

AREA MODEL TO SUPPORT STUDENTS TO SENSE DURING THE TRANSITION FROM BINOMIALS MULTIPLICATION TO FACTORIZATION

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Abstract

Algebra, seen as a gateway to success in higher education, remains as one of the most important topic in early secondary school. However, many studies have shown that students often find difficulties in learning algebra. One of the difficulties is the transition from arithmetic thinking to algebraic thinking. Hence, this study aims at supporting students' understanding during this transition. Employing design research as the root of this research, a series of instructional activities is design. There were two cycles in the main study. However, this paper is based on merely the first cycle. Realistic Mathematics Education is used as the approach in this design and area model is used as the tool to support students' understanding during the transition. The mathematical activities are made around the topic of binomials multiplication, a starter topic in quadratic equations in secondary school. Four students of 7th grade of SMP N 1 Palembang were involved in the teaching experiment as the participants. Data collection of this study includes students' written work, video registration, students' interview and field notes during the teaching experiment. As the result of this study, the students showed not merely ability in binomials multiplication, but also a sense of understanding the basic knowledge to do factorization.

Keywords: algebra, area model, binomials multiplication, factorization, secondary school, design research, Realistic Mathematics Education

INTRODUCTION

Algebra is often seen as a gatekeeper to success in higher education, college preparatory and many career paths. In this regard, Gamoran and Hannigan (2000) claimed that algebra benefits all students, regardless their mathematical abilities. To be more specific, many studies (Capraro & Joffrion, 2006; Edwards, 2000; Erbas, 2005; Gamoran & Hannigan, 2000; Stephen, 2005) have shown the superiority of secondary school algebra.

In contrast to the prominence of secondary school algebra, students are reported to have many difficulties in learning algebra. Al Jupri et al. (2014) noted that common difficulties for students in learning algebra are related to how algebra is being taught in school. In this case, the common way of teaching often leads to students' difficulties in understanding of variables, the arithmetic processes and the strategies to solve the algebraic problems.

Kieran (2004) also found that students often encounter difficulties during the transition from arithmetic to algebraic thinking. The term transition mainly refers to students' way of thinking, viewing and expressing notations and arithmetic processes. Moreover, school algebra in secondary school level usually starts at a formal level and gives less opportunity for students to reason, make sense and build up a bridge to relate arithmetic process with solving algebraic problems. Hence, many students are lost in understanding, which results in a gap between high- and low- achiever students (Susac, et al., 2014).

Many studies have been conducted concerning students' difficulties towards school algebra as well as finding a good instructional design to teach the materials. Yet, most of them focus on introducing the concept of variables and finding solutions. A topic like multiplication of two algebraic factors is likely still being taught in a traditional way using FOIL (First, Outer, Inner, Last) strategy. Usiskin (1988) noted that one of the fundamental issues in school algebra is the limited amount of strategy that the students learnt. Hence, the present study is to provide more alternative strategy as well as to investigate whether some models will work on supporting students' understanding in this topic. The traditional teaching and learning processes provide less opportunities for low-achiever students to understand the concept and wider the possibility for them to fail. Furthermore, students' lack of understanding in this topic will leads to more difficulties in its reverse topic, factorization.

Based on the aforementioned discussion, the present study aims at developing a local instruction theory to support students' understanding and learning process during the transition from binomials multiplication to factorization. Area model is used in this study as a tool to bridge students' understanding during the transition. Therefore, the researchers pose the following research question:

How does area model support students understanding during the transition from binomials multiplication to factorization?

THEORETICAL FRAMEWORK

Secondary school algebra

Kriegler (2004) has defined algebra as (1) generalized arithmetic, (2) a language and (3) a tool for functions and mathematical modeling. Usiskin (1988) has added two more definitions of algebra, which are study of relationships among quantities and study of structures. Besides, the National Council of Teachers of Mathematics (2008) has defined algebra as "a way of thinking and a set of concepts and skills that enable students to generalize, model, and analyze mathematical situation."

According to studies by Silva et al. (2006) and Capraro and Joffrion (2006), studying algebra for the seventh and eighth grades is a critical point as the fundamental preparation for using advanced algebraic concepts as well as other domains in education. Therefore, students' success in this level will impact in their higher education level. Moreover, Susac et al. (2014) claimed that algebra is the first domain of school

mathematics where students first encounter abstract mathematical reasoning. Therefore, many students find difficulties in this topic.

Traced by the errors made by the students, Booth (1988) categorized students' difficulties in learning school algebra into four aspects: (i) the focus of algebraic activity and the nature of the "answer", (ii) the use of notations and conventions in algebra, (iii) the kinds of relationships and methods used in arithmetic, and (iv) the meaning of letters as variables. This study, however, will focus on the third aspect.

Factoring a quadratic polynomial

A factor is something that when multiplied to another factor will result a product, in this case a new polynomial. In this sense, students first encounter binomials multiplication, which will result to a polynomial. Then, then they are asked to factor the polynomial. However, factorization is seen as one of the problematic topics that mathematics teachers are concern about (Roebuck, 1997).

Rauff (1994) claimed that the errors made by students when solving factorization problems are mainly due to differences in viewing algebra. In this respect, students often have false understandings. One example given by Rauff, which is suitable with the factoring issue in Indonesia, is that factoring means to "UNFOIL." Students are used to do FOIL algorithm to multiply binomials without knowing what they are doing, and this leads to the understanding that factorization means the reverse of doing FOIL strategy.

Area model for binomials multiplication

Outhred and Mitchelmore (2004) stated that the rectangular area model is the basis to model multiplication. Furthermore, Ball et al. (2001) argued that when using the area model as a representation for multiplication, one has to pay attention to the units and the differences between linear (side lengths) and area measurements. In this case, area model can show how a variable represents unknown as a unit. The area model can also connect the multiplication of numerical numbers using the area of a rectangle with algebraic multiplication.

Realistic Mathematics Education

In this study, *Pendidikan Matematika Realistik Indonesia* (PMRI), an adaptation of Realistic Mathematics Education (RME), is employed as the teaching and learning approach. PMRI holds the same tenets with RME, with some Indonesian signatures. The present study emphasizes on the *emergent modeling* to support students' learning process. Through emergent modeling, students build up the area model as a tool to grasp the concept and to solve the problems. For more information about PMRI, see Zulkardi (2002).

METHOD

Research approach

Aiming at developing local instruction theory of how to support students' understanding during the transition from binomials multiplication to factorization, design research was

chosen as the research method of this study. Design research aims at developing theories by designing instructional activities and study of how the design supports students' understanding and learning process. In this regard, the design consists of a sequence of mathematical activity and a Hypothetical Learning Trajectory (HLT) (van den Akker et al., 2006).

There are three phases of design research: (1) preparation, (2) experimenting the design, and (3) conducting a retrospective analysis. During the first phase, the researchers studied literatures, developing mathematical instructions and HLT.

Data collection

During the teaching experiment, the collected data include video registration, field notes by an observer and the students' written work. The whole study consists of two cycles. In the first cycle, the mathematical instructions and the HLT were applied to merely four students and the researcher played a role as the teacher. Furthermore, the second cycle engaged a natural classroom condition with the whole students participated and the initial teacher carried the lessons. This paper is a part of the whole study and zoom in merely an activity about factorization in the last meeting of the first cycle.

Data analysis

The collected data were analyzed qualitatively focusing on students' learning process and how the area model supports students to make sense and reason factorization based on their existing knowledge.

RESULT AND DISCUSSION

The preparation phase

Before conducting the teaching experiment, all students of 7.3 of SMP N 1 Palembang did a pre-test. The content of the pre-test was to test students' mathematical abilities about the prerequisite knowledge and to check students' abilities and understanding about binomials multiplication. The prerequisite knowledge includes finding a rectangle area and the formula for the rectangle area, addition and multiplication of integers, and students' ability to simplify linear equation with one variable. As the result, most of the students joined the pre-test were able to do all the prerequisite knowledge problems and merely one student was able to multiply binomials.

Four students, Aldi, Ghifary, Alifia and Diana, were chosen based on the result of the pre-test and suggestion from the teacher. The four students were students with average mathematical abilities in the classroom and represent common mistakes done by all of the students during the pre-test. During the teaching experiment, the four students worked in two groups. Group 1 consists of Aldi and Ghifary whereas group 2 consists of Alifia and Diana.

The previous lessons

Before using *area model* to do binomials multiplication, the students were first introduced two notions, *rectangle formula* and *pieces formula*, and explored the area

model. The explanation about the definition of the *rectangle formula* and the *pieces formula* can be seen in the following figure.

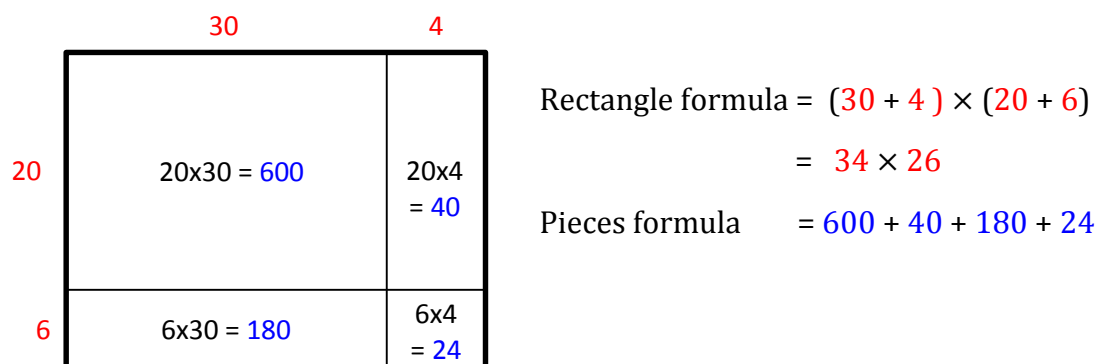


Figure 1: The rectangle formula and the pieces formula in the area model

The students then explored and discussed about the use of area model in associated with multiplication. In this case, the rectangle formula represents the multiplication and the sum of the pieces formula represents the product of the multiplication. After that, by using a context, variable was emerged into the area model to represent unknown lengths. The following figure shows how a variable (x) was emerged in a house plan context.

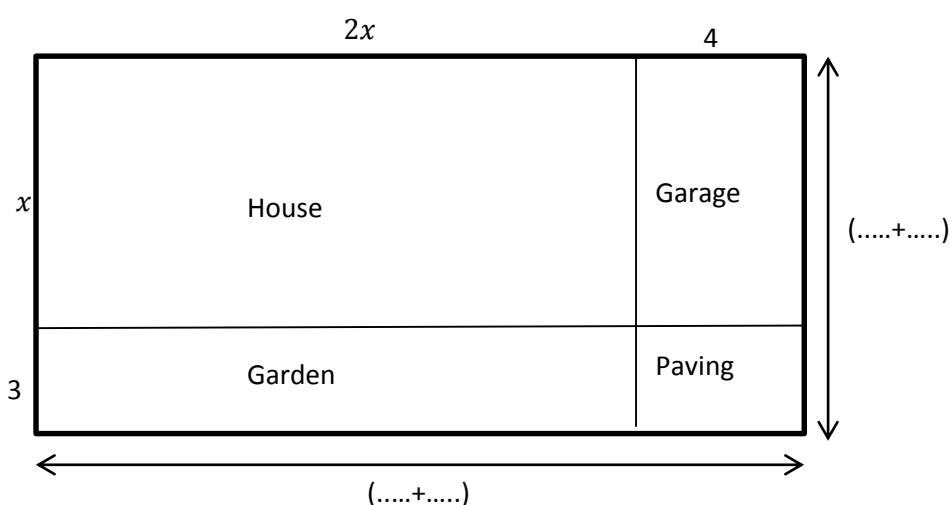


Figure 2: House plan, which emerges variable to represent unknown length

Since the students had a good understanding of using area model to do multiplication of integers,

The factorization

After the students were able to use area model to find the product of binomials multiplication, the researchers started to switch the problems into factorization problems. The following figure shows the written work when the second group encountered factorization problem for the first time. This problem was basically a

factorization problem, but not in formal or straight form. The known was the *pieces formula* or the product of the multiplication and the students needed to find the *rectangle formula*, which was the multiplication.

Rectangle formula = $(x+1) \times (x+6)$
 Pieces formula = $x^2 + 1x + 6x + 6 = x^2 + 7x + 6$

Figure 2: Students' written work on the first factorization problem

Alifia and Diana from the second group realized that this problem was similar to the previous problems where they need to do binomials multiplication. Hence, they sensed that this problem hold the same principles. As the consequence, in solving this problem they related with the previous problems and faced no meaningful difficulty in making sense and finding that x^2 was made of x times x . Furthermore, both students agreed to put 6 in the bottom right of the inside of the area model, and got 6 and 1 in the sides of the area model. However, both of them agreed to put $7x$, taken from the *pieces formula*, in the rest spot, the right upper in the area model. In this case, since both students had found the 1 and 6, the researcher confronted their answer ($7x$) with the question of finding the piece area for that spot in regard to the multiplication of 1 and x . Both students answered that the piece formula for that was $1x$, which was different from their initial answer. However, when the researcher tried to ask them to further think about that, both students found difficulty. Hence, since the first group turned out to be able to understand and make sense this problem themselves, the researcher conducted a whole class discussion asking the first group, consists of Aldi and Ghifary, to explain to the second group.

Fragment 1: Whole class discussion in understanding the principles of factoring quadratic polynomial

- 1 Researcher : (asking to Aldi and Ghifary) Diana and Alifia were confused whether it should be $1x$ or $7x$. How do you think?
- 2 Aldi : $1x$.
- 3 Researcher : $1x$?
- 4 Ghifary : Because this (pointing at $6x$ and $1x$ inside the area model) should be sum up. $6x$ and $1x$ equals $+7x$.
- 5 Diana : Repeat please.
- 6 Researcher : How, what's needed to be sum up?
- 7 Ghifary : $6x$ (pointing at $6x$ in the table) plus $1x$ (pointing at $1x$ in the table) equals $7x$ (pointing at $7x$ in the *pieces formula*)
- 8 Diana : Why it is "sum up"?
- 9 Ghifary : This is "plus"
- 10 Alifia + Diana : Hah? Where is the "+"?

- 11 Aldi : no. *pieces formula* is the sum of all of these (pointing at the x^2 , $6x$, $1x$ and 6), right?
- 12 Alifia + Diana : (clapping hands)
- 13 Researcher : Aldi said *pieces formula* means you need to sum up all of these (pointing at the x^2 , $6x$, $1x$ and 6). So what's the effect here, to decide whether it should be $1x$ or $7x$?
- 14 Aldi + Ghifary: if it is $6x$ plus $7x$, the result is more than the expected answer.
- 15 Ghifary : The result is (counting with fingers)
- 16 Alifia : $13x$.
- 17 Reseachrer : $13x$. And here (pointing in the *pieces formula*) is...?
- 18 Ghifary : that's the answer. $7x$ (pointing at $7x$ in *pieces formula*)

The line 8 to 11 shows that Aldi and Ghifary were able to relate this problem with their previous understanding about area model and the notions of *rectangle formula* and *pieces formula*. This ability led to decision to choose $1x$ instead of $7x$. The reasoning of Aldi and Ghifary about their choice was shown in the line 4 to 7 as well as line 14 to 18.

CONCLUSION

To answer the research question about how does the area model support students' understanding during the transition from binomials multiplication to factorization, we may consider the contribution of the understanding of using area model for multiply binomials to solve the factorization problem. In this regard, the students built up understanding and made sense of the principles of factoring quadratic equation by relating the problem with the principles of using the area model to multiply binomials. In other words, the area model supports the students in the context of understanding the principles of factorization. This understanding was derived from their understanding of how the area model worked to find the product of binomials multiplication. Once the students hold the understanding of the principles of factoring quadratic equations, they would develop strategies, using the area model as a tool or visualization, to do factorization.

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