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USING ARRAY REPRESENTATIONS TO SHOW THE COMMUTATIVE PROPERTY OF MULTIPLICATION

Ismi Ridha Asy-Syifaa
State University of Surabaya
ismiridhaa@gmail.com

Abstract

The commutative property of multiplication is an important property to be taught since it helps students in memorizing the basic multiplication facts; multiplication from 0×0 *to* 10×10*. The students do not have to memorize all the facts; they just have to memorize half of the facts if they know the commutative property of multiplication. In Indonesia, learning multiplication usually focus on "group of" model. Thus, the commutative property of multiplication is also introduced using the same model. However, previous researchers state that it is clearer to show commutative property of multiplication using array representation. This representation can show the commutative property of multiplication naturally compare to the "group of" model. Considering the situation, a design of a teaching-learning process to support second grader elementary students to understand the commutative property of multiplication is needed in Indonesia. This design is made to provide a sequence of activities in a mathematics lesson emphasizing in stimulating students to understand the commutative property and the use of the property to solve multiplication problems. The use of use of context and model (array representation) is considered in this design since the activity is design based on PMRI approach. After the design is implemented in class 2A of SD LAB UNESA Surabaya, the result shows that the array representations do not really help all the students in the class experiment to see the commutative property of multiplication.*

Keywords: *multiplication, commutative property, array representation*

INTRODUCTION

The commutative property of multiplication is an important property to be taught since it helps students when they have to memorize the basic multiplication facts; multiplication from 0×0 to 10×10. Based on Indonesian curriculum, the second grade elementary students are asked to memorize the basic multiplication facts. Thus, by understanding the property, the students do not have to memorize all facts. They just have to memorize half part of multiplication table. Besides, the commutative property can be used to solve a multiplication problem that is easier to be solved if the factors are commuted.

In learning multiplication, there are three models can be used: "group of", number line, and array. In Indonesia, learning multiplication usually focus on "group of" model so that the commutative property of multiplication is also introduced using the same model. However, it is clearer to show the commutative property using the array representation (Anghileri, 2000). Barmby et all (2008) also suggest to describe

multiplication strategies using array since it emphasize in the binary nature of multiplication and also making clear the commutative nature of operation.

Considering the situation that described before, a design of a teaching-learning process to support second grader elementary students to understand the commutative property of multiplication is needed in Indonesia. This design is made to provide a sequence of activities in a mathematics lesson emphasizing in stimulating students to understand the commutative property and the use it to solve multiplication problems. The use of use of context and model (array representation) is considered in this design. The design is used in this research to answer the following research question: *How can array representations help students to understand the commutative property of multiplication?*

THEORETICAL BACKGROUND

Some concepts related to multiplication and its learning process used for this current research and Realistic Mathematics Education (RME) as an approach in designing the mathematics lesson are described.

Multiplication as repeated addition

For elementary students, key concept of multiplication is repeated addition. The students are taught that a multiplication form can be written as a repeated addition form that represents its multiplication. For example, some multiplication can be written as

$4 \times 8 = 8 + 8 + 8 + 8$
$5 \times 8 = 8 + 8 + 8 + 8 + 8$
$6 \times 8 = 8 + 8 + 8 + 8 + 8 + 8$

In multiplication, it is known that $4 \times 3 = 12$ and $3 \times 4 = 12$. Although 4×3 and 3×4 are equivalent expressions in value but are different in representation. Because,

$4 \times 3 = 3 + 3 + 3 + 3$
$3 \times 4 = 4 + 4 + 4$

Most teachers in Indonesia teach multiplication in the way it is described above since it is written in the many mathematics books.

Learning sequence in multiplication

Ter Heege (1985) described three stages for learning the basic multiplications. In the beginning of the learning process, students are introduced to multiplication situations. They have to get acquainted to situations where they have to use multiplication in order to solve the problem. By giving ample time, it is expected that student learn to think multiplicatively. After building students' understanding about multiplication concepts and situations, students are supported to find strategies to derive requested multiplication facts from familiar facts. Other studies (Heirdsfield, 1999; Mulligan et al, 1997) have shown also that children discover and develop their own thinking strategies when they solve simple multiplication problems. Thus, it provides an opportunity for students to have better understanding of multiplication, especially the commutative property of multiplication.

Array representation

There are three models to represent multiplication problem: "group of", number line, and array representation. From those three models, the array representation has an advantageous for learning multiplication because it shows the commutative property of multiplication clearly (Anghileri, 2000). Barmby et all (2008) also suggest to describe multiplication strategies using array since it emphasize in the binary nature of multiplication and also making clear the commutative nature of operation. This representation also provides more opportunities for students to construct their understanding of multiplication, especially the commutative property. Providing array representation will also lead students to count by group and use repeated addition (Tasman et all, 2011).

Pendidikan Matematika Realistik Indonesia (PMRI)

Mathematical activities in this research are designed based on PMRI approach. The problem is designed so that the students are able to imagine the situation clearly. After understand the situation of the problem, the students can imagine the problem into a representation that support their argumentations and reasons. The use of array representation is helped them to see that the number of thing in a row is the same with the number of thing in a column, for different pictures. So, in the end, the students are expected to see that, for example, $6 \times 5 = 5 \times 6$.

METHOD

The approach used to conduct this current research and how data collected and analyzed to answer the research question is described below.

Research approach

Design research is used to answer the research question by conducting three phases in design research:

Preparing a design experiment
Conducting a design experiment
Retrospective analysis

Data collection

In order to answer the research question, the whole processes and activities in the classroom is video recorded. One static camcorder is used to record the whole activities and learning-teaching processes during the lesson. In the design, a small group discussion is planned. One video recording is used to record the whole process of students' discussion in the group. All students' written work is collected to support the analysis process related to students' thinking process. The different types of data collected are supported the internal validity of the research. The data is also recorded by two video recorders which mean that it offers the possibility to retrace the process of the study systematically by the other researchers, so it is supported the reliability of the research.

Data analysis

The data of the whole lesson is analyzed to find out whether the lesson can support students' learning process. The data collected, video and written data, are analyzed using the HLT as a guideline. The analysis is focused on the learning process, the group discussion, and the students' activity in solving the problems. Video and

written data are analyzed in order to make some interpretation of students' thinking process and conclusion of the activities that can answer the research question.

HYPOTHETICAL LEARNING TRAJECTORY

There are three activities designed in learning the commutative property of multiplication. These activities are given in one meeting.

Activity 1

The teacher shows a picture (see Figure 1) in a short time, and then asks the students to determine the total numbers of things that present in the picture. After the students are able to determine the answer correctly, the teacher rotates the picture, and then asks the students to determine the total number of things in the picture again. After the students are able to determine the answer for the second question, they have a discussion class.

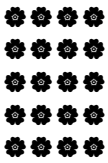

Figure 1. A picture shows things in an array representation before rotated.

From this activity, the hypothesis of students' answer is students are trying to calculate one by one. However, since the teacher shows the picture in a short time, the students will not able to count one by one. Then, it is expected that the students find another way to determine the total number of the things in the picture. There are some solution can be used:

Students will group the things in a row or a column, and know that the rest of rows or columns are the same. So, they determine the total number of the things by using repeated addition $4 + 4 + 4 + 4 + 4$ or $5 + 5 + 5 + 5$.

Students will group the things in a row or a column, and realize that the rest of rows and columns are the same. So, they do skip counting, 4, 8, 12, 16, 20; or 5, 10, 15, 20 to determine the total number of things.

Students will count the number of thing in sides, row and column. Then, they multiply those numbers to get the number of things in the picture.

There will be another solutions appears from the students when this activity is conducted in a real experiment. When the teacher asks the students to determine the total number of things in a picture that already rotated, it is expected that the students will directly realize that the total number of the things are the same by giving the same answer.

In this activity, the teacher' role is as a facilitator. The teacher gives the problems and orchestrates the discussion. In the end of the activity, the teacher encourages the students to have a conclusion: two multiplication problems have the same answer if they have the same factors. If there is no student who can mention the conclusion, the teacher can state it.

Activity 2
The students are given a problem that show a picture of two car collections and asked to figure out which collection that is having more cars, Race's collection or Race brother's collection (see Figure 2). After the students finish their work, they have a discussion class

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Figure 2. The second problem about car collections

From this activity, the hypothesis of students' answer is the students will try to count one by one at the first time. However, they will find counting one by one is not a good way to find the solution. Thus, the students will try to find the other solutions to solve it, such as

Students will count the first row or column for each car collections, and know that the rest of rows or columns are the same. So, they determine the total number of the cars by using repeated addition or skip-counting and then make relation to the multiplication form.

Students will count the number of thing in sides, row and column. Then, they multiply those numbers to get the number of cars.

After they have found the number of cars in each collection, they decide who has more cars. The activity ends by discussing the solutions that the students use to get the answer.

Activity 3
For this last activity, the students are given some bare problems to measure their understanding of the commutative property of multiplication. The students are asked to find the product of 5×7, 32×2, and 15×17. They have to work individually to solve those problems. After the students finish their work, the teacher is expected to discuss the answer of the solutions. It is expected that the students will use the commutative property as a strategy to solve the problems.

RESTROSPECTIVE ANALYSIS

The experiment was conducted in second grade SD LAB UNESA, Surabaya, East Java, Indonesia focusing on three students that become a focus group, named Rizal, Ranuh, and Divan. In this part, it will be explained how those three children solve the problems in every activity. However, the interesting findings of whole class activities will be explained also to support in answering the research question.

Activity 1

The teacher showed the picture of things that arranged in the first array representation, and then asked the students to determine the number of things in the picture. From the video recording, it could be seen that some students were trying to count the things one by one. However, since the teacher showed the picture only for some time, the students who counted one by one were not able to finish their counting and determined the total number of things in the picture. The students who were able to answer the problem using multiplication to solve the problem were asked to write it in the whiteboard. The teacher did not ask them to explain how they got to the answer; the teacher who explained it to the rest of the students. The similar situation was happened when the teacher asked the students to determine the total number of things from the picture that had been rotated.

After the students answered both questions, the teacher asked the students to determine the solution together. They counted in a row and then represented in multiplication form. In the end, they concluded that 4×5 and 5×4 are the same. The teacher mentioned it as the commutative property of multiplication. From the video recording, it showed that some students were able participate to answer the question in this activity. The students also listened to the teacher's explanation. In the end, when the teacher asked them weather they understand about the commutative property of multiplication, the students answer that they understand it.

The students in the focus group were participated in this activity. Two of them were asked by the teacher to write the answer in the whiteboard. They were able to use multiplication to solve the problems. They were also pay attention to the teacher's explanation and mentioned that they understood to the explanation.

Activity 2

For the second activity, the students were asked to answer the cars collection problem. They had to determine who has more car collection, Race or his brother. The teacher gave every student a worksheet about the car collection problem, and then asked them to work in group. After they answered the question, the teacher conducted a class discussion to compare the answers.

Each student in the focus group tried to solve the problem individually first, and then they will compare the answer with the others. From the video recording, it could be seen that all of them tried to count the cars one by one. Although their strategy used was counting by ones, they could not get the correct answer. They answered that Race's brother who has more cars. However, from their worksheet, they were able to give correct answer; the both collection has the same number of amount. It could not be seen from the recording the process how they changed the solution. It was possible that they changed the answer after the class discussion because the worksheet was not collected first.

From the video recording, it also could be seen that there were other students who solved the problem by counting one by one. However, not all of students used this strategy, there were some students only count the number of cars in a row and a column, and then multiply them directly. Then, they concluded that the numbers of cars are the same; it could be seen from their worksheet. There was a student, named Shafa, who wrote a conclusion the activity by saying that: if there are two

multiplications and the factors of multiplications are commuted, the products are the same.

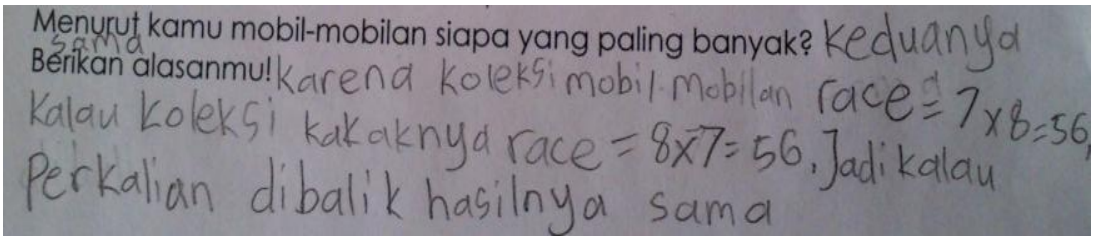


Figure 3. Shafa's worksheet

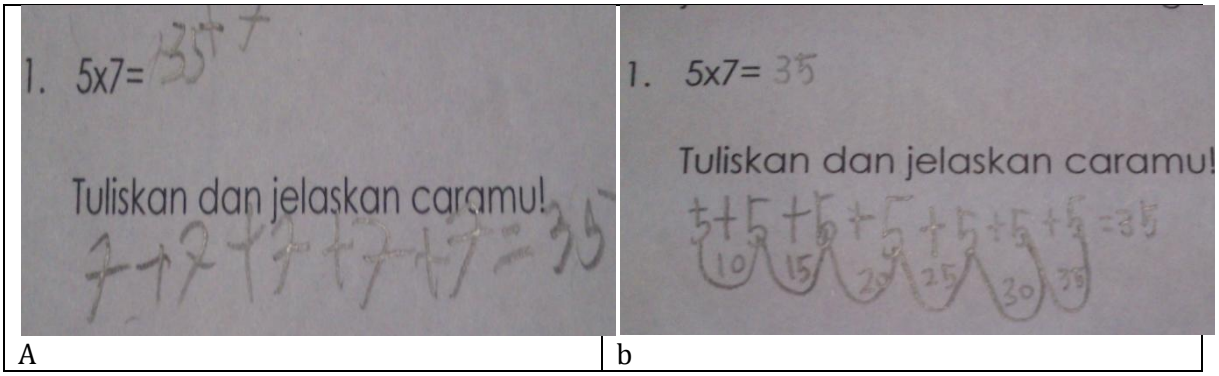
Activity 3

In the last activity, the students were asked to find the product of three multiplication problems individually. The teacher gave every student a worksheet, and then asked them to work individually. After the students finished their working, the teacher collected the work. There was no time for the discussion.

In solving the problems in the third activity, the students in the focus group did not really use the commutative property of multiplication; only Divan who use the commutative property in the second problem. It can be seen from the video recording that they used the finger technique to solve the first problem. For the rest problems, they used repeated addition as the strategy.

However, from all of the students' worksheet, it could be seen that most of the students did not use commutative property to find the product for the first problem, 5×7. However, they commuted the factors to find the product of 34×2 in the second problem. For the last problem, all of the students did not use the commutative property to solve the problems, 15×17.

The possible reason why most of the students do not use the commutative property to solve the first problem is because the problem is a basic fact. Thus, they are familiar with this fact, they know the repeated addition form of this fact, and even they have already memorized it. However, there are few students who use the commutative property, it might be happened that students have not remembered the multiplication fact yet.

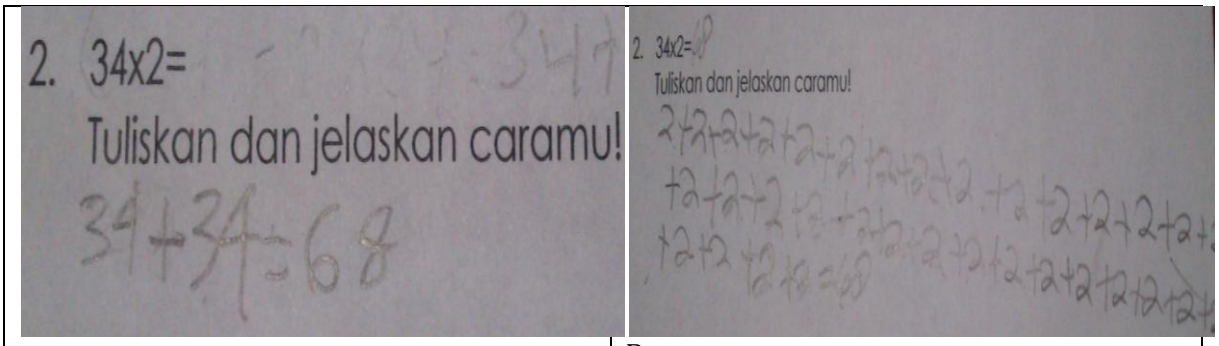


A b

Figure 4. The students' worksheet representing the students' solution (a) without commute the factors and (b) with commute the factors

For the second problem, most of the students used the commutative property since the problem made them to write long repeated addition of 2 if they did not commute the factors; it was tedious and troublesome. Also, they would have long calculation to

do in finding the answer. For the students who did not use the commutative property, it was probably that they still did not understand the commutative property of multiplication. Or, they understood the use of the property but hardly to write it as the solution since it will be different with the solution in the book or given by the teacher before the experiment.



a B

Figure 5. The students' worksheet to solve the second problem
(a) with commute the factors and (b) without commute the factors

In the third problem, the students were hardly to use the concept of commutative property since the two factors were both the big numbers; they were not ones. So, the students still had to write the long repeated addition if they commuted the factors. However, from this problem, it could be seen that the students still do not see the number sense of the problem. They did not realize that if they commuted the factors, they could do addition easier.

CONCLUSION

After conducting the activities that focus on the commutative property of multiplication, there are some important remarks that are concluded from the retrospective analysis. For this lesson, it can be concluded that the students in the focus group do not fully understand to the commutative property of multiplication. Although they participated to the activity and were pay attention to the teacher's explanation, they do not use the commutative property to solve the problems in the second and third activity. From the video recording in the second activity, they still used the counting one by one as a strategy to solve the problem. Thus, they still did not see multiplication in the array representation. As the result, they could not see the commutative property of multiplication in the array representation that presented in the second problem. So, although they participated in the first activity, it is not a guarantee that they understand the commutative property of multiplication presented in array representation. Since they did not see the commutative property of multiplication in the array representation, as the result, they could not solve the problems in the third activity using the commutative property of multiplication. Although there is a student in the focus group who could solve a problem using the commutative property, it cannot be concluded that the student understood the commutative property of multiplication since the student did not use the property for the problem in the second activity and the rest of problems in the third activity. Thus, the use of array representation still not helps those three students in the focus group to understand the commutative property of multiplication by using the property to solve multiplication problems.

However, there were some evidences showing the other students were able to see the commutative property of multiplication in the array representation. It was showed in the retrospective analysis part; there was a student who wrote: "if there are two multiplications and their factors of multiplications are commuted, the products are the same". It can be concluded that this student are able to see commutative property of multiplication in the array representation since the student write the conclusion after the student answer the problem in the second activity. Moreover, from the other students' worksheet in solving the third problem, there are students who used the commutative property to solve the problems. It is expected that they could use the commutative property of multiplication after they were able to see the commutative property of multiplication in array representations from the previous activities. Thus, it can be concluded that the array representations helps some students to see the commutative property of multiplication.

From the retrospective analysis, it can also be concluded that there are three major reasons why the array representations do not help all of the students to see the commutative property. First, the students still do not see the multiplication in array representations. Thus, the students are failed to see the commutative property of multiplication. Second, the time allocation for every activity is being conducted. For example, for the first activity, it has to be conducted longer and given more problems so the students have time to understand and see the commutative property of multiplication that represent in array representations. Third reason is the chosen multiplication problem given to the students for the third activity. The multiplications have to be problems that are easily to solve using the commutative property.

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