ISBN : 978-602-17465-1-6

Code: P-48

EDUCATIONAL DESIGN RESEARCH: DEVELOPING STUDENTS' UNDERSTANDING OF AREA AS THE NUMBER OF MEASUREMENT UNITS COVERING A SURFACE

Susilahudin Putrawangsa[1], Agung Lukito[2], Siti M Amin[3], and Monica Wijers[4]
STKIP Hamzanwadi Selong Lombok[1], Surabaya State University[2], Surabaya State University[3], Utrecht University the Netherlands[4]
sis.putrawangsa@yahoo.com[1], gung_lukito@yahoo.co.id[2], amin3105@yahoo.com[3], m.wijers@uu.nl[4]

Abstract

One of the foundational students' misconceptions of area measurement is conceiving area as the length of a line, instead the size of a surface. They do not see area as the number of measurement units covering a surface. Therefore, this study intends to develop learning activities and materials that support students in developing their understanding of area as the number of measurement units covering a surface.

This research is conducted based on the view of the design research approach consisting of two cycles. The subjects of the study are taken from the third grade students of two primary schools in Surabaya. Meanwhile, the learning activities in this study are based on the view of PMRI (Indonesia realistic mathematics education).

We found that student need to understand physically quantity of area and the measurement unit of area before they are introduced to the meaning of area as the number of measurement units covering a surface. Learning activity that provides students with the experience of covering activity and comparing area problems could lead to the discussion on the physically quantity of area, the measurement unit of area, and its properties (no gap, no overlapping, and unit consistency); meanwhile, learning activity that engaging students in comparing area of tiled floors can be used to develop students' understanding of measuring area as the process of finding the number of measurement units.

Keywords: *Area, area measurement, the unit of area measurement, design research*

INTRODUCTION

Area is defined as the number of measurement units needed to cover a region; and measuring area refers to a process of finding the number of measurement units contained within a boundary (Fauzan, 2002; Clement and Sarama, 2009). Meanwhile, Baturo and Nason (1996) propose the idea of the physical quantity of area. They equate area with an amount of region (surface) that is enclosed within a boundary and the notion that this amount of region can be quantified.

Measuring area is one of the most commonly utilized forms of measurement that is closely associated with real world applications, science, and technology (Hirstein, Lamb, & Osborne, 1978; Martin & Strutchens, 2000). Children at an early age need to comprehend this skill to help them to see the usefulness of mathematics in everyday life (Reys et al 2007). However, research in the field of mathematics education often reveals a problem that students have a poor understanding of the processes used for area measurement (Baturo & Nason, 1996; Simon, 1995; Tierney, Boyd, & Davis, 1990). The emphasis placed on the use of area formula (length x width) mainly attributes to students' difficulties in measuring area (Zacharos, 2006; Bonotto, 2003; Fauzan, 2002). Students just recite the formula without understanding it (De Corte, Verschaffel, & Van Coillie, 1988; Nesher, 1992; Peled & Nesher, 1988; Simon & Blume, 1994). According Zacharos (2006) the formula leads students to conceiving area as the product of two lines (length and width). They could not see area as the size of a surface with in a boundary. Consequently, students tend to say that a surface that has not obvious length or width has no area. Moreover, they tend to compare perimeter when they are asked to compare area of two surfaces (Yuberta, 2011).

Considering the problems above, helping students to see area as the number of quantitative measurement units covering a surface is necessary. Therefore, this study aims to develop learning activities and materials that support students in developing their understanding of area as the number of area measurement units covering a surface. We then formulate the following research question: *"How can we develop students' understanding of area as the number of measurement units covering a surface?"*

This study is a part of our research project on developing students' understanding of the multiplication strategy in area measurement for the third grade students of Indonesian primary school. Design research approach is used in this study which is based on the three phases of conducting design research according to Koeno Gravemeijer and Paul Cobb (see van den Akker et al, 2006) such as: preparation phase, classroom experiments, and retrospective analyses.

There are two cycles in this study. The subjects of the study in each cycle are different. There are 6 students (the third grade) of Al Ghilmani Islamic primary school of Surabaya are involved in the first cycle and 21 students (the third grade) of Unesa Laboratory Primary School are involved in the second cycle.

The data are collected by observing student activity during the experiment (video recording), interviewing students (video recording), conducting pre and post test (written tests), collecting students' works (worksheets), and writing important findings during the observation (field note).

In analyzing the data, we start by looking at the whole data chronologically, such as watching the whole video observation. We then select some important data relating to our conjectures in the local instructional theory. We then analyze the data by comparing the actual learning trajectory and our hypothetical learning trajectory. We describe the actual learning trajectory and provide explanation on students' responses by building our assumptions and conjectures which are based on the related data.

MAIN SECTION

After revising and improving the learning trajectories and materials based on the findings of the first cycle (the pilot study), we then implement the revised learning trajectories and materials in the second cycle. The learning trajectories and the learning materials are implemented in two lessons as below:

 ISBN : 978-602-17465-1-6

Lesson 1
The purpose of this lesson is to support students to see the physically quantity of area. To reach the purpose, we design a learning activity as below:

(1) Using palms as the measurement units

In this problem, the students are asked to determine the number of palms needed to cover the surface of their own desks. In our hypothetical learning trajectory, the students will come up with different results of measurement (although those desks actually have the same size) and use the difference to discuss about the unit measurement of area and its consistency and the idea of gap and overlapping.

Figure 1. Students' different size of palms and ways in covering the identical desk

As our conjecture, the actual learning trajectory shows that the students came up with different result of measurement. The difference is quite large. The smallest number is 26 palms and the biggest number is 46 palms.

Teacher: Why do you have difference answers?
Who want to answer?
Rizki: Because the palms (their hands) are different.
Student: Because the sizes of the palms are different.
Teacher: Well, because the sizes of the palms are different.
What else?
Bila: Because the different ways in putting the fingers (palms) on the desk.
Teacher: Yes. Because the different ways of measuring.

The conversation shows that the students could see why they came up with different results of measurement. They knew that the differences are due to the different size of palms used to cover the desks and the different ways of doing measurement where some of them measure the desks by leaving gap between two palms and some others leaving overlap (see figure 1).

In this problem we expect that the teacher used this finding to discuss about unit measurement and its consistency and the idea of gap and overlapping. However, the teacher failed to lead the students to the discussion on those concepts. This is probably because of the teacher guide that does not properly inform the teacher how to orchestrate the discussion.

Hence, the covering activity in this problem potentially leads to the discussion on the measurement unit of area and its consistency and the idea of gap and overlapping on the unit. To support the teacher in conducting the discussion, a clear teacher guide telling how to orchestrate the discussion is necessary.

(2) Using note-books as the measurement units
In the next activity the students are engaged in using note-books as the measurement unit to measure the area of some surfaces. This problem is treated as the follow-up problem after the students discuss about the unit consistency, gap and overlapping.

Moreover, it intends to provide the students with the experience of using another measurement unit in measuring area to enrich their understanding of the measurement units of area. In actual learning trajectory, students show different ways of iterating the units (note-books) and different ways of counting the units.

In iterating the units, there are three different ways showed among the students. First, most of the students took a certain number of note-books and then transposed each note-book by continuously changing its position on the remainder of the whole surface without leaving gaps or overlap (see figure 2a). Second, some students used as many of the note-books as are required to cover the whole surface (see figure 2b). Third, some students including our focus group started by finding the number of note-books needed to cover the width and the length of the surface being measured and then used their understanding of the structure of units to determine the whole note-books needed to cover the whole surface (see figure 2c). These students seem to see the structure of the area units in columns and in rows, where in rectangular surfaces the number of units covering each length or each width is always the same.

(a) (b) (c)

Figure 2. Different ways of iterating measurement units

We assume that the differences occurred due to the position of the surfaces being measured. In the horizontal surfaces, such as teacher's table and students' desks, the students used as many of the note-books as are required to cover the whole surface. This position allows the students to put the note-books without holding them together (see figure 2b). Therefore, they could put as many of the note-books as are required to cover the whole surface. Meanwhile, In the vertical surfaces, such as the whiteboard, the door, and the announcement board, the students could not use as many of the note-books as are required to cover the whole surface because they have a limitation on the number of the note-books that are able to put and hold on the surfaces (see figure 2a and 2c). For example, if they are three students in a group, so they could hold six note-books maximally on the surfaces. This limitation leads the students to take a certain number of note-books and then transposed each note-book by continuously changing its position, without overlapping or leaving gaps, on the remainder of the whole surface. Our focus group measured the surface of classroom door and used this way in determining the number of the note-books needed to cover the door.

In counting the units, some counted the units one by one as they iterated the units one by one. Only one group which is our focus group counted the units by considering the structure of units in columns. They knew that in each column contains the same number of units and then by repeated addition they counted the whole units. They got that they need four note-books to cover each column of the surface being measured and there were ten rows. By repeated addition they added four note-book ten times as they move in each columns and got 40 note-books for all.

 ISBN : 978-602-17465-1-6

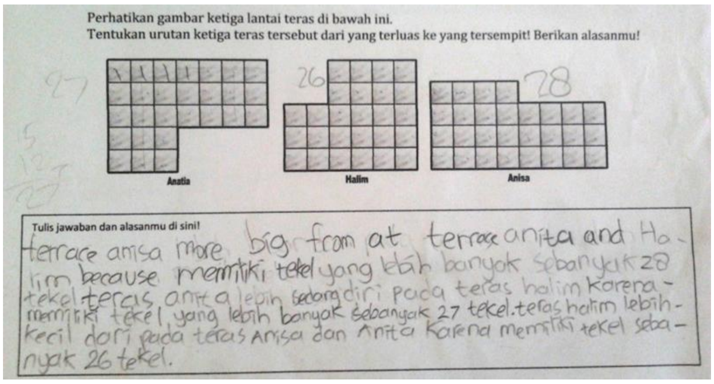
We conjecture that the way the students counted the units (note-books) is influenced by the way they iterate the units. Our conjecture is supported by the findings that the students who iterate the note-books column by columns counted the units column by column too. The students who iterated the units by considering the structure of the note-books in columns and rows counted the note-books by considering the structure of the note-books. Meanwhile, the students who iterated the note-books by taking as many of the note-books as are required to cover the whole surface counted the units without considering the units in columns or in rows.

In the end of the activity, the students could determine the larger and the smaller surfaces by considering the number of note-books needed to cover the surfaces. The students even knew that two surfaces have the same area if they have the same number of note-books. Here the students could see the physically quantity of area in terms of the number of units (note-books) needed to cover the area.

(3) Follow-up problem: Floor problem

In this problem students are given three tiled floors and they are asked to sort the floors based on the area of the floors. The purpose of the problem is to check students understanding of the physical quantity of area and the role of units in telling the area of a surface.

In general, almost all of the students sort the floors correctly. They considered the number of tiles covering the floors. They knew that the more tiles occupied by a floor, the bigger the floor is. Anas, Zaki, and Guinot, for example, answered that Anisa's floor is the largest since it has 28 squares (tiles), then Anita's floor since it has 27 squares, and the smallest is Halim's floor since it contains 26 squares. Here, they sort the area of the floors based on the number of the squares (tiles) occupied by the floors. They treated the squares as the measurement units. It seems that the students see the area as the number of units covering a surface. Let's see the following students' answer of our focus students.



Anisa's floor is bigger than Anita's and Halim's because it has more tiles which are 28 tiles. Anita's floor is bigger than Halim's because it has more tiles which are 27 tiles. Halim's floor is smaller than Anisa and Anita because it has fewer tiles which are 26 tiles.

Figure 3. Student's answer

The students' answer shows that they could sort the area of the floor from the largest to the smallest by considering the number of tiles. They treated the tiles as the measurement units. Here, the area as the physical quantity is conceived by the students. It seems that the previous problems, hand problem and note-book problem, help them to deal with this problem.

Hence, how can we support students to see the physically quantity of area?

 ISBN : 978-602-17465-1-6

Grounding to the findings above, we conclude that the learning activity that provides students with the experience of covering activity and comparing area problem could lead to the discussion on the physically quantity of area and also the discussion on the measurement unit of area and its properties, such as the idea of unit consistency, gap and overlapping.

Finding the area of students' desk using students' palms as the unit measurement in this lesson is one of examples of covering activity in measuring area as well as comparing area problems. In telling the area of the desks students will count the number of palms needed to cover the desks. Here, the physically quantity of area is emerged. This problem also leads to the discussion on unit consistency, gap and overlap since it leads the students come up with different result of measurement due inconsistency of measurement units (each student has different size of palm) and the existence of gap and overlapping units (students potentially cover the desk improperly).

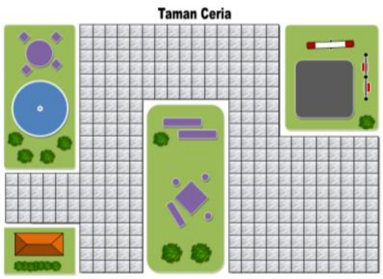
Lesson 2

The purpose of lesson 2 is to support students to develop their understanding of measuring area as finding the number of measurement units covering a surface. Here, the students are engaged in two activities as below:

(1) Comparing area problem

In this problem, the students are asked to determine the larger parking lot of two parking lots covered by tiles. As our hypothetical learning trajectory, we conjecture that will treat the tiles as the measurement units and count the tiles to obtain the area of the parking lots. To determine the larger parking lots, they will compare the number of tiles in both parking lots. The parking lot occupying more tiles is the larger parking lots.

(a)



(b)

Figure 4. Two parking lots (the tiled surfaces) which are being compared their area

Here is what happens in the whole classroom. We found that almost all of the students compare the area of the parking lots by comparing the number of tiles in each parking lot. They determine the area of the parking lots by counting the number of tiles covering the parking lots. In counting the tiles, almost all of the students used the multiplication strategy. Some of them counted the units one by one before they turned to use the multiplication strategy. In the parking lot of Taman Gembira for example, the students counted the tiles covering the width and the length of the parking lot which is 10 tiles and 33 tiles respectively. Then, they multiply those numbers (10 x 33) to obtain the whole tiles. They understood that the product of 10 x 33 refers to the number of tiles covering the parking lot. The use of the multiplication strategy is also occurred when they counted the tiles in the Ceria parking lot. Before

 ISBN : 978-602-17465-1-6

applying the strategy, they split the parking lot into some rectangular surfaces and then apply the strategy in counting the tiles covering each surface.

As well as the majority of the students, our focus students show the same strategy in solving the problem. Let's consider the following transcript between focus students and the teacher.

Teacher: Please explain your solution!
Widya: Well. The smaller (parking lot) is Taman Gembira since it has 330 tiles.
Teacher: Yes.
Widya: Here the way. It is from 10 times 33.
Where those (10 and 33) come from?
From here to the bottom, there are 10 (tiles).
[Pointing the tiles covering the width of the parking lot]
And here are 33 (tiles).
[Pointing the tiles covering the length of the parking lot]
And then we multiply them (10 and 33).
Teacher: Well. What 330 is?
Widya: The number of the tiles.
Teacher: Yes.
So, what about these?
[Pointing other calculations on student worksheet]
Widya: These are for Taman Ceria.
Teacher: Ooo…
Widya: Here is surface 1.
[pointing one of five surfaces. They split parking lot into some smaller rectangular surfaces]
In the surface 1, to the bottom (its width) are 6 (tiles) and to here (its length) are 16 (tiles). It is equal to 96 (tiles).
Teacher: Yes.
Another student, Mita, then pointed the number of tiles (334 tiles) covering the parking lot in Taman Ceria, and said:
Mita: So, the larger parking lot is the parking lot in Taman Ceria.
Teacher: Yes.
Meanwhile, Widya kept counting the tiles covering other surfaces using the multiplication strategy. In the end, she got 334 tiles covering the parking lot in Taman Ceria. In the student discussion, this group agreed that the parking lot in Taman Ceria is larger since it occupies more tiles.

The transcript shows that the idea of the multiplication strategy in counting the tiles is understandable by these students. They knew that the product of 10 times 33, which is 330, refers to the number of tiles covering the rectangular surface with 10 tiles as the width and 33 tiles as the length (see figure 5). We assume that the number of tiles which is relatively a big number for the students and the shape of the parking lot which is rectangular shape trigger the emergence of the multiplication strategy.

Their understanding of the multiplication strategy is even verified by their strategy in dealing with the second parking lot, Taman Ceria. They split the parking lot into some rectangular surfaces and then applied the multiplication strategy in counting the tiles within each surface. It seems that the students knew the multiplicative structure of units in rectangular surfaces. However, their worksheet shows that they counted the

 ISBN : 978-602-17465-1-6

units one by one before they apply the multiplication strategy. The students counted the units one by one until a certain number of tiles and then turned of using the multiplication strategy. It seems that counting one by one was not effective. In this case, we assume that the shape of the parking lot (see figure 4b) as well as the number of tiles being counted (which is relatively big) trigger the emergence of the splitting strategy and the multiplication strategy in determining the area of the parking lot.

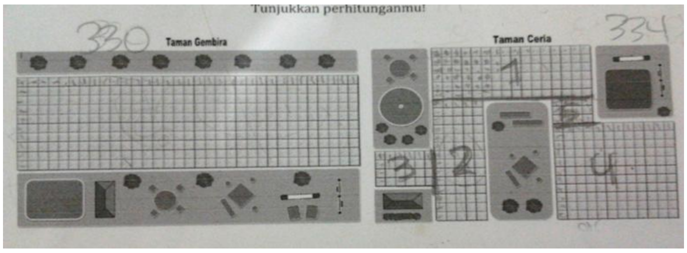


Figure 5. Solutions proposed by the focus students

The transcript also shows that the students could compare the area of two surfaces by considering the number of units (tiles) covering the surfaces. Here, they treated the tiles as the measurement units. They knew that the parking lot in Taman Gembira is smaller since it contains a fewer tiles, which is 330 tiles, if it is compared to the parking lot in Taman Ceria, which contains 334 tiles. It seems that the students conceiving area as the number of the measurement units covering a surface. They knew that the more units need to cover a surface, the larger the surface is.

Hence, based on the findings above we can conclude that the understanding of area as the number of measurement units is conceived by these students in this problem.

(2) Measuring area of a surface

In this activity the students are asked to determine the area of a floor covered by a carpet (see figure 6). The purpose of this problem is to help students seeing measuring area as finding the number of units covering a surface. In our hypothetical learning trajectory, the students will treat the tiles as the measurement units and find the number of those tiles to tell the area of the surface.

In the actual learning trajectory of the whole students, most of the students started drawing gridlines (tiles) on the carpet as the same size as the gridlines (tiles) on the outside of the carpet (on the floor) to help them finding (counting) the whole tiles needed to cover the floor. They then of used the multiplication strategy to count the tiles. They split the floor into two rectangular parts and then counted the tiles in each part by using the multiplication strategy. We conjecture that students' strategy in dealing with this problem is influenced by the strategy they used in dealing with the previous problem, comparing the area of two parking lots.

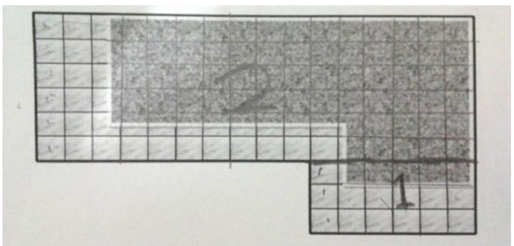


Figure 6. Solutions proposed by the focus students

As well as the majority of the students, our focus students also drew gridlines (see figure 6) and counted the tiles using the multiplication strategy. Their strategy is shown in the following transcript.

Widya: The area of surface 1 is 3 times 6 (see figure 6).
Teacher: Where they (3 and 6) come from?
Widya: Here is surface 1 (pointing surface).
To the bottom is 1, 2, 3, (pointing the units covering the width of surface 1). Here, where the three come from.
Teacher: Yes.
Widya: and to this right is 6 (pointing the units covering the length)
That is 3x6.
Teacher: Yes.
Widya: So, 3x6 which is 18.
Teacher: Yes, and then?
Widay: In surface 2, to the bottom 1, 2, 3, 4, 5, 6 (pointing the units covering the width of surface 1)
Teacher: Yes.
Widya: So, in surface 2, to the bottom (its width) is 6 and to the right (its length) is 16.
So, it is 6 x 16 equal to 98 (actually, she wanted to say 96 because 96 is what written on her worksheet but she mentioned incorrectly).
Teacher: Yes.
Widya: So, the total is 96 + 18 which is equal to 114.
Teacher: What is 114 refers to?
Widya: The number of the tiles.
Teacher: Is that the area of the floor?
Widya: Yes. It is.

The transcript shows that the students used the multiplication strategy in counting the whole tiles after they split the floor into two rectangular surfaces (see figure 6). In surface 2, for example, he knew that the whole tiles covering the surface are 6 times 16 which are 96 tiles. They also knew that the area of the floor is the sum of the area of surface 1 and surface 2 which is 114. They looked at 114 as the number of tiles covering the whole floor and it is the area of the floor. We conjecture that the used of the multiplication strategy as the way to find the tiles is influenced by their strategy in solving the previous problem (the parking lot problem). Here, we can say that the students could see area as the number of measurement units covering a surface.

They also drew gridlines on the carpet, but there was no clear purpose of drawing the gridlines. There is no finding showing that the students used the gridlines to verify their calculation since they started by drawing the gridlines and then doing calculation; or using the gridlines to allow them counting the tiles one by one. We assume that they drew the gridlines as they tried to situate the floor as it is looked as in the previous tiled floors (the parking lots) where all their tiles are visible; or they just follow what the majority of the students did since these students were not the ones who initiate drawing gridlines.

By considering the findings above, we conclude that our focus students could see the area as the number of measurement units covering a surface.

Hence, how can we support students to develop their understanding of measuring area as finding the number of measurement units covering a surface? Grounding to the findings above we conclude that the learning activity that engaging students in comparing and measuring the area of tiled floors is potential enough to building their understanding of measuring area as the process of finding the number of measurement units covering a surface. Students see the tiles as the measurement units and tell the area of a floor by telling the number of units covering the floor. Students compare the number of the measurement units in comparing the area of two surfaces.

CONCLUSION

In this section, we would like to answer our research question "How can we support students in developing their understanding of area as the number of measurement units covering a surface?"

Grounding to the findings elaborated in the previous section, we conclude that students need to have an understanding of physically quantity of area before they are introduced to the meaning of area as the number of measurement units. Understanding the physically quantity of area could support students in developing their understanding of the measurement units of area and understanding area as the number of measurement units covering a surface.

REFERENCES

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, *31*(3), 235–268.

Bonotto, C. (2003). About students' understanding and learning of the concept of surface area. In D. H. Clements, & G. Bright (Eds), *Learning and teaching measurement*, 2003 NCTM Yearbook (pp. 157-167), Reston, Va.: National Council of Teachers of Mathematics.

Clements, D. H. & Sarama, J., (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York: Routledge.

De Corte, E.,Verschaffel, L.,&Van Coillie,V. (1988). Influence of number size, problem structure, and response mode on children's solutions of multiplication word problems. *Journal of Mathematical Behavior*, 7, 197–216.

Fauzan, A. (2002). *Applying Realistic Mathematics Education (RME) in Teaching Geometry in Indonesian Primary Schools*. Enschede: PrintPartners Ipskamp.

Hirstein, J., Lamb, C., & Osborne, A. (1978). Student misconcepts about area measure. Arithmetic Teacher, 25, 10-16

Martin, W. G., & Strutchens, M. E. (2000). Geometry and Measurement. In E. A. Silver, & P. A. Kenney (Eds.), Results from the seventh mathematics assessment of the National Assessment of Educational Progress (pp. 193-234). Reston, VA: NCTM

Nesher, P. (1992). Solving multiplication word problems. In G. Leinhardt, R. Putnam, & R. A. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 189–219). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Peled, I., & Nesher, P. (1988). What children tell us about multiplication word problems. *Journal of Mathematical Behavior*, 7, 239–262.

Reys, R. Lindquist, M. M., Lambdin, D. V., & Smith, N. L. (2007). Helping Children Learn Mathematics 8th Edition. New Jersey, NJ: John Wiley & Sons, Inc.

Simon, M.A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114–145.

Simon, M., & Blume, G. (1994). Building and understanding multiplicative relationships: A study of prospective elementary teachers. *Journal for Research in Mathematics Education,* 25, 472–494.

Tierney, C., Boyd, C., & Davis, G. (1990). Prospective primary teachers' conceptions of area. *Proceedings of the Fourteenth Psychology of Mathematics Education Conference* (Vol. 2, pp. 307–315). Mexico.

Van den Akker, J., Gravemeijer, K., McKenney, S., Nieveen, N. (2006). *Educational Design Research.* USA and Canada: Routledge

Yuberta, K.R. (2011). Developing students' notion of measurement units for area. *IndoMS. J.M.E Vol. 2 No. 2 July 2011, pp. 173-184.*

Zacharos, K. (2006). Prevailing Educational Practices for Area Measurement and Students' Failure in Measuring Areas. *Journal of Mathematics Behavior Ed. 25* (pp.224-239). Elsevier Inc.

This paper can be cited as Zulkardi(2013). Proceeding The First South East Asia Design/Development Research (SEA-DR) International Conference, Sriwijaya University, Palembang, April 22nd-23rd.