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SUPPORTING STUDENTS' FIRST CONCEPTION ABOUT ADDITION OF INTEGERS THROUGH NUMBER LINE ACTIVITIES FOR THIRD GRADE PRIMARY SCHOOL

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Abstract

Students' first conception about negative numbers should be supported by a meaningful way of learning experienced by the students themselves. In Indonesia, the notion of negative numbers sometimes complicated because not every common contexts experienced by students in Indonesia. However, there are still some contexts can be used in understanding this concepts. Students may grasp the idea of negative numbers if it is learned through a familiar context to stimulate their common sense and intuition. Mostly in Indonesia, negative numbers is taught in formal way. This study aimed at investigating the students' understanding about the notion of negative number through number line activities within the context of a 'dice' game. Design research was chosen to be the method to accomplish the goal of the study. Realistic Mathematics Education is used in designing the contexts and series of activities. Our findings are that the students can understand the concept of negative numbers through the number line activities and can relate the context to more formal way of learning negative numbers.

Keywords: *negative numbers, number line, dice game*

INTRODUCTION

During school life, students will experience the process of expanding their mathematical knowledge to include negative numbers. The students are challenged to use their existing concepts about natural numbers to make sense of negative numbers. The students may accept that negative numbers as the inverse of positive numbers. However, this idea is insufficient. There are many ideas about the notion of negative numbers which should be discussed with students. Many students are confused about different types of numbers and do not understand that all different types of numbers are part of the system of real numbers (Bruno, et. al.,1999). Therefore, the notion of negative numbers must be well-understood to make children prepare to learn and understand the bigger numerical system.

Children aged nine to ten years old should know and understand the reason why they learn negative numbers. They may find the notion of negative numbers surround them and know that there is another type of numbers. They try to interpret negative numbers by considering what they already know about natural numbers. Their previous knowledge about natural numbers can help as well as confuse them because they must become aware that the sign in the numbers gives meaning to it. Children

wonder why 5 is not equal with -5. They should be able to reason and describe this understanding by themselves before they experience more complicated situations relating to negative numbers.

Some contexts in learning negative numbers had already been tried and attempted by researchers such as the context of the temperature, abacus and the dice (Linchevski and Williams, 1999), assets and debts (Stephan and Akyuz, 2012), and the use of the number line to represent the operation in negative numbers (Heefer, 2011). They use contexts to give meaning to negative numbers that students need to understand. Context must be carefully chosen by considering the local culture. At some points, some of the contexts mentioned above could help the learner to better understand negative numbers. However, some of those are addressed for higher grades, and they contain some abstract calculation which can be complicated for children aged 9-10 years old and probably it is not replicable for lower grade. Also, the students need to know why negative numbers appear in their life. They have to experience by themselves that negative numbers appear in their life in a particular situation. Therefore, there is a need to put them in that kind of situation.

Negative numbers are the numbers included in numerical system that should be learned and understood by the students to expand their numerical knowledge. The students' first conception about negative numbers may appear by doing subtraction of unsigned numbers result as signed numbers. This extension of number domain could be the indication of the acceptance of negative difference (Kilhamn, pg. 199). This extension of numbers could probably be introduced to students with the use of a number line by extending the positive number line to the left showing the representation of negative numbers. Supporting the idea about number line, other researcher have describe number line as a representation of numbers on a straight line where points represent integers and the distance between points matches the arithmetical difference between the corresponding numbers. (Heffer, 2011). The number line is widely accepted as a model to learn the operation involving integers.

The approach in designing the sequence of learning activities is Realistic Mathematics Education (RME). The key idea of RME is mathematics as human activity (Freudenthal, 1973). Freudenthal revealed that the process of doing mathematics is more valuable than mathematics as a ready-made product (Gravemeijer and Terwel, 2000, pg.780). Through the learning process, students are guided to reinvent mathematics by organizing and interpreting realistic context. (Gravemeijer, 1994; Stephan, 2012,pg. 433). This process will help the students to reason the mathematical aspect within the context. Later, they also realize the usefulness of mathematics concept that they can explain why they should learn mathematics. In RME, modeling as an activity is further elaborated in a didactical sense meaning that students reorganizing the subject matter with the emergence of informal ways of modeling and symbolizing. (Gravemeijer, 2002, pg.2). The design will emphasize the context of the money and dice game using number line model that students can organize their thinking through this context because the case of negative numbers is problematic because this extension of numbers involving formal mathematical operation, and because they are 'symbolically written' (Freudenthal, 1999, pg. 432).

By considering this issue, in the study I intend to investigate students' understanding about negative numbers through number line activities. The present study aims at developing the learning activities on negative numbers with the use of games which

can improve students' conceptual understanding on negative numbers. Design research was chosen to be the method to accomplish the goal of the study. The research approach to use in implementing these learning activities is Realistic Mathematics Education (RME) with the aim that students can improve their understanding with support and guidance from the teacher in the learning process. I pose the research question: *How can number line activities support students' understanding about addition of negative numbers?*

In Indonesian curriculum, the topic of negative numbers included in the chapter of integers are taught in the second semester, in fourth grade (based on the interview with the mathematics teacher, it was not possible for me to conduct this study in fourth grade because they already learned integers before, therefore I tried out the design in third grade classroom). Based on the mathematics textbooks used in the classroom (BSE), there are four main topics students should understand about negative numbers. They are (1) Introduction to integers; (2) addition of integers; (3) subtraction of integers; (4) mixed operation (addition and subtraction) of integers. However, in this study, I intend to make the learning less formal because at some points, for examples subtraction of negative numbers, which involve two minus sign, (one for operation; one for numbers sign) are presented formally in the mathematical textbook. Because that concept has no relation to the context of my study, it will be difficult to understand by the students. I limit my study not to elaborate the rules of sign in subtraction of two negative numbers. Therefore, my expectation is that, with the help of the context using number line activities, students can understand about addition negative numbers by describing it within the context.

Research Methodology

a. Preliminary Design

The first phase in conducting this study is to formulate the Hypothetical Learning Trajectory (HLT) which consists of learning goals of instructional activities, planned instructional activities, and the conjectures on how the students' possible responses when the instructional activities used in the classroom. The HLT will be developed during the pilot experiment and teaching experiment. The HLT will be tested in pilot experiment. The improvement of HLT will be prepared and implemented in the teaching experiment.

b. Pilot Experiment and Teaching experiment

In pilot experiment, the researcher conducted a sequence of learning to small group of students (6 students from SD IT Ghilmani Surabaya). The researcher compares what happened during the classroom to the expectation of the teaching and learning process in Hypothetical Learning Trajectory. The purpose of analyzing the data of the whole lesson is to test the conjectures of students' responses to the actual learning process. The transcript of the selected fragments will be used to test the conjectures of the HLT. The researcher will use the analyzed data to revise the original HLT for the teaching classroom experiment. In experimenting the second cycle, the participant is different from those in the previous cycle. In this phase, the researcher tests the HLT which has been designed in the preparation phase. The purpose of doing the cycle is to test the HLT and the conceptual understanding of the students about negative numbers. The participation of this experiment is 19 students of the

third grade of SD IT Ghilmani Surabaya. During the experiment, the researcher pays attention to one group of students as the focus of the observation. The focus group consists of the students which have been selected by the teacher.

c. Research Subjects
The subjects of this study was 19 students of the third grade of SD IT Ghilmani in Surabaya. They are all girls and from different level (low, middle, and high level students).

MAIN SECTION

Number line is known as a great model to represent number counting and as a powerful tool to do calculation of numbers. With this characteristic, number line is useful to introduce the children about the early concept of calculating numbers starting with easy number for later they may understand how to use it for greater numbers.

Overview of the activities

The first number line activity involves the money context following the previous activity about money accepted and paid by the students in a game. The goal of this activity is to introduce number line as the representation of numbers and as a tool for calculating numbers.
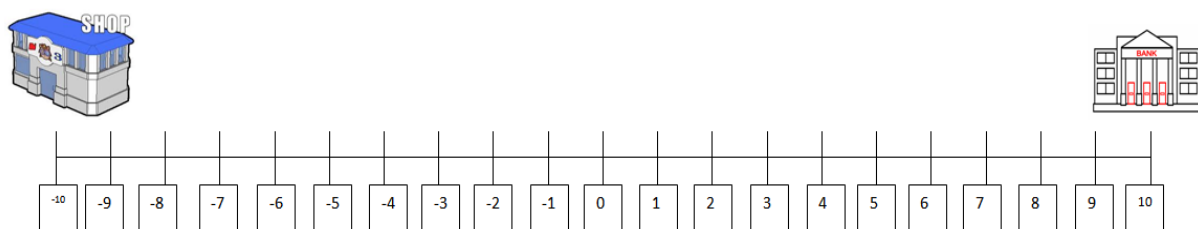
Figure 1. The number line used in the activity

The students were challenged to represent the amount of money to the numbers according to picture shown in on the number line. There are two kinds of pictures on the number line, the picture of 'a bank' show that the students should move to the right if they accept money from the bank. The Picture of 'a shop' shows that the students should move to the left to represent the amount of money they have to pay to the shop. This activity helped the students to give the meaning to negative number and encourage them to use the number line as the model to operate the number. Although in this activity, they will not count big numbers. Instead, they are given some 'dollar' money, which is small in amount and they make jumps based on the amount of the money. In this activity, probably it is the first time for the students to use a number line as calculating tool. Therefore, the teacher could explain briefly to the students why they should use a number line and what is the usefulness of using number line to calculate number and understand the problem of the context.

The second number line activity (see figure 1) is the activity with the game context called 'the dice game'. In this game, students will be given two dices, red and blue

dice. They have to move along the number line given from the start. The number on the blue dice is the number of moves to the right. The number on the red dice is the number of moves to the left. The learning goal of this activity is to introduce the students about the operation of integers. The teacher may explain to students that through this game, the students will learn about addition of two integers. The numbers involved in this operation are the numbers on the red dice and the number on the blue dice. One important thing that teacher should tell to the students is that the number on the red dice, which shows the number of moves to negative direction should be written by the students as negative numbers (signed with negative sign in front of the numbers). Through this game, students will not directly do the addition of numbers, but what they really do in this game is that they record each moves they take, and in the end they can see the pattern of the final position after right and left moves and probably could find out that the final position is the result of the addition between two moves. It is also important to notice that in this game that in each turn the players should always back to starting point.
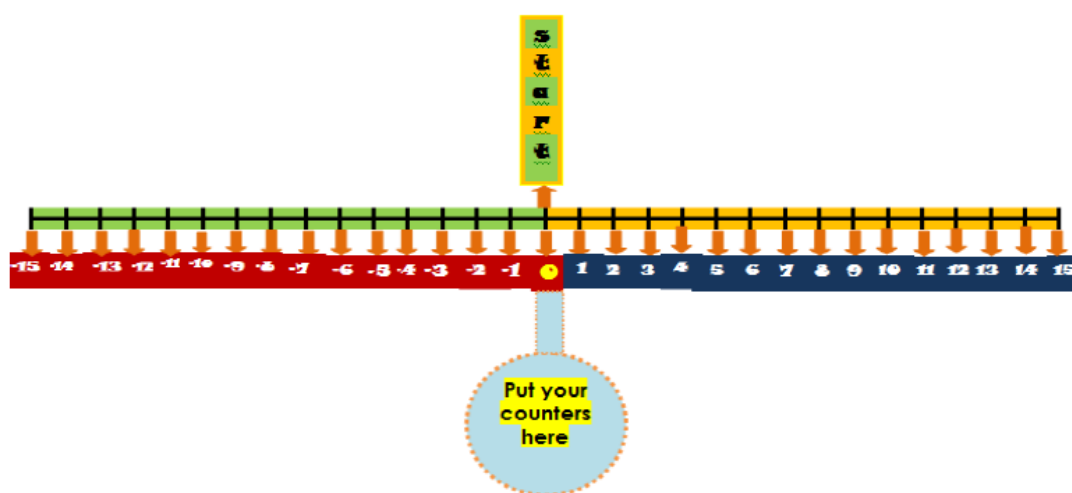

Figure 2. The number line model used as a context as well as the model for thinking

The last number line activity discussed in this paper is the visualization of the dice game in the form of worksheet. In the worksheet, there are some problems relating to the game played in the previous activity. The challenge is that the students must operate bigger numbers and start using the number line as calculating tool. Moreover, the number line given in the worksheet is the empty number line that students should really understand how to put those numbers in the right position. Also, in this activity, students will also be challenged to write numbers of the operation in more formal way. These three activities will be done by the students in group.

Hypothetical Learning Trajectory

In each activity, students are expected to reach the goal of learning and experience a meaningful lesson and can be used for an advance learning of negative numbers. In the first number line activity which involves the money context, students may experience some of these conjectures: (1) The students will read the problem, for example "I have $6, I want to buy a school bag which costs

$8. The students know that 6 is the first position on the number line, because they have to pay 8, so they make 8 jumps to the left. The students realize that this situation will give them a negative numbers, which is -2 as a result. Some students also know that they have to subtract a bigger number from a smaller number. (2) Students can interpret numbers on the problem correctly and put the numbers in the correct position on the number line (3) Students are able to explain the meaning of the numbers (How they say the number if it is debt or money). In the second activities, some possible responses from the students were also predicted: (1) Students are able to make correct movements based on the number on the dice and reach the final position (2) Students can predict the right final position without making jumps on the number line (see the numbers in more formal way). In the third activity, the expectation of students reactions are: (1) Students can make jumps on the empty number line (2) With the help of the number line, students can add the integers (3) Students can write down the number in more formal way (students understand that the combination of two movements is the addition of two integers).

Result and Analysis

In each activity, students showed various responses. Some of them seemed understood the problems, some others did not. We will explain the some of the written work and the discussion among the students. From the first activity, students struggled to work with the number line. Before doing this activity, they did not know anything about negative numbers and never worked with a number line. Therefore, they tried hard to find the meaning of problem and figured out the position in which they should put numbers on the number line. The money context was given with the aim that students could understand it easily because they can relate to their daily activity and can recall some information they got from the previous activity. See the excerpt of a student' discussion with the researcher below:

Student 1   : "Having debt of 4??"
Researcher : "What does it mean to you? Where should you start?"
Student 1   : "4?" (pointing on the number line)
Researcher : "Are you sure? How do we write debt from the previous activity?"
Student 1   : "oooh.. minus 4 (pointing -4), and then get money 7. 1..2..3..4…5…6..7 (make jumps to the right) . I stop at 3."
Researcher : "and what is 3?"
Student 1   : "I have 3"
Researcher :" money or debt?"
Student 1   : "I have money 3"

The learning goal of this activity is that students can give meaning to integers based on the context. The context will help them to understand the notion of negative numbers more easily. We can see that from the dialogue between the student and the teacher, the activity help the students to imagine the situation in which the integers involved.

The excerpt below is the discussion between two students in second number line activity who confused about which move should be done first.

Students 1: "This one first, right?" (pointing number 2 on the red dice)

Students 2: "Hmm..we have to move to right or left at first?" (asking question to
　　　　　the teacher)

Teacher　　: "What do you think?"

Student 1 : " We have to move to the left and then to the right. So, we will stop
　　　　　on number 1"

Student 2 : "okay, what is the number on the red dice? We agree that we have
　　　　　to move to minus and then to the left."

Student1 : "Yes, I know. We stop on number 1."

Student 2 : "Hmm..1..2..3, 1..2.. I think the same. We can move to the left then
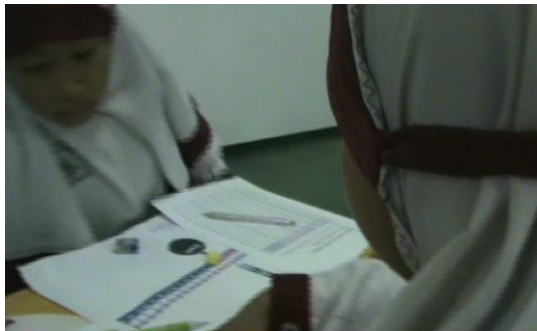　　　　　right or to the right and left."

Figure 3. Two students discussed about the number line activity

We see in the discussion that students had confusion about the moves. One
student realized that it did not matter whether they had to move to the left
then right or vice versa because in the end they will go to the same number.
This thinking will guide them to the understanding about the operation, in this
case addition of two integers. Although in this activity, they only wrote
numbers, but the purpose of the activity is that they will conclude something
about this activity relating to the operation of two integers. The other
responses also shown from these excerpt:

Researcher : "Whose turn?"

Student 1 : "My turn" (toss the dice)

Researcher : " So, what do you get?"

Student 1 　: "The red dice is 5, the blue one is 5(while writing on the
worksheet), so the final position is 0 (answer convincingly)

Researcher : "Wow, how do you get the answer so quickly?"

Student 2 　:"Yes, you have to check it"

Student 1 　: "Okay, 1..2..3..4..5 (pointing on the number line and change the
move) 1..2..3..4..5. yes it's 0."

The conversation above shows a brilliant response of a student who already
can predict the final position after two movements. It is not clear if she really
understand that the meaning of the numbers operation in the context.
However, she must know something about it because she is really convinced
about her answer. From third activity, we observed a written work from one
group of students. From this work, we can see that students can answer the
problem correctly. However, it seemed that the number lines provided on the
worksheet were not used by the students to help them calculate numbers. One
student admitted that they did not use the number line because the number
line was too small and it was difficult for them to draw on it. Probably, it is

better for the improvement for the design to make the number line quite bigger that student may use them freely as a tool for calculation. From the written work, what students did that they put the number of the red dice in the first box and put the number of the blue dice in the second box. We expect that they understand the meaning of the plus sign linking those numbers and interpret that as an addition of two numbers.
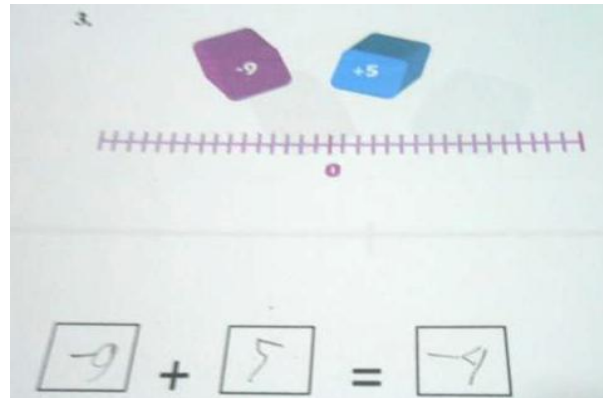

Figure 4. Students' work in more formal problem

The other students work is in figure 4, as shown in the picture that students were challenge to put the number which was already written on the worksheet on the appropriate dice. Here, students should understand which number should be put in the red dice or in the blue dice. The ability of students to put the correct number on the correct dice could indicate that they already understand how to translate the formal way of writing the numbers to the previous context, which was about the dice game. Another thing can be observed from the written work is the way students used the number line. It was different from the previous task (see Figure 3) that students did not really use the number line, given this empty number line, students feel that the number line is big enough and they feel free to write numbers and draw jumps on that. However, instead of estimating the position of numbers which are used in the problem, the students write all the numbers. There is some reason why they did that way, the first reason is those students probably feel that they need the exact position of each numbers and the second reason is that because by writing all numbers on the number line made them easier to make jumps and count the numbers.
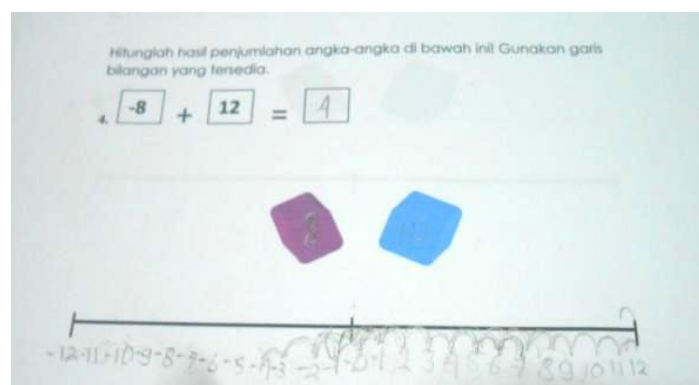

Figure 5. Students used a number line as a tool for calculation

CONCLUSION

The sequence of learning consists of some meaningful number line activities can help students to understand and give meaning to negative numbers. The abstractness of negative numbers could also be bridged by giving a context in which the students can explain and imagine. Also, this study is aiming to promote number line as a powerful model to teach students about negative numbers. This can be seen that the introducing to number line can be done in such a way students can enjoy the learning. We can conclude that the sequence of lesson can support the students in understanding the notion of negative numbers because the student can explain something about negative numbers and can relate the concept to the context given in the lesson. Some written work of the students also tells us how they struggled to used number line and in the end they could achieve the understanding even in more formal level. However, this study is not perfect yet. We want to give some suggestion to some researchers who want to conduct further studies about learning negative number to develop some context which familiar to students to make the learning less formal. Also, we recommend Realistic Mathematics Education (RME) as the basic approach in teaching and learning process especially for topic negative numbers which has been considered as difficult concept to learn for students in primary school.

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