

# DEVELOPMENT OF TEACHING MATERIALS IN MATHEMATICAL PROOF THROUGH COOPERATIVE LEARNING TO DEVELOP STUDENTS' MATHEMATICAL COMMUNICATION AND DISPOSITION

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## Abstract

This study has the main objective producing of teaching materials in mathematical proof. Through the stages of activities that have been designed, the study is expected to obtain relevant teaching material in mathematical proof to develop mathematical students' communication and disposition. Borg dan Gall (Ghufron, 2011: 12) set out 10 steps research and development as follow: 1) research and information collecting, 2) planning, 3) develop preliminary form of product, 4) preliminary field testing, 5) main product revision, 6) main field testing, 7) operational product revision, 8) operational field testing, 9) final product revision, 10) dissemination and implementation. Adapted from Borg & Gall version, the research is conducted in three phases: preparation, implementation, and refinement. The data are obtained through observation instruments such as observation sheets, video record, and lecturing activities documentations. The data analysis is done by constant comparative analysis. This analytical technique enables researchers to review data repeatedly. The observed data are analyzed by two persons separately, and then the result is discussed in routine meeting. In addition to teaching materials and assessment instruments; studies lead to the conclusions: 1) Instructional materials that are designed to be discussed in the group and should be accounted for on an individual basis, can foster students' mathematical communication and disposition; 2) Learning climate that gives comfort to the students to think and express opinions expected to develop students' mathematical communication and disposition; 3) Lecturer teaching style characterized by guidance toward self-regulated learning is expected to develop students' mathematical communication and disposition.

**Keywords:** mathematical disposition, mathematical communication, mathematical Proof

## INTRODUCTION

### Background

Proof is a part of mathematics that are considered difficult job done. Based on the observations of the researcher as a lecturer, this condition occurs in a long time. Students do not only have difficulty in writing proof, but also still have weakness in understanding the frame of evidence.

Constraints experienced by students in the proof can be: 1) lack of understanding of the concept of proof, so that students tend to work in a mechanistic, modeled after lecturer,

inductive, and perceptual; 2) weak in the high order thinking; and 3) the difficulty in understanding and writing notation. From another point of view, this difficulty can be a weak ability to write the language of mathematics, or better known as the weaknesses in mathematical communication. Another aspect that also contributes to the lack of ability to prove is the view of the proof itself is minor. The proof involves many concepts to be a prerequisite. If one of prerequisites is not favored a student, then most likely mathematical proof also includes subject that is not favored as well. In other words, the response to face constraints is influenced by students' views on the subject of proof.

Meanwhile, disposition provides a strong impetus to persist in doing the proof. Students' attempt to work (thinking) in mathematics can contribute good ability to prove. Students no longer works mechanistic, but they use the power of reason to prove. But when the urge to do mathematical thing is weak, then contribution to an improved understanding of the concept will also be low. Thus, mathematical disposition plays an important role in the proof.

The parts that might help in the proof are an effort to increase mathematical communication and disposition. Both of these capabilities have an important role in mathematical proof. If the both of these capabilities evolve, the ability to prove may also increases. Instructional materials can provide or even become facility in improving the both capabilities. Through teaching materials, lecturers can derive the trajectory how the proof learnt. Good management of teaching materials can help students in understanding mathematical proof properly. Therefore, these teaching materials become another important element in the lecturing process of mathematical proof.

## RESEARCH QUESTIONS

In accordance with the foregoing background, the problems that are going to be solved in this study are:

- a) What kind of teaching materials that can improve students' mathematical disposition and communication competencies?;
- b) How are students' learning profile that can improve their mathematical dispositions and communication competencies?; and
- c) How are lecturers' teaching profiles that can improve students' mathematical disposition and communication competencies?

## LITERATURE STUDY

### Mathematical proof

Mathematical proof subject is carried out in order students have basic ability to proof mathematical statements: theorem, lemma, and so on; properly. The main prerequisite for this course is Introduction to Basic Mathematics, especially logical mathematics as the basis of thinking or reasoning. Logics are a method and principles that can separate between correct and wrong reasoning. "as studying mathematics and other sciences, reasoning becomes very important and decisive." (Markaban, 2004:2). Another prerequisite is material that relates to mathematical context, such as number theory, trigonometry and others.

Reasoning and communication competencies are parts of prerequisite, and at the same time are results that can be achieved in a mathematical proof. Logics reason becomes the basis for proving, the facility is logics material. After practicing through the proof, logics reason becomes better. Similarly, the ability to write mathematics or

mathematical communication develops into a part of prerequisite and the intended results.

### **Cooperative Learning**

Students are part of the social environment both from the aspect of where they live and in terms of the community in which they live. As social beings, they have an important role as self existence in their environment. This existence turns into important as self-identity that makes them meaningful in their environment. Moreover, existence becomes a students' driving to thrive in the environment. Learning community (including in classroom) is a social environment for every individual who involves in learning.

Like the social life in general, each individual requires the existence in the environment. Every individual needs a role to keep his or her identity, thus learning is meaningful from the social point of view. Interaction in learning is needed to complete required information in constructing knowledge. Empty chain in the cognitive structure allows charged or connected to the discussion, thus giving assistance in the integrity of constructed knowledge. This interaction is important in learning to maintain the integrity of mathematical knowledge.

### **Cooperative Learning Models**

According to Bainbridge (2004:1), "Cooperative learning is a method of instruction that has students working together in groups, usually with the goal of completing a specific task. This method can help students develop leadership skills and the ability to work with others as a team." Co-operative learning methods fall into two main categories: formal and informal.

Structured and facilitated learning, which also monitored by teacher over time and is used to achieve group goals in task work, called formal cooperative learning. Meanwhile, the informal ones covers methods more focused on social dynamics, projects, and discussion than on mastery of well-specified content.

There are many different types or forms of co-operative learning. However, all of forms involve having students work in small groups or teams to help one another learn academic material. In this paper we only discuss the three types of cooperative learning, namely: Student Teams Achievement Division, Teams Games Tournament, and Jigsaw.

Student Team Achievement Division (STAD). Typically, STAD is a co-operative learning programme in which students work in 4 member heterogeneous—performance level, sex and ethnicity—teams to help each other master academic content and teachers follow a schedule of teaching, team work, and individual assessment. The teams receive certificates and other recognition based on the average scores of all team members on weekly quizzes. For example, teams that average 15 to 19 improvement points receive a GOOD TEAM certificate, teams that average 20 to 24 improvement points receive a GREAT TEAM certificate, and teams that average 25 to 30 improvement points receive a SUPER TEAM certificate.

Teams Games Tournament (TGT). "Teams Games Tournament is Cooperative learning strategy where students work together in groups and are responsible for their

teammates learning as well as their own and compete in tournaments against other teams to test for learning." (Bingham and Rivera: 11). TGT uses the same steps: teacher presentations and teamwork as in STAD. Different from STAD, TGT replaces the quizzes with weekly tournaments. The simply Procedure is as follow: 1) Teacher presents the material; 2) Students work in teams of 4-5 to prepare team members for a quiz; 3)Tournaments are held usually at the end of the week where content-relevant games are played at a table (3 students) with one member from different teams; 4) Teams are recognized for highest score each week.

Jigsaw is a strategy that emphasizes cooperative learning by providing students an opportunity to actively help each other build comprehension, and help them create their own learning. "Teachers arrange students in groups. Each group member is assigned a different piece of information. Group members then join with members of other groups assigned the same piece of information, and research and/or share ideas about the information. Eventually, students return to their original groups to try to "piece together" a clear picture of the topic at hand." That's the simple overview of Jigsaw according to Education World (2013: 1).

### **Mathematical Communication**

Mathematics can be meaningful as a communication tool. Shadiq (2004) argues that the need for the students to learn mathematics are the reason that mathematics is a powerful, conscientious, and clear communication tool. If students do not necessarily talk about mathematics naturally, teachers have to help them learn how to do. As a language, mathematics is more universal than some other languages. That is, the language of mathematics is not claimed as a language of a particular area, but applied universally. Mathematical language is even more practical in its use. Mathematical notation and symbols are generally accepted in almost in every country.

Communication is an important part in learning mathematics. NCTM (David et al. 2003: 238) argues that, "communication helps build meaning and permanence for ideas". Meanwhile, according to Kusuma (2005), a mathematical communication skill is the ability to communicate mathematical ideas. When mathematical communicating occurs, ideas presented are not only a mathematical context, but also sentence with mathematical structures. Ministry of Education (Sadiq, 2004) states that many of the issues or information conveyed by the language of mathematics, for example, present the issue or problem into a mathematical model which can be diagrams, mathematical equations, graphs, or tables. Based on these statements, communicating in mathematics may transform problems or information conveyed by the language of mathematics in the form of presenting it in the form of diagrams, mathematical equations, graphs, or tables.

NCTM (2003) suggests a mathematical indicator for students' mastery of communication as follows: 1) communicate their mathematical thinking coherently and Cleary to peers, teachers, and others; ; 2) use the language of mathematics to express mathematical ideas precisely; 3) Organize and consolidate their mathematical thinking though communication; 4) Analyze and Evaluate the mathematical thinking and strategies of others.

With a slightly different formulation, Kusuma (2005: 2) reveals that students' ability to communicate mathematically can be shown as follows: 1) Reflect and clarify students' thinking about mathematical ideas and relationships; 2) Formulate a mathematical definition and generalization through the method of the invention; 3) Declare mathematical ideas both in oral and written form; 4) Read the discourse of mathematics with understanding; 5) Clarify and expand questions toward the mathematics that is being studied; and 6) Appreciate the beauty and power of mathematical notation and its role in the development of mathematical ideas.

Furthermore, the research team agreed to refer to Kusumas' statements in determining indicators of mathematical communication. The six indicators are used as a reference in preparing the evaluation instrument; in this case it is a test of mathematical communication.

### **Mathematical Disposition**

Sumarmo (2012: 2) states that, "mathematical disposition is the desire, awareness, inclination and strong dedication within students to think and act mathematically". Based on the statement, mathematical disposition can be classified as a component of a person's psychology to mathematics. These elements are able to encourage someone to work based on mathematical thinking. Furthermore, mathematical disposition is internalized into problem solving skill and it becomes students' character gradually.

Mathematical disposition appears when students complete the task of mathematics, if it is done confidently, responsibly, diligently, showing persistence, challenged feeling, willingness to find alternatives and reflect on their way of thinking that has been done; or vice versa. This is in line with the NCTM (1989: 233), which states that: "*The assessment of students' mathematical disposition should seek information about their: 1) confidence in using mathematics to solve problems, to communicate ideas, and to reason; 2) flexibility in exploring mathematical ideas and trying alternative methods in solving problems; 3) willingness to persevere in mathematical tasks; 4) interest, curiosity, and inventiveness in doing mathematics; 5) inclination to monitor and reflect on their own thinking and performance; 6) valuing of the application of mathematics to situations arising in other disciplines and everyday experiences; 7) appreciation of the role of mathematics in our culture and its value as a tool and as a language.*"

Mathematical disposition is one of the factors that will determine the success of students' learning. In order to develop good habits in doing mathematics, and have responsibility for everything about learning, they require a disposition that would make them persistent facing more challenging problems. Challenging issue is not synonymous with much homework, but do not give any assignment—by reason for reducing the burden of tasks—it is clearly omitting something important for students.

In this study, the team agreed to use indicators of disposition according to Maxwell (2001), which consist of: 1) inclination, which is how the students' attitudes are toward the tasks; 2) sensitivity, which is how a student's readiness is in facing tasks; and 3) ability, which is how students' focus is on completing the task is complete; and 4) enjoyment, which is how the students' behaviors are in completing the task.

## METHODOLOGY

### Research Methods

The used method is research and development formed experimental instruction in the classroom by focusing on the development of teaching materials. "Method of research and development is a research method that is used to produce a particular product" (Sugiyono, 2009: 407). The purpose of the research is to develop teaching materials authentication mathematics and rediscover hypothesis of students' learning process. Research studies that investigated the quality of relationships, activities, situations, or materials are frequently referred to as qualitative research. (Fraenkel & Wallen, 2007). Borg dan Gall (Ghufron, 2011: 12) set out 10 steps research and development as follow: 1) research and information collecting, 2) planning, 3) develop preliminary form of product, 4) preliminary field testing, 5) main product revision, 6) main field testing, 7) operational product revision, 8) operational field testing, 9) final product revision, 10) dissemination and implementation.

Adapted from Borg & Gall version, the research is conducted in three phases: preparation (designing prototype), implementation, (and prototype development) and refinement. Previously, it requires initial hypothesis, i.e., the alleged design of instructional materials in this study is referred to as Design of Mathematical Proof. The design of teaching materials develops into an instrument as instructions for carrying out each stage of the research.

During the preparation phase (preliminary design and limited experiment) the design of teaching materials serves as a guide in designing next instructional materials. In the implementation phase (first experiment) the design of teaching materials serves as a guide in teaching and observation for lecturers. While on the stage of completion (final design implementation) the design of teaching materials serves as a guide for decision-making at the time of the final analysis before the instructional materials are redesigned into the design of instructional materials for mathematical proof.

First year consists of two stages. First stage is the stage of prototype orientation activities including: literatures review, designing teaching materials, validation of experts' team, and improvements design. Second stage is the stage of the implementation and prototype development, pilot testing a design of instructional materials in the real classroom. Third stage is implemented in the second year, a developing stage. Activity at this stage is: workshop that lead to the design validation, field trials, analysis of the test results, design revisions, and determination of the design as a mathematical proof of teaching materials, as the end product of research. At this stage also scientific publication is conducted as a result of the study.

### DATA COLLECTION

Data collection techniques uses observation sheets, expert validation and test sheets supported by interviews. Observation sheet is used to observe the situation of research subjects to collect the needed information. Observation sheet is supported by media such as video recording and digital cameras. This way is expected to obtain the information in detail and accurately. Additionally, observation sheet is also used to obtain lecturers' and students' profile information during the experiment of teaching materials. Observation sheets are analyzed to obtain a proper design of instructional

materials and lecturers' profiles in learning. In addition, the results of the analysis of observations provide information for students' attitude during learning.

Validation sheet is used to gather experts' advice in designing teaching materials based on the results of experimental information or literature that have been implemented. Validation sheet becomes one of the basic revisions or decision-making in the design of teaching materials. The test is used to gather information on growth of students' character and its impact on test scores. The assays are designed by researchers after the assessment by institution.

### **DATA ANALYSIS**

The data are analyzed using constant comparative technical analysis. This method was originally developed for the use in grounded theory methodology, and is now applied more widely as a method of analysis in qualitative research. Constant comparative technical analysis method provides an opportunity to perform analysis according to the observed focus repeatedly. If there is an interesting section to be analyzed, the data can be examined repeatedly. The initial stage of analysis is the set list from the temporary answer to each question based on the initial analysis and validation. The analysis is continued by consolidating the list of temporary answers and the result of observation data. The observation data are analyzed by two persons separately. The aim is to consolidate the results of the analysis. The results of the analysis are then discussed in regular meetings in the research forum. The analysis is conducted repeatedly until obtaining a sample saturation point theory.

### **LOCATION AND SUBJECTS RESEARCH**

The study is conducted at the Nusantara Islamic University. The subjects are sixth semester students of the Department of Mathematics Education, Nusantara Islamic University School of Education. The selection of subjects is based on teaching materials that will be developed as well as the ability of the prerequisites that need to be owned by the student. To be able to communicate well, students need the ability of mathematical process. It means that to be able to communicate well, students must really understand the mathematical process. Therefore, it becomes important to consider the prerequisites in an effort to improve ability in mathematical communication. Therefore, the selected students are the sixth semester in assuming that they have the necessary prerequisites.

### **TARGET ACHIEVEMENT**

This study is successful when the goals are achieved. In other words, the target achievement in the study is the achievement of the objectives. Thus, the target achievements of this research are: 1) the creation of teaching materials to improve the students' mathematical disposition and communication skills; 2) described profiles learning that can improve students' mathematical dispositions and communication; 3) described lecturers' profiles that can improve students' mathematical disposition and communication skills; 4) obtained documents of improving students' mathematical disposition after attending mathematical proof course; and 5) obtained documents of improving students' mathematical communication after attending mathematical proof course.

## DISCUSSION

### Development of Teaching Material

Broadly speaking, this study is conducted in three phases over two years; namely preparation (prototype design) and implementation stage in first year, as well as refinement in the second phase. The preparation stage includes: preliminary design, validation, and limited test; implementation stage is an experimental stage core, while the refinement phase includes validation and revision of experts, completed with scientific publications.

The first stageresearch and development begins with studying curriculum to determine the aim, and competence of teaching materials that will be developed; as well as identify the main material that needs to be taught. The next step is the literature review; carried out by collecting and selecting relevant material, putting it back together systematically, so that eventually it is acquired the design of teaching materials and research instruments. Design and instruments are then validated, tested on a limited basis, analyzed and revised, resulting in revised design I.

As with phase I, phase II research and development are conducted in the first year. Design revision I produced at stage I is tested on real classroom, sixth semester students of the Department of Mathematics Education, Nusantara Islamic University School of Education. The trial results, such as: recording lectures, observation sheet activities of lecturer and students, as well as formative and summative test scores; then are analyzed, and the results are used to improve the design of teaching materials so obtained Design revision II.

At the time this report was written, the study runs just for a year. Thus, the resulting teaching materials already have received validation from experts and users, while the effectiveness and practicality have been tested through action research. Nevertheless, research products will still be perfected through trial in the second year.

At the beginning of the analysis, list of hypothesis has been alleged in designing teaching materials of mathematical proof that can improve mathematical disposition and mathematical communication competencies. The alleged list is obtained from the literature review, input validator, and undertaken preliminary studies. The alleged list are: 1) teaching materials provide facilities that encourage learning proof cooperatively, 2) instructional materials provide facilities so that students are individually accountable for his thoughts, and 3) teaching materials facilitate students' full potential at all levels in thinking ability. This list becomes the orientation analysis of the development of teaching materials for mathematical proof.

The observed data of teaching materials are analyzed repeatedly. The analysis is performed until the fourth cycle. Categories which appear at the beginning of the analysis are consolidated with the observed data. At the end of the analysis, only two categories remain consistently. The two categories are set out as the answer to first research question. The categories are: 1) teaching materials provide facilities that encourage learning proof cooperatively, 2) instructional materials provide facilities so that students are individually accountable for his thoughts.

### Student Learning Profile

The analysis of the students' learning profile in mathematical proof subject is conducted up to five cycles. In the initial analysis, it is determined that two categories of learning can improve students' mathematical disposition and communication skills. The categories are: 1) learning provides an opportunity for class discussion, and 2) learning provides the opportunity for students to deliver presentation in front of class.

The both categories are then consolidated with the observed field data. The observed field data are analyzed by two analyzers separately. The result is discussed in regular meetings of the research team. The data analysis results are consolidated repeatedly to set out the samples theory as follows: "Learning climate that gives comfort to students to think and express opinions".

Students who dare to express his ideas, they have better mathematical communication. Although there is an error possibility in the concept of the idea, but the error can be determined directly, and lecturer can provide assistance to overcome the students' weaknesses in proving. In contrary, students who do not dare to express their ideas, their weaknesses cannot be detected directly so that these create difficulty in improving.

### Lecturer' Profile

Lecturer is the main actors in lecturing. However, students' learning is still influenced by how the lecturer manage the lecturing. The lecturer can determine the direction and outcome of the course. Therefore, the lecturers' teaching style can be an indicator of students' successful learning. The lecturer has many opportunities to manage learning well. The lecturer also can serve as mentors for students about how they should learn. Besides, the lecture can be a patternfor students, so he should be able to give good overall impressions for students.

The observer scrutinizes the activities during lecturing. The goal is to obtain description of lecturer' teaching style in improving students' mathematical disposition and communication. The description can be seen from the obtained data from the research. The data are analyzed repeatedly to obtain an overview of the lecturer's teaching style in improving students' mathematical disposition and communication. Based on the analysis, it is concluded that, "lecturer teaching style that is characterized guidance toward self-regulated learning, has a great opportunity to develop students' mathematical dispositions and communication skills. "Lecturer can optimize students' capabilities through learning process that can generate motivation to perform mathematical proof, so that the lecturer can put in motivation in the lecturing. Likewise, it is likely to insert motivation in teaching materials.

### CONCLUSION

1. Instructional materials that are designed to be discussed in the group and should be accounted for on an individual basis, can foster students' mathematical communication and disposition;
2. Learning climate that gives comfort to the students to think and express opinions expected to develop students' mathematical communication and disposition;
3. Lecturer teaching style characterized by guidance toward self-regulated learning is expected to develop students' mathematical communication and disposition.

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