can we support students' understanding of the addition of fractions?

To be more specific, this study tries to answer the following sub research questions.

1. *How do paper strips and bar model promote students' understanding of the addition of fractions?*
2. *How does estimation skill lead students to avoid the incorrect procedure 'top+top over bottom+bottom' in solving the addition of fractions problems?*

In this paper, the researcher focuses on and elaborates results that will be used to answer the first sub research question.

THEORETICAL FRAMEWORK

Addition of Fractions

Addition of fractions is an operation of fractions in which students regard as difficult. Numerous studies have documented that a common mistake in adding two fractions is adding across the numerators and the denominators, or 'top+top and bottom+bottom' procedure (Howard, 1991; Young-Loveridge, 2007). Teachers need to take into account the knowledge that can support students' understanding of the addition of fractions. Some studies revealed that the concept of equivalence is helpful in constructing the understanding of a fraction as a single number and the understanding of the addition of fractions (Charalambous & Pitta-Pantazi, 2007; Reys et al., 2009; Pantziara & Philippou, 2012). The concept of equivalence is useful in finding the common denominator in the addition of fractions with unlike denominators.

Realistic Mathematics Education (RME)

Many studies, such as one that was conducted by Charalambous & Pitta-Pantazi (2007), revealed that teachers should pay attention more on students' conceptual understanding rather than on formal algorithms. Teachers have to move from the conventional approach to an approach that support students' meaningful learning. As has been suggested by the theory of RME, mathematics teaching and learning should start and stay in reality and support students' meaningful learning (Gravemeijer, 2004). In addition, models are a useful tool to be used in the lessons to explore mathematical ideas and to solve problems. The contexts and models are important to support students to shift from informal knowledge to a more formal knowledge. According to van Galen et al. (2008), paper strips and a bar model can be helpful for students to construct their understanding of the concept of fractions. By exploring the paper strips and the bar model, the students will have a mental image for fractions and have a fractions sense. The mental image for fractions and the fractions sense is very useful in developing the understanding of the idea of the equivalent fractions and the common denominator.

METHOD

The purpose of this study was to support students' understanding of the addition of fractions by integrating the use of contexts and models. To reach this goal, the researcher designed instructional activities and carried it out to find out how the instructional activities help students in understanding the concept of the addition of fractions. By doing this, the researcher intended to contribute to an innovation of the teaching and learning of the addition of fractions. Thus, the researcher applied design research approach. Design research is an approach that balances both, the theory and the practice (Bakker and van Eerde, 2013). Designing learning activities grounded on theories and examining how the activities support students' learning is the core of design research.

In the preparation, the researcher designed learning materials and Hypothetical Learning Trajectory (HLT). HLT contains conjectures of students' thinking. The process of creating the learning materials and HLT was grounded on theories about students' learning in the topic of fractions. During the teaching experiment, the HLT was used as a guideline for the teacher in conducting the lessons. The data collected in during the experiment were video of classroom observation, students' work, interview, and field notes. Then, in the analysis, the researcher compared between the conjectures and the actual students' learning by referring to the HLT.

There were three cycles in this study, which each cycle consisted of three meetings. In the first meeting, the goals were grasping the idea of partitioning, the notation of fractions, and the equivalence of fractions. The second meeting focused on the comparison of fractions and the estimation of the sum of two fractions. In the last meeting, students reviewed the estimation of the sum of two fractions and explored the idea of common denominator in adding fractions by using paper strips and bar model. In this paper, the researcher elaborates the results of the first cycle, and focuses on the first and the third meeting to investigate how paper strips and bar model support students' understanding of the addition of fractions.

This study was conducted in a school in Surabaya, namely SDN Laboratorium Unesa, with a teacher and students of the third grade involved in this study. The participants in the first cycle were five students of 3A, namely Tya, Sam, Diva, Nanda, and Nabil. In this cycle, the researcher became the teacher. Then, in the second cycle, the researcher took 3B as the participants. In the last cycle, the participants were the students of 3A exclude the five students that have participated in the first cycle. In the second and third cycle, the regular teacher became the teacher who carried out the lessons.

RESULTS

Students' Preliminary Knowledge

The pre-test and the interview result showed that the five students were able to represent fractions in the pictures, and conversely, they could label the fraction of given pictures. All students could represent a fraction in a bar, in which they knew that the denominator of a fraction correlates to the number of partitions, and the numerator represents the shaded area. Moreover, some of them had been able to find the equivalence of fractions as can be seen in the Figure 1. The figure below shows Samuel's and Nanda's answer in finding the equivalence of fractions.

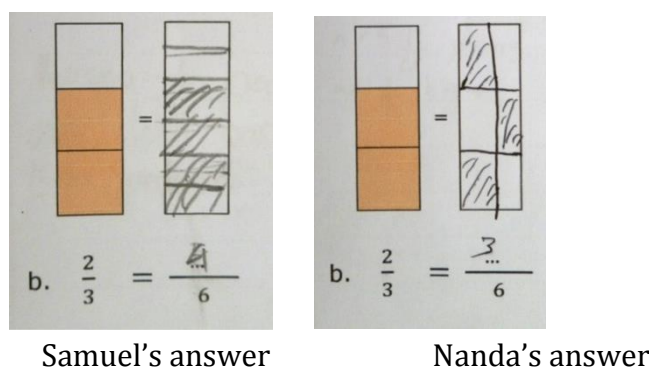


Figure 1. Samuel's and Nanda's Answer of the third Problem

In adding fractions with unlike denominators, as has been documented by many researchers, most of the students did 'top+top per bottom+bottom' strategy. While in adding fractions with the same denominator, most of the students were able to solve it and represent it in the bar. The figure below is the example of students' answer in adding fractions.

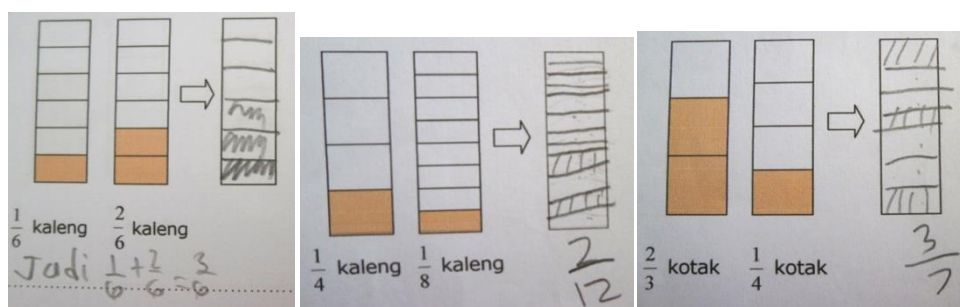


Figure 2. Various Students' Answers in Adding Two Fractions

Based on the Indonesian curriculum, the third grade students have learned about the concept of fractions, about how to compare unit fractions, and how to compare fractions with the same denominators. In line with what they have learned, the result of the pre-test above indicated that students were able to label the fractions of given pictures and represent fractions in a regional model. Some students were able to find the equivalence of fractions while the other students were confused how to do it. Furthermore, they did not know how to add fractions with different denominators. Therefore, there is a need to engage the students to learn more about the concept of fractions to get the fractions sense, so that they know the reasoning of the equivalence of fractions and the addition of fractions.

Meeting 1

The aims of this meeting were (1) students are able to partition into equal parts, (2) students understand the notation of fractions, and (3) students understand the idea of equivalent fractions. In this activity, the researcher, as the teacher engaged students to help Pak Doni, as the committee of the competitions in the Independence Day, to figure out how to measure the water in the competition 'memindahkan air' if only different buckets are available (as in the Figure 3). Then, the teacher engaged the students to make measuring strips with various numbers of partitions by using paper strips to help Pak Doni to measure the water of each participant in the tube.



Figure 3

In the discussion of how to deal with different buckets, as has been expected in the HLT, the students argued that Pak Doni should use a transparent bucket or use buckets with the same size. Combining these two answers, the teacher said that Pak Doni uses a transparent tube. Thereafter, the teacher demonstrated how Pak Doni uses paper strip to measure a tube, which a half part of it is filled with water. Then, the teacher divided the students into two groups. One group that consisted of Nanda, Sam, and Nabil got a tube which $\frac{1}{3}$ part filled with water, and the other group that consisted of Tya and Diva got a tube which $\frac{1}{4}$ part filled with water. The students were asked to make two measuring strips with different partitions.

After figuring out what parts of the tube filled with water, the students made other measuring strips with different partitions that can be used to measure the water in their tube. Nanda's group made measuring strips with 4 and 8 partitions, while Tya's group made measuring strips with 3 and 6 partitions. After they finished making two different

measuring strips, they put and arranged it together on the poster paper and then labeled the fractions of each partition, as in the figure below.

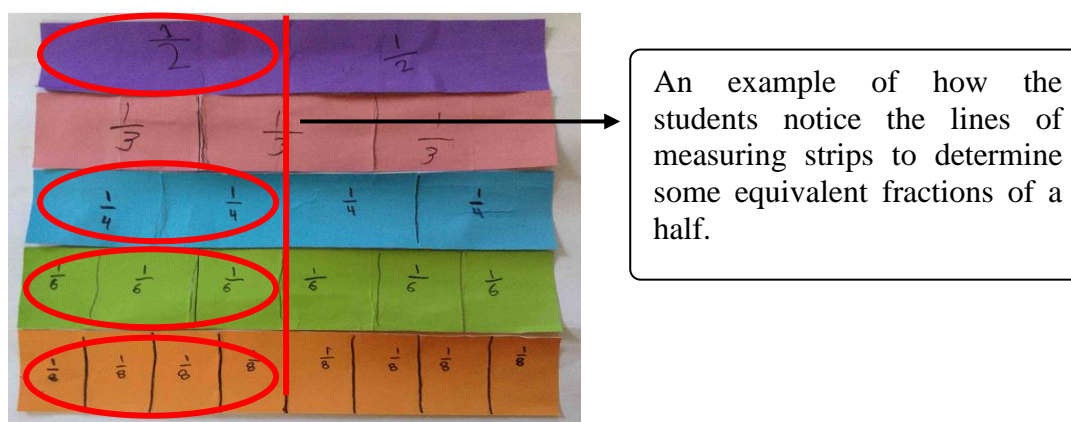


Figure 4. The Measuring Strips that the Students Made and How They Find the Equivalent Fractions

Afterwards, the teacher and the students discussed the equivalence of fractions by noticing the lines of each measuring strip they made as in the Figure 4. After engaging students to find equivalent fractions, the teacher engaged the students think about what they notice from some examples of equivalent fractions. Below is the transcript of the discussion.

The teacher : Now look at the measuring strips. Can we represent a half with other fractions? (*showing the measuring strips that students made*)

Sam : $\frac{2}{4}$ (*looking at the measuring strips*)

The teacher : Is there other fractions? Look at the line (*pointing the line of a half*)

Students : $\frac{3}{4}$, and $\frac{6}{8}$ (*looking at the measuring strips and then extending the line of a half into other measuring strips as in the Figure 4*)

The teacher : What about $\frac{2}{3}$?

Diva : $\frac{4}{6}$ (*looking at the line of $\frac{2}{3}$ in the measuring strips and extending it into the six-partitioned measuring strip*)

The teacher : Let's write it down. Look at $\frac{3}{4}$ and $\frac{6}{8}$. Do you notice why 3 can be 6 and 4 can be 8? (*pointing the numerator and the denominator on $\frac{2}{3}$ and $\frac{4}{6}$*)

Tya : I know I know.

The teacher: Why? (*looking at Tya*)

Tya : Because 3 times two is six, so the bottom must be multiplied by two too (*by pointing the number*).

The teacher : What about this, how come a half equals to $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$? (*showing the fractions*)

Sam : Because $\frac{4}{8}$ is 4 divided by 8... ehm, four.. is a half of eight.

In this discussion, students tried to find the equivalent fractions by looking at the lines of each measuring strips as in the Figure 4. Then, from some examples of equivalent fractions, the teacher engaged students to notice the pattern of the equivalent fractions.

In the HLT, the researcher expected that the students will notice the pattern of the equivalence of fractions. As expected in the HLT, Tya and Samuel were able to notice the pattern of the equivalence of fractions after noticing the pattern of some equivalent fractions. Tya figured out that to get equivalent fractions, the numerator and denominator should have the same factors of multiple. For example, if the numerator is multiplied by 2, then the denominator also has to be multiplied by 2. Different from Tya, Samuel noticed the relation between the numerator and the denominator. As can be seen in the fragment, he noticed that 1 is a half of 2, so the numerator of the equivalent fractions is also a half of the denominator.

After the students got the idea of equivalence of fractions, the teacher asked them to name some equivalent fractions of $\frac{1}{5}$. The teacher posed the question to check students' understanding of equivalent fractions. In answering this question, all students could answer it without looking at the measuring strips. They doubled the numerator and the denominator of $\frac{1}{5}$ for instance $\frac{2}{10}$ and $\frac{4}{20}$.

The description of the activity above indicates that students are able to partition the paper strips into equal parts. Moreover, as can be seen in the Figure 4, students also can find the equivalence of fractions by noticing the lines of each measuring strip. From some examples of equivalent fractions, some of them notice the pattern of equivalent fractions from some examples. They notice that the numerator and the denominator of equivalent fractions are multiple of each other. For instance, when the teacher asked them to find the equivalent fractions of $\frac{1}{5}$, they were able to answer it by doubling the numerator and the denominator.

Meeting 3

The aim of this activity was engaging students to experience and explore the idea of common denominator in adding fractions. In this activity, the students experienced pouring water from two different tubes into a tube, and then explored the fractions representing the parts of tube filled with water before and after being poured.

In the beginning of the activity, the teacher demonstrated in pouring a tube, which a half of it is filled with water, to another tube, which a quarter part of it is filled with water. Then, the teacher engaged the students to think what parts of the tube is filled. Thereafter, the students experienced by themselves in pouring a half and a third tube of water. After they poured 2 tubes which $\frac{1}{2}$ and $\frac{1}{3}$ part of it filled with water respectively,

they explored why they got $\frac{5}{6}$ as the result. After that, the students together with the teacher discussed it. Below is the transcript of their discussion.

- The teacher : Now, pay attention. $\frac{1}{2}$ and $\frac{1}{3}$, how can the result be $\frac{5}{6}$?
(showing the measuring strips)
- Tya : Because a half and a third can be represented in sixth
(while notice the lines at the measuring strips as in the Figure 4)
- The teacher : Why do we choose a sixth?
- Nabil : Because it's the same.
- Tya : We can multiply it.

In the discussion above, the teacher and the students discussed the result of $\frac{1}{2} + \frac{1}{3}$ by noticing the measuring strips they made in the first meeting. By using their insight of equivalent fractions in the first meeting, the students look at the lines of each measuring strips to find the equivalent fractions of $\frac{1}{2}$ and $\frac{1}{3}$.

The transcript above shows that students get the idea of common denominator. As has been expected in the HLT, they use the idea of equivalence of fractions, which they try to find a fraction (or partitions in the bar) that can represent a half and a third. By saying 'because it's the same', Nabil means that both fractions can be represented in sixth. Furthermore, Tya finds that she also could multiply the denominator to get the common denominator.

After the discussion, in the third activity the teacher gave a set of problems about the addition of fractions in the bars. The figure below is the example of students' work in solving the problem.

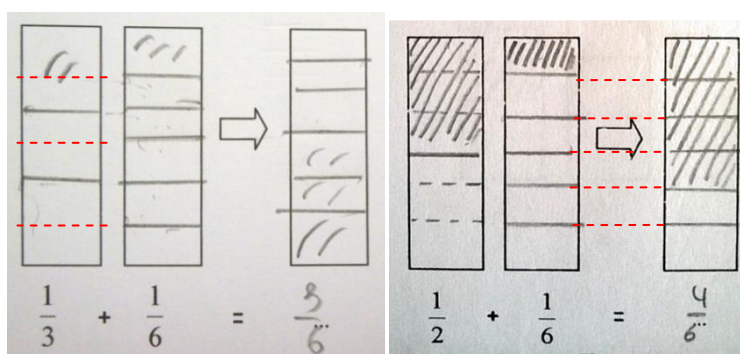


Figure 5. Examples of Students' Work in Adding Two Fractions by Using bar model

It can be seen in the Figure 5 that students use their experience in extending the lines of paper strips to get the equivalent fractions. They use their knowledge of equivalence of fractions that they learned in the first meeting by exploring the paper strips to find the common number of partitions (the common denominator). The discussion and the students' work indicate that they are aware that to add fractions with different denominator, they need to find a common number of partitions in the bar that can

represent two fractions being added. In other words, to add two fractions, the denominators of the fractions have to be equal.

DISCUSSION AND CONCLUSION

The description above reveals that the students start to grasp the idea of common denominator. The students explore the pattern in the measuring strips that they made by using paper strips to get a sense of the equivalence of fractions. They notice in the measuring strips that equivalent fractions have the same extension lines and have the same size of shaded area. By using the insight of equivalent fractions in the measuring strips, they try to find the common number of partitions in the bars in adding fractions with different denominators. In this case, the common number of partitions represents the idea of common denominator. They extend the lines in the bar to get the same number of partitions. Then, they shade the total parts as many as the total number of parts being added.

In regards to answering the first sub research question 'How do paper strips and bar model promote students' understanding of the addition of fractions?', it can be seen from the description of the results that students begin to understand the idea of equivalence of fractions after exploring the paper strips that have been partitioned into various numbers of partitions to represent fractions. Then, they use that idea to find the common number of partitions in adding fractions in the bars. The bars help them to visualize how to get the common denominator. In conclusion, paper strips and bar model help students to grasp the idea of common denominator.

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