

## PIAGET'S THEORY IN A DEVELOPMENT OF CRITICAL THINKING

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### Abstract

This paper presents a study of cognitive structure of critical thinking in constructing a conic section for Indonesian students. Context of this research is using Piaget's theory of cognitive development when students do critical thinking. Revolutionary Piaget's theory of cognitive development of children has focused on the development of logic. When someone thinks logically, it means that he/she is in a higher-level. This kind of thinking is a high level of critical thinking. In order to get the schemata of students' critical thinking, the students are given open mathematical problems. The approach is a qualitative exploration of students' performance in doing problem-solving. In order to see their thinking processes, it is used a think-load, which is expected to be an overview of the activity of students' thinking. Furthermore, critical thinking is observed through as simulation and accommodation of Piaget's theory, so the cognitive structure of students' schemata can be illustrate. The results showed that there are three characteristics of critical thinking process, (1) The existence of a sub-structure perfection of thought that will be used in generalizing the solution, (2) an ability to reflect toward the problem fully, and (3) The existence of consciousness to explore solutions, although students do not have the ability to explore the possibility of another solution

**Keywords:** Piaget' theory, Critical thinking, think-aloud, logical thinking

### INTRODUCTION

Learning perspective currently incorporates three important assumptions as Anthony (1996) said:

- (1) Learning is a process of knowledge construction, not of knowledge recording or absorption;
- (2) Learning is knowledge-dependent; people use current knowledge to construct new knowledge; and
- (3) The learner is aware of the processes of cognition and can control and regulate them.

Each child builds on the previous stage of cognitive development increasing the child's ability to solve more complex problems (Oxford, 1997: p. 189). The fundamental basis of learning was a discovery. Understanding is a discovering or a reconstructing by rediscovery, and such conditions must be compiled with if in the future individuals are to be formed who are capable of production, creativity and not simply repetition.

There are some researchers who uncover cognitive structures associated with the construction of detailed knowledge about the mastery of new knowledge. Piaget portrayed the child as a lone scientist, creating his or her own sense of the world. Then individual will interpret and act accordingly to conceptual categories or schemas that are developed in interaction with the environment. The knowledge of relationships among ideas, objects, and events is constructed by the active processes of internal

assimilation, accommodation, and equilibration (Oxford, 1997, p. 39). Until children can construct a certain level of logic from the inside, they are non-conservers because they can judge on the basis of what they can see (Kamii & Ewing, 1996, p. 261).

Some theories (such as that of Piaget, the SOLO Model, or more broadly, the enactive-iconic-symbolic theory of Bruner, 1966) incorporate both aspects. Others such as Lakoff & Nunez (2000) and situated learning Lave & Wenger (1991) paint a broader brush-strokes showed biological and social structures involved. It has been developed for different purposes. The SOLO Model, for example, is related to the performance assessment through learning outcomes were observed. Other theories such as Davis (1984), Dubinsky (Czarnocha et al., 1999), Sfard (1991), and Gray & Tall (1994) concerned with the order in which concepts are built by an individual.

So in recent years, various theories have emerged to explain and predict cognitive development in mathematics education. Authors identified two types of theories of cognitive growth are:

- 1) Global theory of long-term growth of the individual, such as Piaget's theory of stages (eg, Piaget & Garcia, 1983).
- 2) The growth of local theories like the theory of conceptual action-process-object-schema

Dubinsky (Czarnocha, Dubinsky, Prabhu, Vidakovic, 1999) or sequence-multi structural-uni structural abstract-relational model extended SOLO (Structure of the observed learning results, Biggs & Collis, 1982, 1991; Pegg, 2003).

Question. How does the cognitive structure of students in a state of critical thinking?

## THEORETICAL FRAMEWORKS

Assimilation, accommodation and illustrations

Learning is an adaptation which has assimilation and accommodation in Piaget's term. To reach an understanding of basic phenomena, children have to go through the stages which Piaget presented (Bybee & Sund, 1982, p. 36). In problems solving, students construct the structure of thinking through the processes of assimilation and accommodation. Working memory capacity (that is, the capacity to hold various pieces of information simultaneously and to use them for further processing) is a critical feature of several models of human cognition, and it is widely recognized that it affects performance on many tasks (Morra, Gobbo, Marini & Sheese, 2009, p. 20). It has also been claimed that individual differences in working memory capacity account well for difference in measures of fluid intelligence (Engle, Tuholski, Laughlin & Conway, 1999; Kyllonen, 2002).

According to Fisher (1995, p. 57), thinking which is visualized and expressed can be observed and communicated. As stated by Gentner (1983) and Morrison, Dumas, & Richl (2010), balancing inhibitory control in working memory and relational representation can be illustrated the process of assimilation and accommodation fundamentally. And then author adopted from assimilation and accommodation of Subanji (2007).

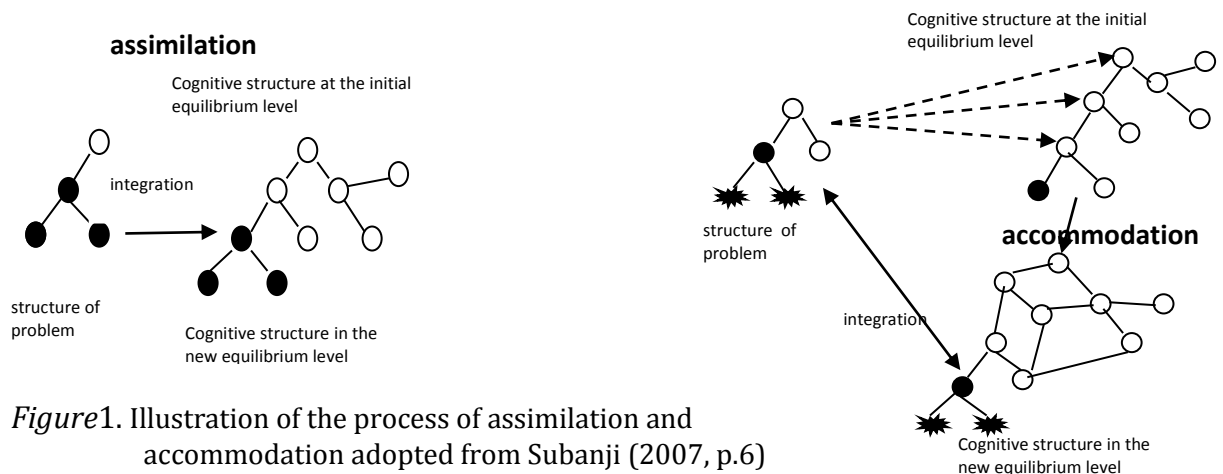


Figure1. Illustration of the process of assimilation and accommodation adopted from Subanji (2007, p.6)

Subanji (2007, p. 39) said that the substructure incompleteness in the process of assimilation is a process of direct interpretation of the problem with more complex structure using a simple thinking structure. This thinking process was preceded by the imperfect assimilation process. The assimilation took place in the process of problem solving, but the complex problem was interpreted to the simple problem. Therefore, it produced an inappropriate answer.

In the process of problem solving (before the reflection), the students only conduct the assimilation process, but did not produce the appropriate structure to the structure of the problem. In this case, their thinking structure was still incomplete; nonetheless it had been used to interpret a complex problem structure. However, it produced an inappropriate answer (wrong). After receiving the answer, the students did not go through the reflection again.

Furthermore, when the opportunity for reflection was given, the disequilibrium took place again in the students' thinking process, with the result that they continued to the assimilation and accommodation process. For the illustrations, see Figure 2.

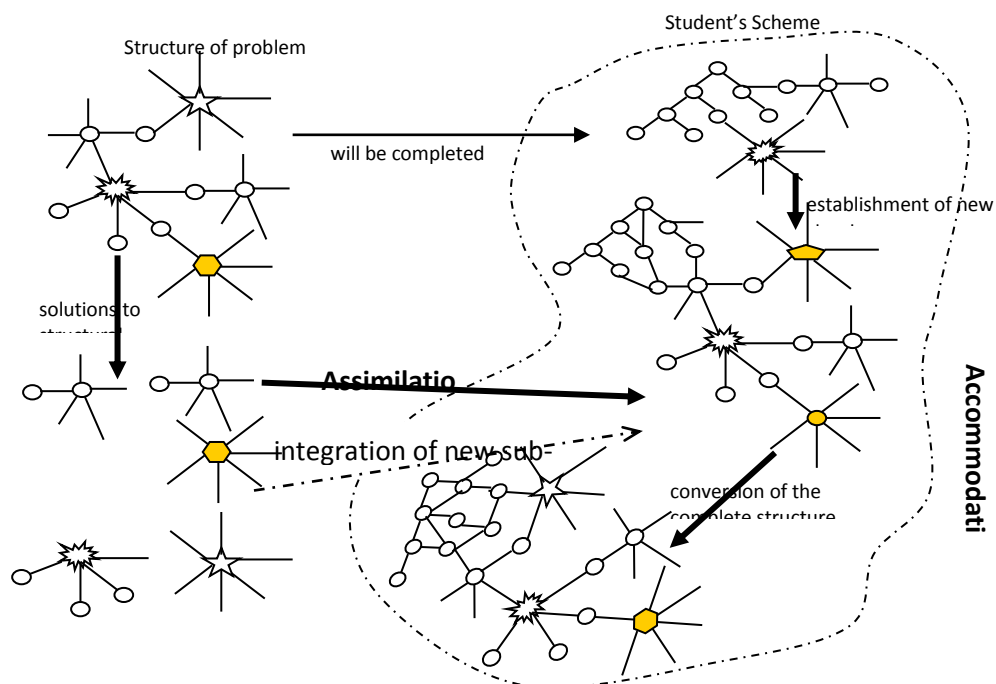


Figure2. Structure of problem-solving adopted from Subanji (2007)

Encoding Process of Thinking

Categorization is done to facilitate the interpretation of the data, simplifying analysis of the problems and the process of thinking of the object of research. It is related to the process of thinking including problems, relationships and strategies. As Gentner & Goldin-Meadow (2003, p. 6) shows the same view in cognitive linguistics that the coupling between language and cognition is strong enough to allow semantic structure to serve as an window on conceptual structure.

Furthermore Forbus, Gentner, & Law (1995) habitual use of a given set of relational terms promotes uniform relational encoding; thereby the probability of transfer between relational situations is increasing. Then performed: when a given domain is encoded in terms of a stable set of relational terms, the likelihood of matching new examples with stored exemplars that share relational structure is increasing. Recoding involves a mental transformation of information into another code or format (Ashcroft, 1994).

Error Assimilation and Accommodation

In solving the problem, if the formation of cognitive structures is not perfect in the sense of the word: a cognitive structure to the structure of the problem is not the same, and then integrated it will produce the wrong answer (Subanji, 2007, p. 49). There is an example problem from Frederick (Kahneman, 2002, p.451): "The price of baseball bat and ball is \$12. Bat costs \$ 10 more expensive than the ball. What is the price of the ball?" Many students answered \$2. Possibility of thought processes occurring imperfections assimilation. Kahneman problem structure can be described in Figures 3a and 3b.

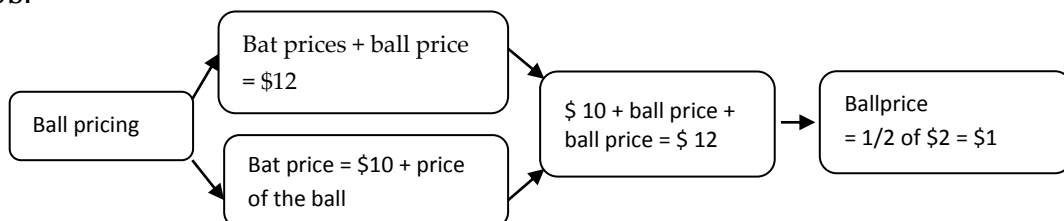


Figure 3a. Suitability: structural problems with the structure of thinking

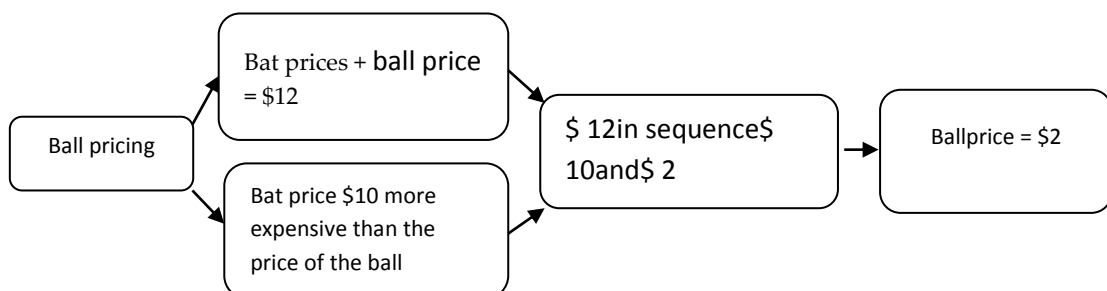


Figure 3b. Mismatches: structural problems with the structure of thinking

From the above illustration, it seems that there is no correspondence between the structures of a problem with the structure of student thinking. However, the assimilation process is already under way obtaining answers \$2. Frederick’s real problem is a simple matter, even been able to use that mindset quickly without any control (reflection) then the answer to be incorrect.

Examples of accommodations mistake on elementary school students: Today is Sunday. What day is it 2011 days later?

Basically, elementary school students are familiar with addition, multiplication, subtraction

and division. However, when they are faced with the problems mentioned above in the absence of changes in cognitive structure namely linking multiple weekly with multiples 7, it will result in a wrong answer. Note Figure 4a.

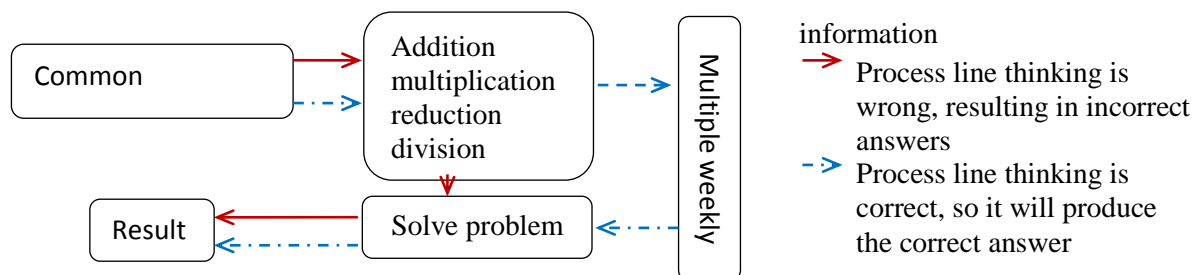
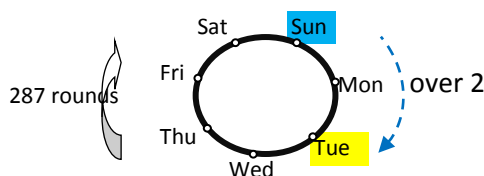


Figure 4a. Right mindset accommodation

On the contrary, when students firstly linked between weekly and multiples of 7, as well as more associated with the addition or day trip, there will be the right answer. The solution like that: 1 round = 1 week = 7 days, so the multiples will fall on the same day Sunday.  $2011:7 = 287$  remainder 2 or  $2011 = 287 \times 7 + 2$ , or  $287 \times 7 = \text{Sunday}$ . There is an excess of 2 days, so the answer is Tuesday. Figure 4b illustrated the problem solving.



Figur 4b. illustration of problem solving

Critical thinking

Thinking can be divided into four categories, including *recall thinking*, *basic thinking*, *critical thinking*, and *creative thinking* (Krulik, Rudnick, & Milou, 2003, p. 89). Krulik et al. (2003) said that critical and creative thinking are higher-order thinking, and basic, critical, and creative thinking are reasoning. Figure 5 presents the hierarchy of thinking from Krulik, at.al (2003).

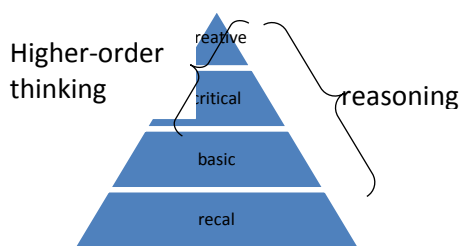


Figure 5. Hierarchy of thinking

*Critical thinking is the ability to analyze a problem, determine if there is sufficient data to solve it, decided if there is any extra information in the problem, and analyze the situation. Such thinking includes recognizing consistant or in consistant data, as well as contacditary data, and being able to draw conclusions from a set of data. Student should also be able to determine whether conclutions are valid or invalid.*

What is critical thinking?

In Department of England Education (2010, p.5) define : (1) Critical thinking is applying well-known criteria to a problem, turning the handle and producing an answer, (2) Critical thinking is when you set out to find faults, (3) Critical thinking is when you balance everything in question to reach a judgement, (4) Critical thinking is when you examine the 'item' in question to find its good and bad points.

Ennis R. H. (2011) Critical thinking is reasonable and reflective thinking focused on deciding what to believe or do. If you feel like you are doing one activity, then switching to another, and you don't know how to pull it together in the end, it may be that the lesson is not aligned. Look back to the objectives and make sure all activities support these objectives and build in critical thinking and challenges (Van de Walle J. A. 2010, P.62)

Halpern (1999) Critical thinking refers to the use of cognitive skills or strategies that increase the probability of a desirable outcome. Critical thinking is purposeful, reasoned, and goal-directed. It is the kind of thinking involved in solving problems, formulating inferences, calculating likelihoods, and making decisions. Critical thinkers use these skills appropriately, without prompting, and usually with conscious intent, in a variety of settings. That is, they are predisposed to think critically. When we think critically, we are evaluating the outcomes of our thought processes-how good a decision is or how well a problem is solved. (p. 70).

Dumke (1980), "instruction in critical thinking is to be designed to achieve an understanding of the relationship of language to logic, which should lead to the ability to analyze, criticize, and advocate ideas, to reason inductively and deductively and to reach factual or judgmental conclusions based on sound inferences drawn from unambiguous statements of knowledge or belief" (p. 3). While this instructional goal is problem solving straightforward, implementing instructional strategies that achieve these ends is a daunting task.

## METHOD

### Participants

A qualitative design was chosen for this study in order to investigate the intricate thinking process (Bogdan & Biklen, 1992). To see it, the data were gathered by the think aloud method (van Someren, Barnard & Sandberg, 1994) which was conducted by asking the research subjects to solve problems and to tell how their thinking process is at the same time. Think aloud was developed by the cognitive psychologists aiming to investigate how someone solves a problem. Using this method, the solver's cognitive process related to the problems can be recorded and analysed. The research subjects were 2 students who were in Mathematics Education academic year 2012/2013. They had not studied a conic section equation, but could express their thought process when they solve the problems.

### Questionnaire

To investigate the critical thinking of students, researchers gave questionnaires, which can open students explore the characteristics of critical thinking to solve problems with a central question: "Finding the set of points where the ratio of fixed distance to one of the lines, the lines are perpendicular to each other, and to the point that lies on the other

line! "

For the complete information about the thinking process of students, investigator conducted interviews to student during students working for the task and after that. In accordance with the opinion of Guba & Lincoln (1994) the received view of science pictures the Inquirer as standing behind a one- way mirror, viewing natural phenomena as they happen and recording them objectively. The researchers called the students one by one to work construction tasks of conic section equation. We exploring several students, until finding at least two students, who were able to answer perfectly, and explained their thought processes when solving problems.

**RESULTS AND DISCUSSION**

After exploring 8 students, we found 2 students, named Subject 3 (S3), and Subject 8 (S8), who were able to answer perfectly. We interviewed them to know their mindset such as ‘what is his way of thought to solve problems’. As for the answer as follows: S 3 and S8 used the comparison distance between two points, for solving the problem. So it does not produce a conic section equation. They has been constructed of conic section equations with various positions, namely:

- (1) The comparison same distance between the PF and PD (e=1) will be obtained equation type one, as shown in Figure 6a.
- (2) The comparison: distance PF < distance PD (e <1, taken e = 1/2) will be obtained equations type two, as shown in Figure 6b.
- (3) The comparison: distance PF > distance PF (e>1, taken e = 2) will be obtained equation type three, as shown in Figure 6c.

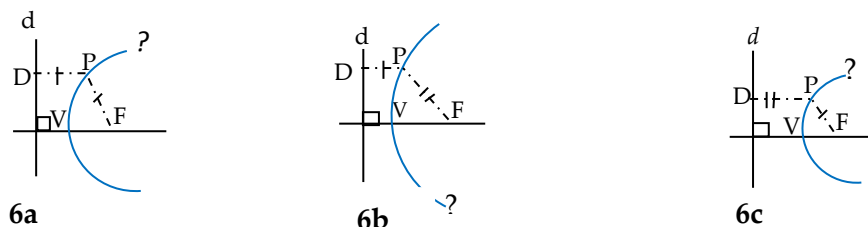
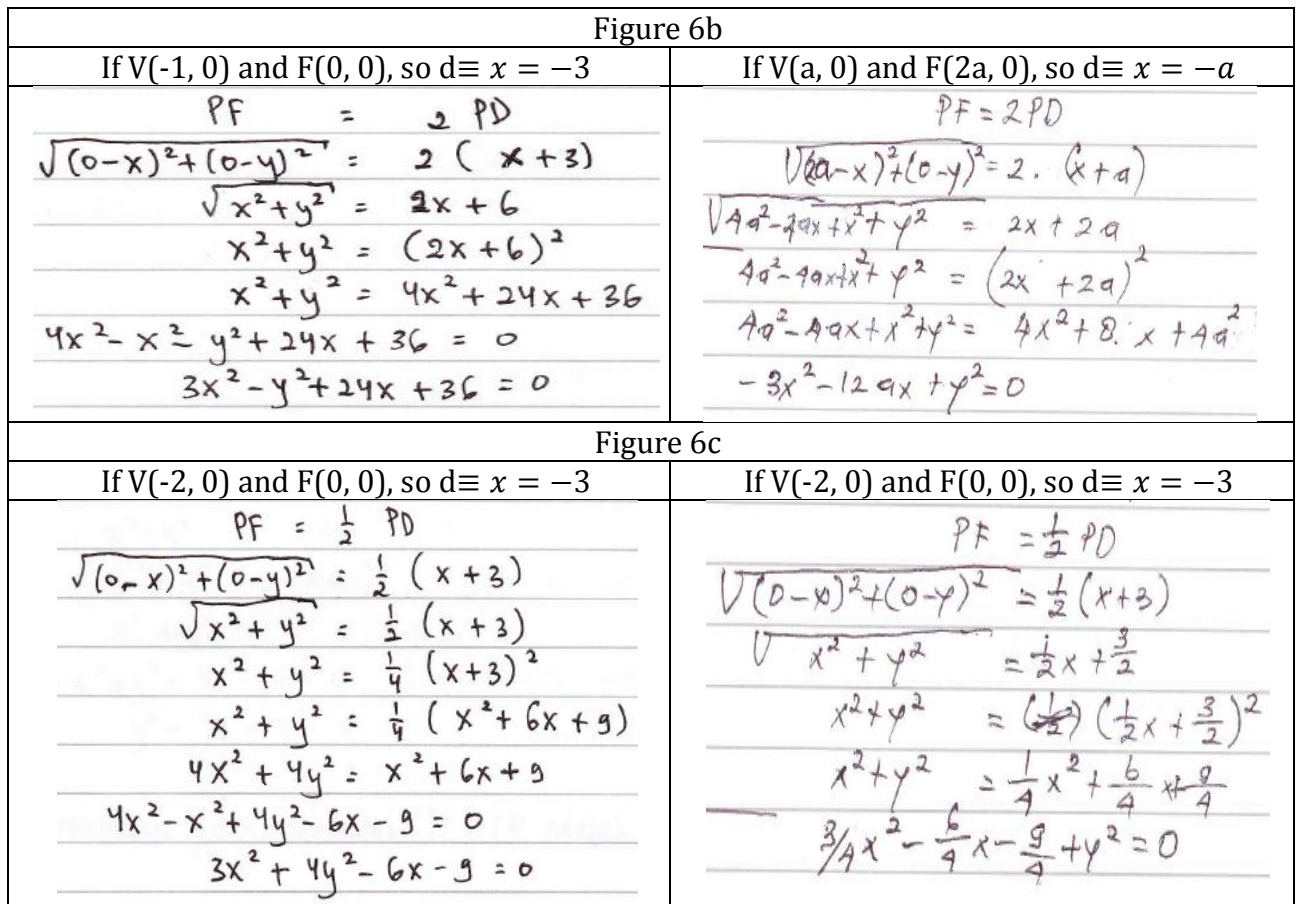


Figure 6. sketch the graph of a conic section generated S3 and S8

Results of calculation of S2 and S8

Figure 6a	
S3	S8
If V(-1, 0) and F(0, 0), so d ≡ x = -2	If V(-a, 0) and F(0, 0), so d ≡ x = -2a
$PF = PD$ $\sqrt{(x-0)^2 + (y-0)^2} = x + 2$ $\sqrt{x^2 + y^2} = x + 2$ $x^2 + y^2 = (x + 2)^2$ $x^2 + y^2 = x^2 + 2x + 2x + 4$ $x^2 + y^2 = x^2 + 4x + 4$ $y^2 = 4x + 4$ $y^2 - 4x - 4 = 0$	$PF = PD$ $\sqrt{(x-F)^2 + (y-F)^2} = (x+2a)$ $\sqrt{(x-0)^2 + (y-0)^2} = (x+2a)$ $x^2 + y^2 = (x+2a)^2$ $x^2 + y^2 = x^2 + 4ax + 4a^2$ $y^2 - 4ax - 4a^2 = 0$



For the structure of critical thinking when building a conic section equation is shown in Figure 7.

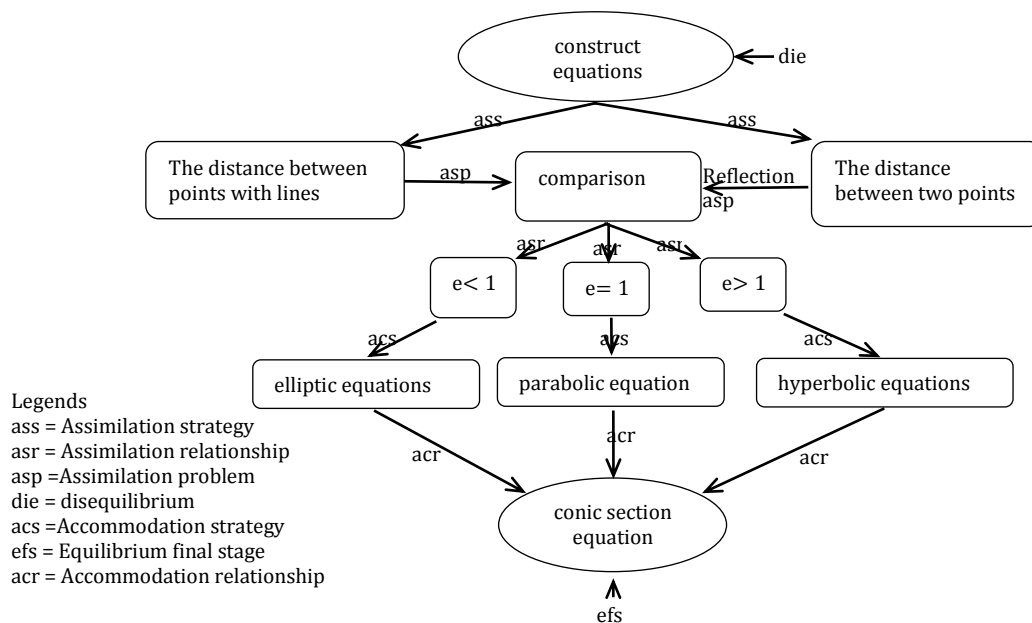


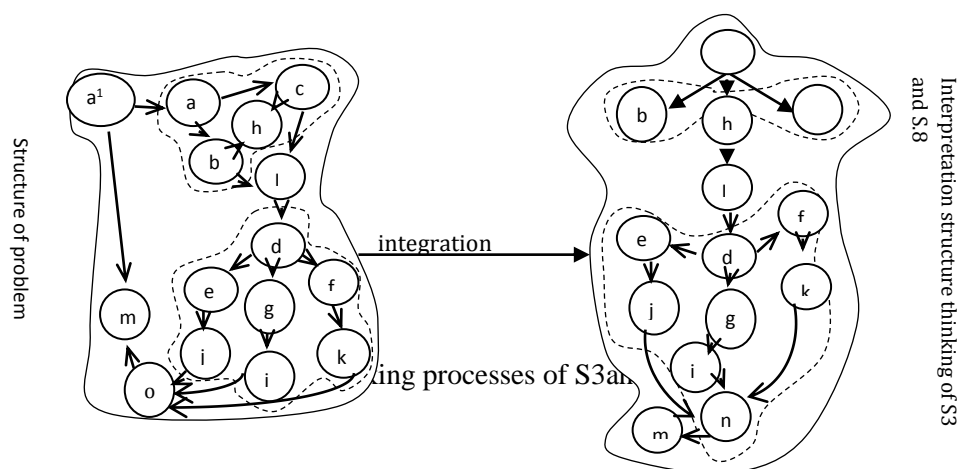
Figure 7. Structure of critical thinking when constructing the conic section

Problem solving means answering a question for which one does not directly have an answer available. This can be because the answer cannot be directly retrieved from memory but must be constructed from information that is available in memory or that can be obtained from the environment. Another possibility is that finding the answer involves exploring possible answers none of which is immediately recognized as the



solution to a problem. Problem solving then means that new information must be inferred from givens and knowledge in memory to accept or reject possible answers. Most of the problem solving involves a combination of these two types of reasoning: constructing solutions and constructing justifications of these solutions (van Someren, et. al, 1994).

In the process of problem solving, S3, and S8 is a process of assimilation, and accommodation, resulting in a structure that corresponds to the structure of the problem. In this case, the structure of his thought is complete; used to interpret the structure of a complex problem. Cognitive structure of critical thinking can be seen in Figure 8.



Legends

○ = Statement

◇ = Question

- |                |  |   |                        |
|----------------|--|---|------------------------|
| a              | Problem, the proposed construction of conic section equation | g | $b < c$                |
| a <sup>1</sup> | Problem, often encountered in daily life                     | h | quadratic              |
| b              | The distance between two points                              | i | elliptic equations     |
| c              | The distance between points on a line                        | j | parabolic equation     |
| d              | compare  | k | hiperbolic equations   |
| e              | $b = c$  | l | graph                  |
| f              | $b > c$  | m | completed              |
| o              | equation of the line for a corner                            | n | conic section equation |

**CONCLUSIONS**

From the results, it can be concluded that the process of assimilation and accommodation when critical thinking starts from students' awareness of the existence of a complex problem. The results showed that there are three characteristics of the process of critical thinking to construct a conic section:

- (1) The existence of a sub-structure perfection of thought that will be used in generalizing the solution,
- (2) The existence of consciousness to explore the possibility of another solution, as found by Subanji (2007) and Supratman (2013)

## REFERENCES

- Anthony, G. (1996). Active learning in a constructivist framework. *Educ. Stud. Math.* **31(4)**, 349–369. ME 2002c.01837
- Ashcroft, M. H. (1994). *Human Memory and Cognition*. New York: Harper Collins.
- Biggs, J. & Collis, K. (1982). *Evaluating the Quality of Learning: the SOLO Taxonomy*. New York: Academic Press.
- \_\_\_\_ (1991). Multimodal learning and the quality of intelligent behaviour. In: H. Rowe (Ed.), *Intelligence, Reconceptualization and Measurement* (pp. 57–76). Hillsdale, NJ: Laurence Erlbaum Assoc.
- Bogdan, R. & Biklen, S. (1992). *Qualitative research for education: An introduction to theory and methods*. Boston, MA: Allyn and Bacon.
- Bybee, R.W. & Sund, R. B. (1982). *Piaget for Educators 2nd ed.* Columbus, OH: Charles E. Merri Publishing Co.
- Czarnocha, B.; Dubinsky, E.; Prabhu, V. & Vidakovic, D. (1999). One theoretical perspective in undergraduate mathematics education research. In: O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education (PME-23)* (Vol. 1, 95–110). Haifa, Israel. ME 2002a.00240
- Davis, R.B. (1984). *Learning Mathematics: The cognitive science approach to mathematics education*. Norwood, NJ: Ablex. ME 1985c.02123
- Department of England Education (2010) Developing critical and creative thinking: in science  
<http://www.education.gov.uk/>
- Ennis R. H. (2011) The Nature of Critical Thinking: An Outline of Critical Thinking Dispositions and Abilities. *This is a several-times-revised version of a presentation at the Sixth International Conference on Thinking at MIT, Cambridge, MA, July, 1994. Last revised May, 2011* [http://faculty.education.illinois.edu/rhennis/documents/TheNatureofCriticalThinking\\_51711\\_001.pdf](http://faculty.education.illinois.edu/rhennis/documents/TheNatureofCriticalThinking_51711_001.pdf)
- Engle, R. W., Tuholski, S. W., Laughlin, J. E., & Conway, A. R. A. (1999). Working memory, short-term memory and general fluid intelligence: A latent variable approach. *J. Exp. Psychol. Gen.* **128(3)**, 309–331.  
<http://psychology.gatech.edu/renglelab/1999/working-memory2c-short3dterm-memory2c-and-general-fluid-intelligence.pdf>
- Fairbrother, R. & Hackling, M. (1997). Is this the right answer? *International Journal of Science Education*, **19(8)**, 887–894.
- Fisher, R. (1995). *Teaching Children to Learn*. London, UK: Blackwell/Simon & Schuster/Stanley Thornes.

- \_\_\_\_ (2013). *Teaching Thinking Philosophical Enquiry in the Classroom*. London, UK: Bloomsbury Publishing Plc.
- Forbus, K. D.; Gentner, D. & Law, K. (1995). MAC/FAC: A model of similarity-based retrieval. *Cognitive Science* **19**(2), 141–205.
- Gentner, D. (1983). Structure-Mapping: A Theoretical Framework for Analogy. *Cognitive Science* **7**(2), 155–170. Available from:  
<http://axon.cs.byu.edu/~dan/673/papers/gentner.pdf>
- Gentner, D. & Goldin-Meadow, S. (2003). *Language in Mind Advances in the Study of Language and Thought*. Cambridge, MA: MIT Press.
- Gray, E. & Tall, D. (1994). Duality, ambiguity and flexibility: a proceptual view of simple arithmetic. *J. Res. Math. Edu.* **26**(2), 116–140. ME **1995c**.01407
- Guba, E. G. & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. In: N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 105–117). Thousand Oaks, CA: Sage.
- Halpern, D. F. (1999). Teaching for critical thinking: Helping college students develop the skills and dispositions of a critical thinker. *New Directions for Teaching and Learning*, *80*, 69–74.
- Kamii, C. & Ewing, J. (1996). Basing teaching on Piaget's constructivism. *Childhood Education*, **72**(5), 260–264.
- Kahneman, D. (2002). *Maps of bounded rationality: A perspective on intuitive judgment and choice*, in *Les Prix Nobel*, Edited by T. Frangmyr, pp. 416–499. <http://www.nobel.se/economics/laureates/2002/kahnemann-lecture.pdf> 25 July 2013
- Krulik, S.; Rudnick, J. & Milou, E. (2003). *Teaching Mathematics in Middle School*. Boston, MA: Allin and Bacon.
- Kyllonen, P. C. (2002). Knowledge, speed, strategies, or working memory capacity? A systems perspective. In: R. J. Sternberg & E. L. Gigorenko (Eds.), *The general factor of intelligence: How general is it?* (pp. 415–445). Mahwah, NJ: Erlbaum.
- Lakoff, G. & Nunez, R. (2000). *Where Mathematics Comes From. How the embodied mind brings mathematics into being*. New York, NY: Basic Books. ME **2002f**.04631
- Lave, J. & Wenger E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge, UK: Cambridge University Press.
- Morra, S.; Gobbo, C.; Marini, Z. & Sheese, R. (2009). *Cognitive Development Neo-Piagetian Perspectives*. New York: Taylor & Francis Group.
- Morrison R. G.; Dumas L. A. A, & Richland, L. E. (2010). A computational account of children's analogical reasoning: balancing inhibitory control in working memory and relational representation. *Developmental Science* **14**(3), 516–529.  
[http://learninglab.uchicago.edu/Publications\\_files/morrison\\_etal\\_DS\\_2010.pdf](http://learninglab.uchicago.edu/Publications_files/morrison_etal_DS_2010.pdf)

- Oxford, R. (1997). Constructivism: shape-shifting, substance, and teacher education applications. *Peabody Journal of Education* **72(1)**, 35–66
- Pegg, J. (2003). Assessment in Mathematics: a developmental approach. In: J.M. Royer (Ed.), *Advances in Cognition and Instruction*. (pp. 227–259). New York: Information Age Publishing Inc.
- Piaget, J. & Garcia, R. (1983). *Psychogénèse et Histoire des Sciences*. Paris: Flammarion.
- Sfard, A. (1991). On the Dual Nature of Mathematical Conceptions: Reflections on processes and objects as different sides of the same coin. *Educ. Stud. Math.* **22(1)**, 1–36.
- Subanji (2007). *Kovariansionalpseudoreasoningprocessof constructingthe graphfunction, incidence ofcontrastdynamics*. Ph. D. Dissertation. Malang, Indonesia: University of Malang.
- Supratman (2013). Piaget's Theory in the Development of Creative Thinking. *Journal Of The Korean Society Of Mathematical Education Series D Reseach In Mathematical Education* Volume 17/Nomor 4 2013
- Van de Walle J. A. (2010) *Elementerary and middle school mathematics Teaching Developmentally*. Allyn & Bacon is an imprint of pearson [www.pearsonhighered.com](http://www.pearsonhighered.com)
- van Someren M.W.; Barnard Y.F. & Sandberg J. A.C. (1994). *The Think Aloud Method: A Practical Guide to Modelling Cognitive Processes*. London, UK: Academic Press. Retrieved October 21, 2013, from [ftp://akmc.biz/ShareSpace/ResMeth-IS Spring2012/Zhora el Gauche/Reading%20Materials/Someren et alThe\\_Think\\_A loud\\_Method.pdf](ftp://akmc.biz/ShareSpace/ResMeth-IS Spring2012/Zhora el Gauche/Reading%20Materials/Someren et alThe_Think_A loud_Method.pdf)