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DEVELOPING STUDENTS' INITIAL UNDERSTANDING OF AREA MEASUREMENT THROUGH THE UNIT: TEACHING EXPERIMENT ON A LESSON WITH CASHEWNUT COOKIE CONTEXT CONDUCTED IN THIRD GRADE STUDENTS OF ELEMENTARY SCHOOL

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Abstract

We examined way of helping fifteen third grade students of class 3A in Elementary Laboratory School of Surabaya State University to understand the area measurement through the unit concept in a first lesson of our instructional sequence of teaching experiment. We used the cashewnuts cookie context and set three kinds of activity. First activity highlighted some unit issues through investigation: overlapping units, gap and leftover. Second activity concerned on the emerging students' insight on relationship between the number of unit and the area of a surface. Third activity concerned on measuring and comparing the surface by the use of units. The results indicated that the students need a clear situation of task in activity 1 through discussion. The part of discussion takes crucial role around the task. We noted that missing this discussion part steered the students to confusion. The confusion leads to the failure of students' acquisition of understanding of units issues: overlapping, gap and the effect of each issue. However, we found the second activity could promote students to finally recognise the unit issue. The students also emerged the connection between the ideal size or area with number of units used through the activity of making ideal size of cookie for given fourteen nuts. The activity three could help students to recognise the use of units to find the area in a strict regard of overlapping and gaps through measuring the cookie with the nuts. The students could also compare the area by the use of those units.

Keywords : *area measurement, unit, overlapping, gaps, measuring, comparing*

INTRODUCTION

Background

Many researchers have described the importance of the unit concepts in measurement (e.g., Battista, 2006; Kamii, 2006; Mulligan, Mitchelmore, & Prescott, 2005 in Barrett, 2011). Reynolds and Wheatley (1996) contend that to understand area a child must construct and coordinate units. This idea implies that once the idea of constructing an area unit is formed, and the coordination of all constructed area units is made, the children are possibly assisted when they are dealing with the area measurement. According to these literatures, the need of understanding area unit becomes more centralized in the topic of area measurement since it takes a worthwhile role during area measurement learning.

There are several consideration related to the understanding of area unit. The first thing is the unit appropriateness; a unit must be related to the aspect of the object to

be measured. Several researchers have described young students' difficulty measuring area (Nitabach and Lehrer, 1996), demonstrating how they sometimes treat the length as space-filling attribute. In this occasion, the students measure the length of a side, then move the ruler a bit and measure the length between the sides again, and so on, and finally treat the length as a space-filling attribute.

Another case will be finding the area by using linear tool which is a ruler. Sometimes, instruction is given by using a ruler. Both these situations clearly describe on how inappropriate it is to use linear tool in dealing with an area measurement. Therefore, the concept of unit appropriateness is broken, the concept implying the approriate use of tools to the obejct being measured. If instruction begins with a ruler, one of the most common mistakes is for children to measure the length of each side and add the two linear measures together, or the students may easily multiply the length and the width of the rectangle to get an area, and still it is a misconception.

Apparently, this misconception is still happening in Indonesia. Not only what presented in schoolbooks, but also in the teaching process, the concept of area is directly given to the students by drawing the plane with a given length and width, then multiplying both linier to get the area.

Aims

To deal with this misconception as well as to promote the concern on the appropriateness of unit feature, a particular learning set is highly needed. This learning also needs to regard students' first experience with area including discussion of the issues of leftover space, overlapping units and precision to name a few (Lehrer, 2003).

Based on this literature study, we designed an instructional activities that adress the unit issues: overlapping and gaps, the emerging connection between the unit and the area through the number of unit and size context, and measuring and comparing area by the use of unit. We regard the cashewnuts cookie context potential enough to pursue our goals

Method

We conducted two cycle - design research. The first cycle is preliminary teaching experiment conducted with six third graders and the second cycle is teaching experiment conducted in another group of students from a third grade class. We focused in the second cycle in this article. We designed three kinds of activity in our part of instructional sequence of teaching experiment and constructed the Hypothetical Learning Trajectory (HLT) for each activity as summarized in the following table:

Table 1. Table of HLT of three designed activities on cashewnuts cookie

Activity	Task	Conjectured students' thinking and actions
Cashewnut cookie man	Finding the number of nuts on top of a fully covered cookie	• Discussion on the possible different result • Discussion on different nuts arrangement • Discussion on overlapping, gap and each's effect to the number of nuts • The estimation of number of nuts
Cookie and fourteen	Observe the nuts arrangement of	• The studenst emerge the state of overlapping nuts and the effect of the number of nuts used

nuts	fourteen nuts on top of both cookie (cookie A the bigger, and cookie B smaller)	• The students emerge the state of gap between the nuts and the effect of this to the number of nuts used • The students discuss the size of given cookie in respect to the number of nuts given • The students make connection between the number of nuts used to ideal cookie size by the strict consideration on unit issues: overlapping and gaps
Measure and order the cookie	Measure the size of cookie by the use of nuts and order the cookie based on the size	• Students used the connection between the number of nuts and the size to this task and measure the cookie by the use of nuts • The students avoid the overlapping and possible gap while measuring • The students obatined the size of cookie by the use of the nuts, and compare the size • Students conclude that more nuts used, bigger the cookie

We conducted these activities in first lesson of our teaching experiment of design research with Pendidikan Matematika Realistik Indonesia (PMRI) approach in Elementary Laboratory School of Surabaya State University. Our research subject is students of class 3A that consists of fifteen students and also the mathematics teacher as the teacher in our experiment. As the sources of the analysis, we used students' work and video during the lesson 1 of our teaching experiment. We analysed the students' work and we made the fragments and the transcription from videos that showed students' learning and thinking process. These both sources are triangulated each other to support us doing the analysis. We compare the result with our HLT.

Research questions

Finally, in this study we search for the answer to our research questions

1. *How does cashewnuts cookie context help students to emerge the issues of unit: overlapping and gaps?*
2. *How does cashewnuts cookie context help students to gain insight into the relation between unit and area measurement?*

RESULT AND DISCUSSION

Activity 1 – Cashewnut cookie man

The goal of this first activity is that the students could recognise the issue of overlapping units, and uncovered part of plane (gap and leftover) and the effect of each issue to the number of unit used through the cashewnut cookie context.

We gave the students the following context

A cookie man will make cookies with the top of each cookie full of cashewnuts. Because cashewnuts is relatively expensive in market, the cookie man does not want to have possible remaining nuts after he finish making all cookies.

Before the class, we set that these following discussion points come after the problem. The teacher asks some questions and the students are asked to respond.

- *What do you think the cookie man will do?*
  Students can be guided to say that the man needs to estimate the number of nuts on each cookie

- *How does the cookie man find the number of nuts for his entire cookies that he makes?*
  Students can be guided to say that the cookie man multiply the number of nuts on each cookie with the number of cookies.
  After students understand the problem, they will work to find out the number of nuts used on one cookie.

What we found in the class about this activity 1

The teacher started the class by asking whether the students like cookie. The teacher also told the students about the topping of cookie and asked the students what topping they usually like. Each student engaged with the situation. Then, the teacher gave the problem above to the students.

A cookie man will make cookies with the top of each cookie full of cashewnuts. Because cashewnuts is relatively expensive in market, the cookie man does not want to have possible remaining nuts after he finish making all cookies.

After reading the problem, she added statement *"the cookie man does not want to use so many nuts, so how do you help him so that he does not experience loss"* which we did not think her to do.

However, she then asked the students what would be the better idea to help the cookie man.

*Teacher : **What is your idea how to help the cookie man to make the cookie (making circle with fingers indicating the cookie) but the nuts used is not so many? ****

Some students : Divide Ma'am
Teacher : Asti
Asti : Divide
Teacher : What is divided?
Asti : So there will be cashewnuts and chocolate for example
Other students raised their hands
Teacher : uhmm, so not overwhole full [she came to board and pictured Asti's idea, the picture was cookie with a cashewnut put on center of cookie and then the chocolate on other place]
Bunga raised hand
Teacher : Bunga, what is your opinion?
Bunga : Split the nuts
The teacher pictured Bunga's idea, other students kept raising the hand, however
Teacher : Keep your answer...

From the conversation, it is implied that what these students had in mind was not *'find the number of nuts on one fully covered cookie'* like we expected them to have, but *'put not too many nuts and reduce the nuts on each cookie as possible as they can to get the small number of nuts'*. In other word to say is that they ignored or just forget the statement of *'full cover'* on the problem. We assume that this happened because teacher's statement (*) on the conversation above was interpreted differently by the students. Hence, that big shift happened.

Furthermore, we noted that it was also because the teacher did not do further discussion we set and she did not direct the students to the important point

The students estimate the number of nuts on one cookie, so that the cookie man will know the total number of nuts used for all of his cookies made.

Hence, she did not sound the problem in which **students need to find the number of nuts on one cookie.** The absence of this important point clearly led the students to the misunderstanding. In addition, the teacher did not do any clarification and further discussion about the important point and the real task instead of rereading the problem and giving stress to the word *'full'*. Hence, the students became even more confused.

We moved around the groups and recorded what students discussed and argued about. The first group we observed was the third group (Hiu, Munawar, Dinda, Ersa, and Olga)

They are putting nuts on top of cookie
Olga : Do not put too many
Hiu : Just put three?
Dinda : Put on the centre
Hiu : But it is asked to be full
*Munawar : **Full but not much, so how?***

The other group we observed was the first group (Ranti, Rafa, Rama, Sasi, and Asti), our focus group.

Asti : Full
Ranti : Full?...so it means like this [putting nuts on top of cookie]
*Asti : **No, not like that, it is full, but not too many***
On the background was sound of teacher rereading the problem "the cookie is full of nuts but he does not want to have many"

We conluded that these students confused about the task poorly given by the teacher. Teacher also did not do discussion about the point we set so that she gave the unclear task. What they had in mind that confuses them about the task was *"full but not too many".* This was not we wanted them to have. We wanted them to just figure out the number of nuts on each cookie so that they can estimate the total number of nuts used for all cookies made to avoid loss of having many reamining nuts in the end of his work. The estimation we conjectured could be different so that some issues: overlapping and uncovered part can be generated. The following are the work of groups.

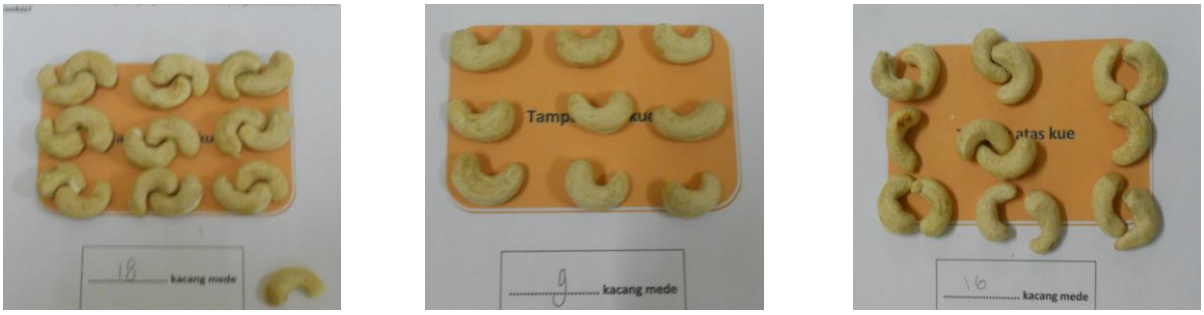


Figure 1. Groups' work in activity 1

The result of activity 1

The teacher and the students did discussion about the difference between the number of the nuts each group used. However, the students argued the difference occuring since that every group has different idea and thinking. They did not even talk about the nuts arrangement, gap for example. We assume that based on groups' work, it was hard to see the issue of overlapping or even discussion about the uncovered part (gap and leftover). Furthermore, we found that the teacher missed asking the students about the arrangement itself that can bring students talk about those issues. We concluded that from activity 1, students did not yet figure out the issue of overlapping, gap, and leftover. Hence, they could not yet 'see' the effect of overlapping and uncovered parts to the number of nuts used to cover the cookie.

However, the students were helped to see the issue of overlapping and uncovered parts in the activity 2 we anticipated.

Activity 2 – Cookie and 14 nuts

The second activity aims to get students recognising the issues of overlapping and uncovered parts (gap and leftover) and also seeing the connection between the number of units to the size of plane through the context of cashewnuts cookie. The groups were given the paper displaying the two cookie outlines different in size. Fourteen cashewnuts for each cookie were given to groups. The students were asked to put the fourteen nuts on top of each cookie and to observe what they obtained from the work. We conjectured students to argue about the issue and also the connection between the number of nuts given and the the size of each cookie.

What we found in the class about this activity 2

We observed students in focus group while they were busy working on the task.

Ranti : Write what you found based on your observation! [reading the instruction] Uhm ... the nuts here [pointing out to the cookie A] do not overlap, but here [pointing out to the cookie B] overlap
Rafa : Because this is small [pointing out to the cookie B]
Ranti : Well, why these overlap? Since the cookie is small. Why these do not overlap? Since the cookie is big
Asti : Yes, that is it. That's what I think. Yes
Rafa : I also think the same.

We found that these students had already connected the idea of the size of cookie given and the effect of number of nuts given. Then, they observed the overlapping nuts. We interprete that these students could see that cookie A is too big for those given 14 nuts so that no nuts overlapping. Meanwhile, they thought that the cookie B was too small for those given 14 nuts so that some overlapping nuts were created. We then observed another group.

Fig 2. Student discussing about the difference

(Busy putting the 14 nuts on top of cookie B)
Ersa : Here
Olga : Put it in between these
Hiu : There is exactly gap between the nuts

This group emerged the term 'gap' while they tried to accomplish putting those fourteen nuts on top of cookie B. They tried to put the nuts between two other nuts as well as made some nuts overlapping since the cookie B size was too small for those fourteen given nuts. We also did small interview with this group after couple of minutes.

Observer : Hiu, have you already finished?
Hiu : Finished
Observer : What if I ask you something?
Hiu : What?
Observer : How many nuts here? [pointing out cookie A]
Hiu and Olga : Fourteen
Observer : How many nuts here? [pointing out cookie B]
Hiu : Fourteen
Observer : So, do you think that the number of nuts here enough for cookie A?
Students : Yes, enough
Observer : Enough? Can I put another nut here [pointing out some gaps in cookie A to put other nuts], can I do that?
Students : You can
Observer : So, fourteen nuts is enough for cookie A?
Dinda : Not really
Observer : Why?
Dinda : Because on some uncovered parts, more other nuts can be put
Observer : That's good, do you understand about what Dinda said?
Students : Yes
Observer : And then, is that enough for fourteen nuts to be put on the cookie B?
Olga : No
Observer : Why so?
Olga : Because, it's already full
Ersa : The place is small
Observer : So, do you think the nuts are too many for the cookie B?
Hiu : Nope, more less
Observer : What do you mean?
Hiu and other students : They fit there
Observe : What if I add one on this one, to be 15, one on this one
Students : 16
Observer : Can I really do that? Do you think that 14 is already too many for cookie B?
Dinda : The cookie A can have no room later on
Ersa : I think it's too many
Observer : So how is it supposed to be?
Students : 11

We conclude that these students understand that if some gaps exist, then the number of nuts can be more than what they have since they can put another nut in between. Hence, they can see the effect of existing gap between the nuts on top of cookie. Based

on small interview, we can see that firstly students did not directly relate the number of nuts to the size of given cookie. We tried to ask to get the students into it. We found that they finally said that fourteen nuts are less for cookie A because they can put other nuts. They found fourteen is little bit more for the cookie B.

We found that this important idea, connection between the number of nuts and the cookie size, worked out while the students set up to the next session, making ideal cookie size for fourteen nuts.

The teacher asked the students to make their own cookie for those fourteen given nuts and informed them about the ideal cookie size and shape for those fourteen given nuts.

From this session we found that each group already understood the situation in which they need to make their own outline of cookie to those fourteen given nuts. They already considered overlapping and gaps not to occur or minimally to occur. We moved around to see each group's work.

In Ranti's group, we found that they made first the outline, rectangular shaped cookie. After that, they put the nuts on top of that cookie and we observed that the cookie they made was little bit bigger for those fourteen nuts. However, we tried to remind the students whether that was the proper cookie size for those fourteen nuts and restated the task. They then shrank the cookie a bit to get those fourteen nuts really fit to the cookie.

In Bunga's group, we found that they did the same way as the Ranti's group. Generally, this was not exactly what we predicted to happen. Nevertheless, the students already got an idea about the ideal size of cookie for those fourteen given nuts with strict consideration to possible gap and overlapping nuts.

Fig 3. Student making a representation of cookie with ideal size of given

We found that Olga's group worked differently amongst others. We saw them firstly arranging the nuts. They seemed to avoid overlapping nuts as well as minimized the gap in between. We found that they then traced the outline of cookie exactly from the edge of nuts, made a circular-liked figure surrounding the nuts and ended up with the shape formed by the arrangement of the cookie. We interpreted that they have created the representation of ideal size of cookie for those 14 given nuts.

The result of activity 2

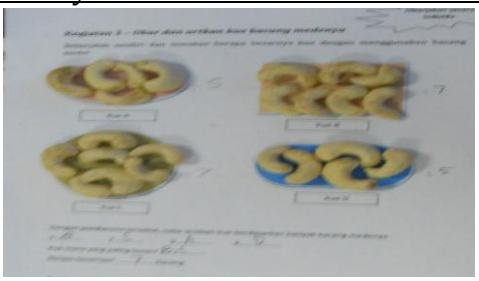
From this activity, we conclude that the students already understand some issues like unit overlapping, and uncovered parts (gap and leftover) through this activity and the effect of each issue to the number of nuts used to cover the cookie. They even could make connection between the number of nuts used to the size of given cookie and did some observation whether the size really ideal for the given number of nuts through the consideration of unit issues: overlapping and uncovered parts (gap and leftover).

Furthermore, they finally could make figure that was approprite in size for the given number of nuts by the strict consideration on overlapping nuts and possible gap and

leftover. We assume that they have now an insight into the size in term of number of unit through the cashewnuts cookie. We assume that they can have those idea now since they understand the issue of overlapping and gaps as well: for particular number of nuts given, the outline of cookie will not be proper for those nuts if some gaps or overlapping occur. They think that the nuts should be in situation such that they are close to each other, no overlapping and no possible gaps. Hence, the ideal size can be formed. We made the next task such that students can really see the clear relation between the unit (nuts) and the area (cookie size). We put the students further to realize the use of nuts to figure out the size.

Hence, we set the students to conduct the activity 3 where they need to find the size of given cookies by the use of nuts. We already used the term 'measure' in this task. We observed the students on this activity, and they seemed grasping the idea of the use of nut to measure the size of cookie.

Activity 3 – Measure and order the cookies



In this activity, the students measured the cookie by the use of nuts. They reported the number of nuts on each cookie and made an order of cookie based on their size.

Figure 4. Measuring and comparing the cookie size

What we found in the class in this activity 3

We found that all the students already avoided overlapping and minimized the gap in between the nuts. They found the number of nuts on each cookie.

We interpreted this situation as finding the ideal number of nuts without overlapping and gaps to fit the cookie. Here, the students clearly already figured out the use of nuts to find the size of cookie.

While asked the order of cookie based on the size, Ranti and some other students argued that the size was regarded based on the number of nuts. She meant that the biggest cookie would hold nuts at the most.

We found that Ranti and her group had already understood the use of nuts to derive the size or area of the cookie. They even could argue that more nuts used, bigger the cookie they have.

The result of activity 3

In the end of the task the students could conclude that the nuts could be used to find how large the cookies were. They also argued that the number of nuts would affect the size of cookie. The more nuts used, the bigger cookie they get under the measurement situation.

CONCLUSION

We conclude that giving clear task and discussion part during the activity 1 is very important to avoid students' confusion around the task. Having the discussion with the students about finding the number of nuts on top of fully covered cookie should be conducted prior to finding the number of nuts itself. It is due to avoid the misunderstanding like we experienced in the class. Hence, the role of the teacher is clearly strong in this phase to make the task clear for the students.

We found it also hard to have students emerging the unit issues: overlapping, gaps under the students' work – full but not many. They could not directly see the phenomenon of overlapping nuts, gaps in between the nuts from the difference of the number of nuts in each group they observed. For this reason, the teacher must be able to orchestrate the discussion around this task and trigger some questions to have students to talk about the unit issues.

However, we successfully anticipated the lack of implementation on the first activity in the second activity. Although in the first activity, students could not easily emerge the unit issues, this concept could be triggered from activity of putting fourteen nuts on each different sized cookies we found succesful. From these the analysis, we found that:

1. The students emerged the state of overlapping nuts and gap in between the nuts from their observation. We noted that they could argue about the connection between the number of nuts and the size of the cookie provided. They also already connected the size to the ideal number of nuts to cover the cookie A and B.
2. We found that students already had in mind the connection between the number of nuts and the possible ideal size of cookie. We interpret this situation as they found the connection between the concept of unit to area measurement.
3. In the activity 3, we found that the students already developed the understanding of the use of the nuts to find how large the cookies were.

Based on this analysis and findings, we summarize the answer to our research questions

1. *How does cashewnuts cookie context help students to emerge the issues of unit: overlapping and gaps?*

   The context supports the students to observe the nuts arrangement on top of cookie and emerge the state of: overlapping nuts, gap in between nuts.

2. *How does cashewnuts cookie context help students to gain insight into the relation between unit and area measurement?*

   The cashewnut lesson enables students to firstly emerge the unit issues: overlapping, gap, and the students use this to argue about the connection of the issue to the size of cookie as well as the number of nuts used. The students treated the nuts as the size determiner and make use the nuts to derive the size and and do area comparison.

Finally, we conclude that the experience with the cookie context helps the students to understand area throught the conception of unit which we found support the description of researchers on importance of the unit concepts in measurement (e.g.,

Reynolds and Wheatley, 1996; Battista, 2006; Kamii, 2006; Mulligan, Mitchelmore, & Prescott, 2005 in Barrett, 2011). We also noted that the discussion on unit issues: overlapping and gap (Lehrer, 2003) could contribute to the students' understanding of the ideal size of particular number of given unit which is the crucial idea to the connection between the number of unit and the area. With some suggestion aforementioned, this lesson can be an alternative to teach area measurement for third graders.

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