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UNDERSTANDING THE CONCEPT OF CONSERVATION OF AREA: RECOMPOSING A SHAPE WILL PRESERVE ITS AREA

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Abstract

In order to figure out how the students understand the concept of conservation of area focusing on recomposing shapes, this study has designed an instructional learning activities for the third grade students of Elementary school in Surabaya. The study conducted before the students learned the concept of area measurement, even without introducing the concept of area directly. Two different classes were taken to involve in the first cycle and second cycle respectively. This study used design research as the methods and Indonesian version of Realistic Mathematics Education, called PMRI (Pendidikan Matematika Realistik Indonesia) as an approach to support the learning process of conservation of area. Real life context based on five tenets in PMRI which is making table-cloth from the cloth-rag and the shape of rice field was chosen in developing a sequence of learning activities to reach the learning goal. In the implementation of learning activities such as making table cloth and comparing rice field could be used to reveal the idea of recomposing in the concept of conservation of area. From those activities, the students build their understanding that recomposing preserve the area of a shape because none of the parts of the shape are wasted or leftover (identity concept). They also found that two shapes can have the same area although they have different form. Somehow, some students were found still struggle with the way to recompose a shape. They showed that it was difficult to figure out which parts of the shape to be cut and where to be placed.

Keywords: *the concept of conservation of area, recomposing a shape*

INTRODUCTION

Piaget tested the conservation task among children on several ages to indicate their cognitive development (Arnett & Maynard, 2012). Piaget did several conservation tasks, including the conservation of area. Based on his experience about the difficulties of children to solve the conservation task, Piaget thought about the way to learn the concept of conservation of area for the students. For example, Piaget, Inhelder & Szeminska (1981, as cited in Kordaki 2003).said that the concept of conservation of area should be learned before the students learn area measurement since it is a fundamental and preliminary aspect in students' understanding of the concept of area measurement.

However, there is still no research that showed how the students' understanding about the concept of conservation of area when it is taught before the concept of area

measurement, especially in Indonesia. So, how to teach the concept of conservation of area as the preparatory for learning the area measurement for students in Indonesia?

This study will show the result of two out of five learning activities which used for teaching the concept of conservation of area before learning the area measurement for students in Indonesia. This study will be focused on the students' development to understand that recomposing a shape will preserve its area.

The concept of conservation of area
Piaget, Inhelder and Sheminska (1981, as cited in Kordaki, 2003) defined conservation of area means a modification in form cannot produce change in area. Whatever happens to a figure, like dividing into some parts and then re-arrangement of the parts to form another figure as long as it is not added or not subtracted, it will preserve the area of the figure.

In the Piaget' conservation task, the concept of conservation of area was tested by asking the child whether more ground is covered by the blocks that are spread out or the blocks that are close together.

Piaget used a geometrical experiment called "cows on a farm" to test conservation of area. To illustrate this, Piaget used green cardboard to represent farmland. Two identical green farms were shown to the children; each had a little wooden cow placed upon it. The children were asked to think whether the cows had the same amount of grass to feed upon. Piaget would proceed to add equally little cubic farmhouses to the models. In one farm yard the houses where all arranged in a tidy row, on the other farmland the houses were all scattered about

Carpenter & Lewis; Piaget et al., and Steffe & Hirstein (as cited in Kospentaris, Spyrou, Lappas, 2011) argued that the fundamental concept to understand the concept of conservation of area is compensation and part-whole relation For instance, to calculate the area of the combination of two simple 2-dimensional shapes, the students have to be able determine what kind of shape used. In order to know what kind of shape used in the combination shape, the students should be able to know where the part to be cut is. So the students could find the area of each dimensional shape then add their area to get the area of the combination of two simple 2-dimensional shapes. Sometime, the students could recompose the combination shape into rectangular shape to simplify the process of counting the area. Both of the strategy, dividing the shape into two and adding both of the area or recomposing the shape into rectangular shape are very important to be mastered by the students.

Recomposing shapes
In the definition of Oxford Dictionary, recompose means compose again in differently. In addition, recompose in this study means rearrange a shape into a new form of the shape with preserving its area. So the new form of the shape was made from its own parts, no parts are wasted or added.

Lehrer (in Clement & Stephan, 2001) said that the students used to have difficulty accepting than when they cut a given region and recompose its parts to form another shape, the area remains the same. Moreover, Douady&Perrin (1986 as cited in Kordaki & Potari 1998) argued the students tend to focus only on the dominant

dimension (perceptual justifications) of the shape or to compare the areas of the shapes by their perimeters

Therefore, this study will give the students chance to explore and to discuss the consequence of recomposing a shape by cutting and pasting strategy in order to preserve its area.

What grades of students should be taught?

Piaget (1960 as cited in Taluomis, 1975) stated the students' construction for understanding the concept of conservation and measurement of area develops gradually through stages and the concept of conservation precedes the measurement. The concept of area measurement usually stressed in grades 3-5 (Clement & Stephan, 2001). However, Clement and Stephan [2001] argued that there are some less formal aspects of area measurement that can be introduced in earlier grades. Also, Clement and Stephan (2001 as cited in Haris, 2011) included the concept of conservation of area belongs to five basic ideas for understanding the area measurement for the early age students.

Therefore, to make the concept of conservation of area become deeper understood by the students, it should be taught shortly before the students learn the area measurement. So the student has been able to catch the notion of area but has not yet disturbing by the knowledge of the formula area and the use of grid paper. The students on age 9-10 years old are appropriate to learn this concept since they have been categorized in concrete operational stages which based on Piaget these students could master most of conservation task and begin to understand the reversibility ("Piaget's Stages of Cognitive Development", n.d)

How to teach?

The aim of this study was to create a learning sequence of instructional activities based on the Indonesian version of Realistic Mathematics Education, PMRI (Pendidikan Matematika Realistik Indonesia) for the third grade students in Indonesia to build students' understanding that recomposing a shape will preserve its area in order to understand the concept of conservation of area.

Indonesian version of Realistic Mathematics Education (PMRI)

The innovative design of the learning sequence on the concept of conservation of area will put more attention into the students learning process, not on the students' result. The learning sequence will be learned by the students through the three principles of PMRI. This study used the context of making table-cloth from cloth-rag and the rice field in Indonesia in two learning sequences (making table cloth and comparing rice field). The context of cloth-rag and rice field has been familiar with students in Indonesia. Moreover, this context will give the students an opportunity to find and develop the cut and paste strategy by themselves. This strategy then will be used as a model for bridging the informal mathematical activities, cut and paste, into the formal mathematical knowledge, the notion of conservation of area.

In order to develop the learning sequence based on PMRI's principles, the instructional activities should be derived from the five tenets on PMRI which are defined by Treffers (1981 as cited in Zulkardi, 2002]. The following description explains how these tenets are adopted in this study.

1. Phenomenological exploration
   A concrete meaningful context, 'making table cloth from the cloth-rag", is elaborated as the starting point for the lesson sequence in this study to bring the students to use the strategy cut and paste for making the table cloth.

2. Using models and symbols for progressive mathematization
   The way to determine which part should be cut and where to place will be used as a model to facilitate the students' progressive mathematization, from the intuitive into more formal mathematical concepts.

3. Using students' own constructions and productions
   The students will produce their own way when cutting and pasting the shape as the meaningful strategy for them.

4. Interactivity
   There will be a class discussion and working group in which students could communicate, compare and justify their ideas with each other.

5. Intertwinement
   The design of this study will connect the notion of conservation of area with the concept of area measurement.

METHOD

Design research
The research approach that used in this study is design research. Design research is an approach that envisions a tighter, more rigorous connection between learning principles and features of the educational innovation (Walker, 2006). The purpose of design research is to improve the mathematics education. In other word, design research is an approach for discovering ways to develop a design, such as learning activity, based on theories and to determine the effectiveness of this design in practice. Hence, the design of sequence activities of this study will help to improve the mathematics education toward the concept of conservation of area in the third grade of students in Indonesia.

Hypothetical Learning Trajectories (HLT)
According to Clement & Sarama (2004 as cited in Daro, Mosher & Corcoran, 2011) learning trajectories is a description of children's thinking and learning in a specific mathematical domain include the relate conjectured through a set of instructional tasks designed to raise the mental processes and to move children levels of thinking. Furthermore, the hypothetical learning trajectories is made up of three components which are the learning goals that defines the direction, the learning activities and the prediction of students' thinking and learning process for understanding the context of the learning activities (Simon, 1995 as cited in Daro, Mosher & Corcoran, 2011 p. 136)

The HLT on this study was used to predict students' response so the teacher could anticipate what he should do. Also, the HLT was made to support the formulation of the learning goals, the conjectured about the learning process that anticipates how students' thinking and understanding might evolve when the learning sequences are employed in the classroom.

Research subject

The learning sequence conducted in to third grade students of SD Al-Hikmah Surabaya. Teaching experiment in this study divided into two cycle, cycle 1 and cycle 2. The cycle 1 used 6 random students with various level of academic achievement of class 3C. The teacher in the cycle 1 is the researcher herself in order to evaluate the implementation of the HLT in the students. The revised HLT based on cycle 1' result will be used for conducting cycle 2 with all of the students in the class 3A by their own mathematics teacher, Bu Lila. This study will provide a lesson plan for the teacher as the combination of the HLT and teacher guide of the learning sequence.

Data Collection

Since this study will put more attention into the students learning process, not on the students' result, so observation using video recorded was the best way to see the students' learning process. The written work and interview of the students could also support the data collection.

Instrument

Actually, there are five activities which purposively made to learn the concept of conservation of area, but in this study we will present the first and the second activities, making table cloth and comparing rice field. These two activities were arranged such that an activity can support the following activity. Therefore, the students should pass the first activity, making table cloth, before learn the comparing rice field.

The first activities, making table cloth consisted of worksheet individual, worksheet for group and exercise. First the students should imagine the possibility to make a given cloth-rag to be table cloth just by looking the Figure. Then they would be given the model of the cloth-rag and table to be tried on (hands-on activity). Later the students will practice their understanding from the worksheet into the exercise.

The second activity, comparing rice field consisted of three problems. The first problem asked the students to compare two rice fields (Pak Darma and Pak Badrun's rice field) that have different shape. The area of the two rice field was same. The second problem asked the students to determine which one is the biggest between the rice fields of Pak Darma, Pak Badrun and Pak Salman. This is designed to make the students used the fact that the area of the rice field of Pak Darma and Pak Badrun were same, so they just need to compare either the rice field of Pak Darma or Pak Badrun with the rice field of Pak Salman. The third problem explored the students' creativity to make another shape of rice field that have the same area with the rice field of Pak Salman.

MAIN SECTION

Retrospective Analysis on the cycle 1

In the making table cloth, the students still used their perceptual justification to determine which one is bigger between the cloth-rag and the table. They focused only into the dominant dimension. For example, some students thought that the cloth-rag and the table do not have the same area since the cloth-rag is longer than the table.

Also, the question in the first activities could not support the students to think about the possibility of two different shapes have the same area. Since the question, *"Which*

one is bigger the cloth-rag or the table?" will force the students to choose either cloth-rag or table. So, if they answered the question, it should be either the cloth-rag or the table. The students on that age, 9-10 years old, would not think another possibilities answer for that kind of question like "*none of them are bigger than each other*". This kind of question needs to be revised to bring out the students' thinking about the concept of conservation of area. The solution is by giving additional option on the question that can lead the students to think that it is possible that two shapes have the same area although the form are different. For example, *"Which one is bigger between the cloth rag and the table? Is the cloth-rag bigger than the table? Is the table bigger than the cloth-rag? Or none of the cloth-rag and the table is bigger than each other?"*

Furthermore, in the making table cloth activities, the students looked struggle to understand the concept of reversibility because based on Piaget concrete operational stage; students at this stage (9-10 years old) have begun to understand the concept of reversibility. The students still could not understand that the area of the cloth-rag before and after cutting will be the same. Some students still thought that cutting the shape will change the size of the shape, so the area of the two shapes would be also different.

Hence, to solve this problem the teacher was helped by two identical papers A4. One of the papers A4 was cut in pieces and was arranged to form different shape. The teacher used the fact of two identical papers A4 to push the students think that before and after cutting, the shapes still have the same area (reversibility). Assume that one paper A4 need 100 ml watercolor to color it. As the two papers A4 are identical, so each of the paper A4 needs100 ml watercolor. When one of the papers A4 is cut into three parts, the students were asked, "*How many ml watercolor needs to color this paper?*" The students knew that to color the paper they still need 100 ml. They answered it confidently. Although when the teacher asked this question for several time, the students still kept their answer and argued that the papers A4 that has been cut have the same space like before just different in form.

Thus, it shows that the intertwined between geometry with numbers helped students to understand the concept of conservation of area. However, sometime students need to learn geometry independently. Therefore, in the next cycle, we recommended the above guidance as the last alternative guidance when the students still could not understand that recomposing a shape will preserve its area.

Furthermore, in the first cycle, the students got difficulties to understand the question of the third problem. They could not grasp the intention of the question. The question actually asked to change the shape of Pak Salman' rice field into different form with the same area. The students cut the given model of Pak Salman' rice field but they tried to make it into the original. But in the class discussion, when the part that have been cut was moved into another side of the remaining part, one group has been able to think that that will be the same with the Pak Salman' rice field just the form is different.

It was the start of students' understanding that recompose a shape will preserve the area of the shape in the cycle 1. After the students understood the concept, it was easier for them to apprehend the other concept in the following activities. This fact is

very important to be considered in the cycle 2. The faster the students grasp the concept recomposing, the better understanding they will get.

Retrospective Analysis of Cycle 2
In the cycle 2, the student did not have difficulties to solve the problem of making table cloth from the cloth-rag. Some students have been aware that it was possible to cover the table with the long cloth rag by cutting the cloth-rag. But there were some students which think it was impossible to make cloth table from cloth rag which size is twice longer but a half less wide from the table. However, after the students were given the model to manipulated, all of them cut the cloth rag into two equal parts and combined it to cover the table.

In order to make the students get the idea that recomposing shape will preserve the area earlier, the teacher gave such a short discussion about the concept of reversibility (see figure 1) like the following transcript.
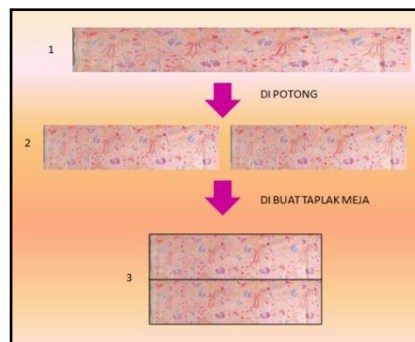

Figure 1. The concept of reversibility for the class discussion

Teacher   : Look that the cloth-rag in number 1 was cut into two equal parts in number 2. Then the parts in the right and left combined to fit table like the Figure number 3. Now the question, does the cloth rag in number 1 have the   same area with the sewed cloth-rag in number 3?
Students   : Same…eh not same
                Some students were talking each other about their difference answer.
Teacher   : Okay, listen. Did the cloth-rag in number 1 fit precisely to cover the table- cloth in number 3?
Students   : Fit
Teacher   : Is there any waste parts of the cloth-rag?
Students   : No
Teacher   : Does the cloth rag is less to cover the table?
Students   : No
Teacher   : Now, my question, does the area of cloth rag in number 1 same with
the   table cloth in number 3?
Students   : Not….eh same.
         The teacher repeated this question twice to stress the students' understanding.
Teacher   : Does the form of the cloth rag in number 1 same with the table cloth in number 3?

Students   : Same….eh not. It is different.

The transcript shows that the students were trying to grasp the idea in the discussion. They used to suddenly change their own answer (see the transcript).

Therefore, the teacher repeated the question in order to make the students keep thinking on the problem. But, in the end of this discussion the students had understood that the cloth-rag in number 1 was same with the table cloth in number 3. The role of the teacher in this discussion was very important to help the students understand that recomposing preserve the area because there is no part that wasted or added.

In the comparing rice field activity, Eggy and Ikmal, the students in the focus group could show the success of the recomposing concept as a tool to think, not just as a tool to prove. They have been able to do recomposing concept just in their mind. They used the concept of recomposing as the argumentation in their discussion. They argued the way to recompose the rice field of Pak Badrun to fit with Pak Darma's without using the cutting the model of the rice field. They just put the model of rice field in top of each other and analyzed which parts should be cut and where to place by using their logical thinking on the length and the width of the parts.

Furthermore, the student could solve all the problems in the comparing rice field activity well. In order to help the students understand the meaning of the third problem in the comparing rice field activities, we gave such an inter-perception, introducing a perception for the students about the shapes which are similar and different. After the students understand how does two difference shapes, they knew how to answer the problems. All of the groups reformed the rice field of Pak Salman into another different form. Also, the students have known that the original and the reformed rice field of Pak Salman have the same area since no parts are wasted or added.

Discussion
The design of instructional activities in this study has been able to answer the research question, how the students could develop the understanding that recomposing a shape will preserve the area of the shape. The two activities, making table cloth and comparing rice field provide a chance for students to support their understanding about the concept of recomposing. The activity to make a table cloth from a cloth-rag gave a reason for the students to cut and paste the cloth-rag to fit the table. When the cloth-rag recomposed to fit the table, the students learned that recomposing shape will preserve its area since no parts are wasted or added. Then in the second activity, comparing rice field, the students not only strengthened their understanding that recomposing shape preserved the area but also developed their reasoning skills in geometry when they were thinking which parts should be cut and where to place.

The problem that asked the students to reform the Pak Salman's rice field into another different from which has the same area actually was the notion of the concept of recomposing. But it will be difficult if we just directly give the students that problem since the students at the age 9-10 years old still struggling to understand the concept reversibility. The students need an introduction activity to bring their understanding there.

Hence, from the result of this study, we knew that making table-cloth and comparing rice field activities are good introduction activities to support the students understanding that recomposing a shape will preserve its area.

The context of making table cloth and rice field was very helpful and meaningful for the students. It shows that the RME approach has been successfully used to support the students' understanding on the concept of conservation of area.

Furthermore, most of the prediction in the HLT occurred in both in cycle 1 and cycle 2. For example, some students have thought to cut the cloth-rag in order to fit the table. The way to cut the cloth-rag also was accordance with the prediction in HLT, like cutting the cloth-rag into two equal parts. The prediction about students' difficulties to determine which to cut and where to place revealed in many students. Not only for the low achiever, but also the high achiever got difficulties on it in some cases.

However, there were some conjectures in HLT which did not occur in the implementation. For instance, the fact that the cloth-rag fit with the table in the first activity still could not reveal the students' understanding that recomposing shape preserve its area without the help of the teacher. The students still need more guidance to understand the concept of reversibility. Somehow, these students still needs more practice to develop their skills to recompose an irregular polygon into rectangular shapes.

CONCLUSION

The most important result is the instructional learning activities of this study could build the students' understanding that recomposing a shape will preserve its area. From the activity making table cloth, the students could understand that recomposing the shape of cloth-rag without waste any part of it will preserve the area of the shape. Although they were not introduced by the concept of area, the students could comprehend the preservation of area. The question, *"Which one is bigger between the cloth rag and the table? Is the cloth-rag bigger than the table? Is the table bigger than the cloth-rag? Or none of the cloth-rag and the table are bigger than each other?"* had an important role to lead the students to understand the concept of conservation of area

The second important result is that the students' skill to recompose a shape in order to preserve the area needed to be developed since some students still got difficulties to find which parts should cut and where to place.

The role of the teacher was being decisive to lead the students to understand the concept, also to lead the discussion on the right track. The ability to manage the class also needs to support the convenience learning process in the classroom. The social norms need to control the class. An ice breaking could refresh the students when they get bored with the learning process.

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