DEVELOPING PISA-*like* MATHEMATICS TASKS TO PROMOTE STUDENTS' MATHEMATICAL LITERACY

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Abstract

Considering the low performance of Indonesian students in PISA (Programme for International Student Assessment) survey in the period 2000-2012, the need of PISA-like tasks promoting mathematical literacyis important to be developed as learning resource for practisioners. For these reasons, this study aims to produce a set of PISA-like mathematics tasks which are valid, practical, and has the potential effect as well as explain the process of developing those tasks. Thus, we used the preliminary stages, and prototyping using formative evaluation (self evaluation, expert review, one-to-one, small group, and field test). A total of 67 students of senior high school students at Palembang and 12 experts were involved in the prototyping phase. Data collection techniques used are walkthrough, documentation, questionnaire, test results, and interviews. This study produced a set PISA-like math tasks as many as 12 items in the category of content, context, and process. The validity came from the experts who reviewed the prototype at this stage, while the practicality, particularly, obtained from the revised tasks in the steps of both 'one-to-one'and 'small group'. From the field test, we conclude that the tasks also potentially effect to the students' mathematical literacy in activating the indicators of each FMC, i.e, communication, reasoning and argumentation, representation, mathematising, problem solving, and using formal/symbolic language and the students' interest and seriousness when solving the tasks.

Keywords: PISA-like mathematics task, mathematical literacy, fundamental mathematical capabilities, mathematical process

INTRODUCTION

The latest PISA mathematics survey results in 2012 shows that Indonesian students, as in several previous results, only reached below level of items assigned and bottom rank compared to the other country participants, i.e. 64 out of 65 countires (see PISA results: www.oecd.org/pisa). This result shows that Indonesian students have not been able to well performed their mathematical literacy when solving PISA tasks. In other words, they found difficulties to use mathematics in a variety of contexts. The low mathematical literacy achievement in Indonesia was also addressed by several studies. Wu (2011) stated that the achievement of many students in Asia are still low in solving problems coming from the everyday world as it is often found in the PISA questions. A study of Edo, S.I., Hartono, Y., & Putri, R.I. (2013)presented that Indonesian students have difficulty in formulating such everyday problems in mathematics, understanding the mathematical structure and evaluate the reasonableness of mathematical solutions to real-world contexts. This study was supported by the Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014) and Lutfianto, M., Zulkardi, & Hartono, Y. (2013), each of which revealed that the failure of students working on the PISA happened when they formulated the problem to formal mathematics and by the time they get results mathematically, which is then not followed up on stage to interpret the situation/context of the desired matter. All forms of this weakness seems to be supported by the fact that it is a lot of test material in question in the PISA assessment was not included in the Indonesian mathematics curriculum (Kemdikbud, 2013).

Considering this fact, Indonesia through this moment has been started to implement a new curriculum in 2013 whose framework of its development is encouraged by the PISA result (Kemdikbud, 2013). Consequently, the need of developing learning resources fostering students' mathematical literacy is important to be done.Stacey (2013) recommended to conduct study in designing teaching program that uses PISA tasks to improve the quality of mathematics teaching and analyze how students make errors in solving PISA tasks. Meanwhile, Zulkardi (2010) suggest to develop PISA-like mathematics tasks as well as use them in instructional practices. To produce PISA tasks which valid, practical, and has a potential effect, research on developing PISA-*like* tasks also were conducted by several researchers with a focus on examining the context (Lutfianto, 2012), level (Kamaliyah et al, 2012), and competence (Edo et al, 2013; Novita et al, 2012).

Therefore, this present study aims to produce a set of PISA-like mathematics tasks which are valid, practical , and has potential effect to the development of students' mathematical literacy.

THEORETICAL FRAMEWORK

Mathematical literacy

One can views mathematical literacy as mathematics which is not only about it as a domain, but also it as application in concrete and practical day-to-day existence (Mbekwa, 2006; Shiel et al, 2007). In PISA 2012, mathematical literacy is defined as an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2013). This definition can then be described into several points, i.e. what kind of problems needed to be solved (challenge in real world context), how mathematical process perform in the solution (mathematical thought and action), and how mathematical competencies are activated in each of the process. See the figure below.

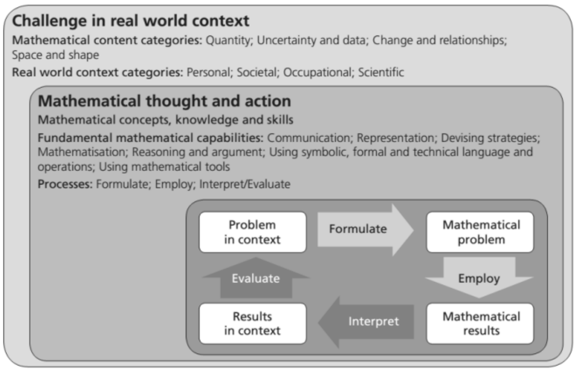

Fig. 1 A model of mathematical literacy in practice (OECD, 2013, Stacey, 2012)

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Mathematical literacy starts from real world problems, which are categorized into category of contexts, i.e. personal, occupational, societal, and scientific, and contents, i.e. change and relationship, space and shape, quantity, and uncertainty and data (see OECD, 2013, Stacey, 2011). The process of mathematical literacy begins from identifying the real world problem and formulate the problem mathematically based on the concepts and relationship inherent in the problem. After getting an appropriate mathematical form of the problem, the next step is to employ certain mathematical procedures to obtain mathematical results, which then interpret those back into the initial problem.

The word 'formulate' refers to the ability of individuals to recognize and identify opportunities to use mathematics and then provide mathematical structure for a problem that is presented in some contextual form (OECD, 2013). Meanwhile, OECD (2013) defines word 'employ' as a person's ability to use the concepts, facts, procedures, and reasoning to solve problems that have been formulated to obtain a mathematical conclusion. In the process of employing, mathematical procedures such as arithmetic calculations, a person should show ability of completing the equation, making deductive reasoning of mathematical assumptions, manipulating symbols, filter information contained on the tables and graphs, forming regularity/patterns, identifying relationships in mathematical unity, and making mathematical arguments. Lastly, the word 'interpret/evaluate' reflect a person's ability to interpret solutions, results, or conclusions of mathematical problems in a real-world context.

PISA framework also mentions some competencies that underlie mathematical process performed by a problem solver. Turner, R (2012) mentions seven fundamental mathematical capabilities that underlie a person's knowledge and skills in using mathematics effectively.

1. *Communication,* covers reading, decoding and interpreting statements, questions, tasks or objects enables the individual to form a mental model of the situation (receptive component); presenting answers or justification of the statement provided.
2. *Reasoning and argumentation*, thinking logically that explore and link the elements of the problem so that can make inferences from these elements, examine the justification given, or provide justification to solutions or statements of the problems.
3. *Mathematising,* covers transforming a problem defined in the real world to a strictly mathematical form (which can include structuring, conceptualising, making assumptions, and/or formulating a model), or interpreting or evaluating a mathematical outcome or a mathematical model in relation to the original problem.
4. *Representation*, selecting, interpreting, translating between, and using a variety of representations (e.g. graphs, tables, diagrams, pictures, equations, formulae, and concrete materials) to capture a situation or interact with the problem.
5. *Devising problem solving strategies,* covers selecting or devising a plan or strategy to use mathematics to solve problems arising from a task or context, as well as guiding its implementation.
6. *Using symbolic, formal and technical language and operations,* covers understanding, manipulating, and making use of symbolic expressions within a mathematical context (including arithmetic expressions and operations), governed by mathematical

conventions and rules; understanding and utilising constructs based on definitions, rules and formal systems.

METHOD

This is a designresearch using the type of development study. The emphasis of this study is on iterative development with formative evaluation in various user settings (Plomp, T., & Nieveen, N, 2007).The formative evaluation consisted of preliminary stage and prototyping phase which includes self evaluation, expert reviews and one-to-one, small group, and a field test (Tessmer, 1993, Zulkardi 2002).

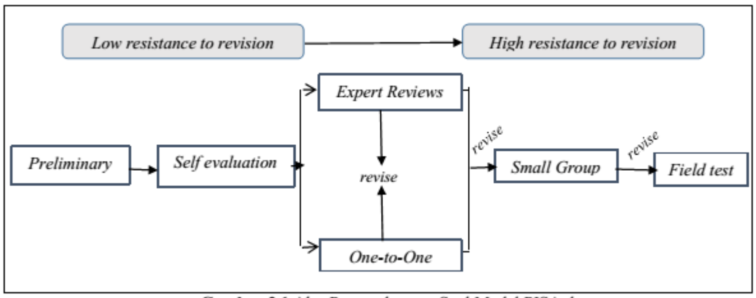

Fig 2. Formative Evaluation (Adopted from Tessmer, 1993; Zulkardi, 2002)

The development process started from preliminary step by grasping with the concept related to developing mathematical literacy tasks then used it to design an initial prototype. This prototype was then self-evaluated before entering the next steps. In expert review, twelve experts were involved to validate the tasks in terms of content (satisfied framework of PISA 2012), construct (develop students' mathematical literacy), and language (grammatical error, obvious limitiation of questions and answers).Simultaneouly with expert review, 4 students in one-to-one validation evaluate particularly on how they understand the information e.g. picture, phrase, etc in the tasks and not focus on how they answer the tasks. These results gave important suggestions to revise the tasks so that those could be re-evaluated in small group. The smallgroup phase involved 12 students with various academic abilities to solve the tasks in 90 minutes. Here, we firstly obtained students' performance in solving the tasks because we scored and analyzed a variety of student's answer. We used this data as a view to assess student's real performance in larger test, i.e. field test. Meanwhile, the field test was conducted on 50 students aged maximum 16 years 2 months from a senior high school at Palembang.

RESULT AND DISCUSSION

Developing tasks

In the preliminary stage, we conducted several steps:(1) examined the literature on developing mathematical tasks, framework of the PISA 2012, mathematical literacy, the relationship between the current curriculum and the PISA survey, (2) designed an initial prototype comprising a set of PISA-like task and its scoring, (6) determined the validators, and (7) determined research subject. At the stage of self evaluation,we examined the initial prototype resulting prototype I. Prototipe I was assessed and evaluated by validators (expert review) and students (one-to-one). The experts are from Mathematics Expert Group of PISA (Prof. Kaye Stacey and Dr. Ross Turner), PMRI lecturers (Prof. Dr. Ahmad Fauzan, Prof. Dr. Ipung Yowono, Prof. Dr. RK Sembiring, Dr. Sugiman, Dr.Yenita Roza , Prof. Dr. Siti M Amin), 1 math teacher (Yanna Sanova, S.Pd.),

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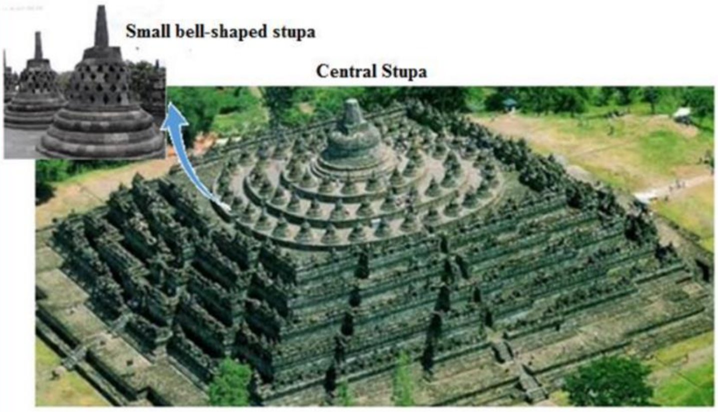
and an Indonesian PISA wes (Ariyadi Wijaya, M.Sc). The following is one of tasks example on this situation.

Before revision

BOROBUDUR

Borobudur, a Buddhist temple located in Central Java, consists of three levels: Kamadhatu, Rupadhatu, and Arupadhatu. Arupadhatu is the topmost level which is a circular platform. On the platform, there are a number of small bell-shaped stupas arranged in three circular platforms surrounding one large central stupa as shown in the following figure.



Source: dimensionsinfo.com

Question 1: Borobudur
How many small stupas on Arupadhatu? Write down your own strategy.

Fig 3. Task 1 before revision

Table 1. Comments from Experts and Students on task 1

Validation	Comments/Responds	Revision
Expert Review	Add information on the question of whether the answer must be exact or result of estimation: *"I like the context. question 1 is good. Maybe you should tell students whether they have to be exact or approximate. I expected to give an approximate answer – in fact students do not really know they are symmetric, so it can only be an estimate. you will need to think about the best wording in bahasa."*	➢ Fix the context of task become a problem from a photographer who wants to redraw a model of the photograph of Borobudur temple vertically taken from a helicopter on the air ➢ Change the tupe of tasks from *open-constructed response* became *selected-response.* Reason: easy to code the students' answers and match with the contextual situation given on the tasks
	Add information how important someone need to know the number of stupas: *"I see from the photograph insert that it is possible to get close enough to the stupa platforms to actually count the stupas. So, why would anyone need to estimate the number from an aerial photograph?"*	
	Consider how to code students' responds: *"I do like the idea of asking for an explanation of the strategy. The problem will be how you will code different responses to the question, and how you will put this with their answer to the 'how many' question."*	

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		➢ Clarify the picture by enlarging photo size
One-to-one	I do not know the exact number of stupas on the back. If there is an additional information about the symmetrical arrangement of the stupa or something like that, surely it will help my calculations.	➢ Add information that the stupas are arranged concentrically which is put in the general information.
	The picture less clear, enlarge the picture	➢ The picture was then enlarged.

After obtaining some suggestions from both experts and students, the prototype 1 was revised become prototype 2. The following is the revised tasks based on the table 1.

After revision

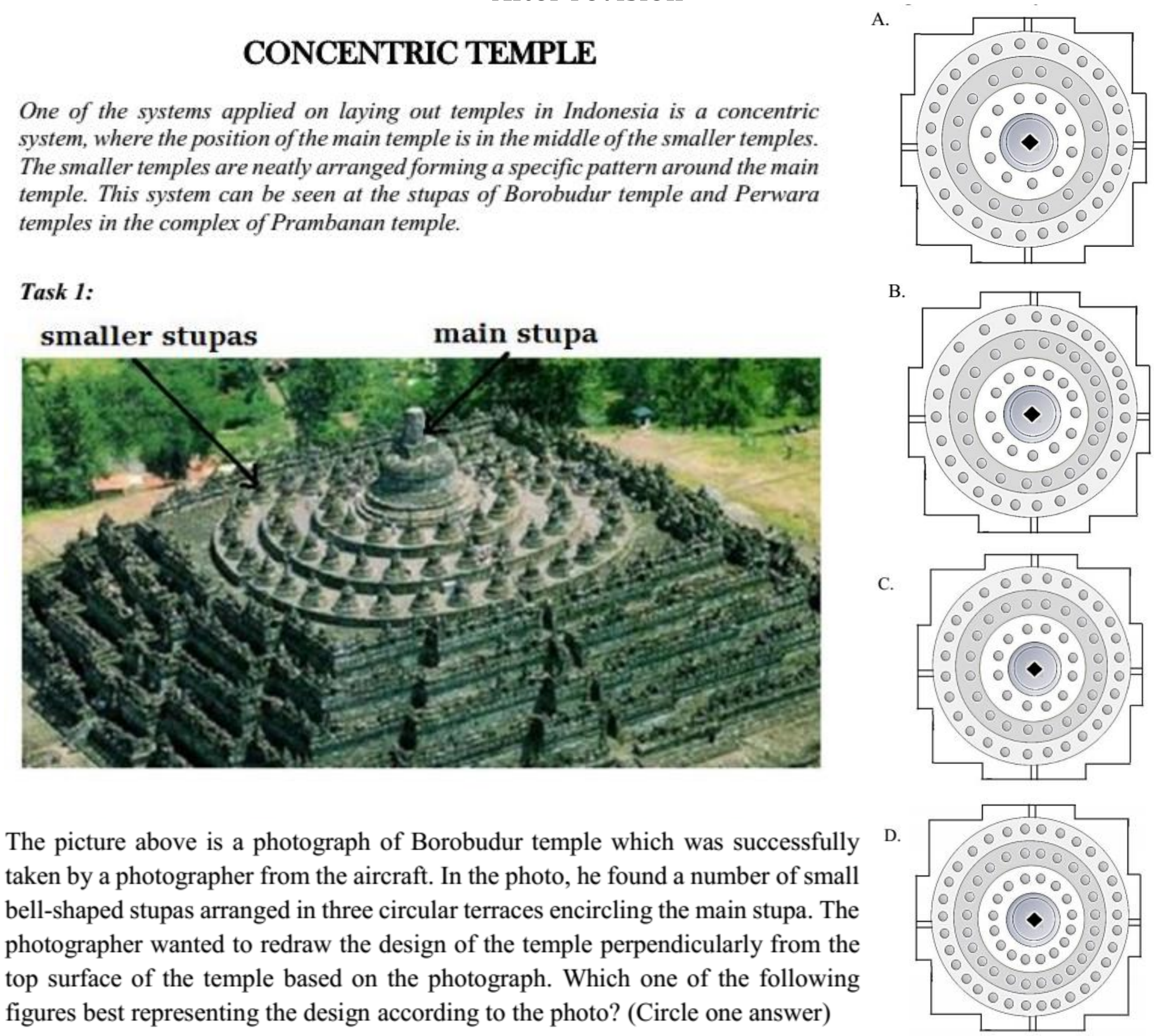

Fig 4. task 1 after revision

Now, we will see another task which is still in same context with the task above.

Before revision

Question 2: Borobudur

Suppose there are 8 platforms on Arupadhatu containing a number of small stupas which are arranged to follow the pattern as shown in the figure. Determine the total number of small stupas contained on Arupadhatu. Support your work by involving some calculation.

Note: The pattern informs that the number of stupas on each platform is increasing with the number of platforms

Fig 5. task 2 before revision

Table 2. Comments from Experts and Students on task 2

Validation	Comments/Suggestion	Decision for Revision
Expert Review	The task is dependent with the previous task." *Here you have a problem of question independence. You cannot really question 2 unless you have the correct answer to question 1. That is unacceptable practice for a test like PISA."*	Remove the task and change the task with another task with similar context: Prambanan temple. The task examine similar mathematical content with the task before revised. The unit of context changed from "Borobudur temple" to "Concentric Temple"
	Offering new idea of changing content of question: *"I'm thinking of a different question – just an idea, it might be too difficult, and it might be too difficult to code the responses. "At another historic site, a similar structure has four rings of stupas containing, respectively, 12, 20, 28 and 36 stupa. Give a mathematical argument to explain why it is that this sequence of numbers increases in steps of the same amount."[I try to get around the independence issue by using the same kind of context, but shifting it a little to a different location. I admit that this might not be realistic.]"*	
	The task is less contextual	
One-to-one	The solution of the answers depend on answers on task 1 so if you can not answer the task no one certainly can not answer task 2	The task is removed and changed with the task using context of Perwara temple in Prambanan temple

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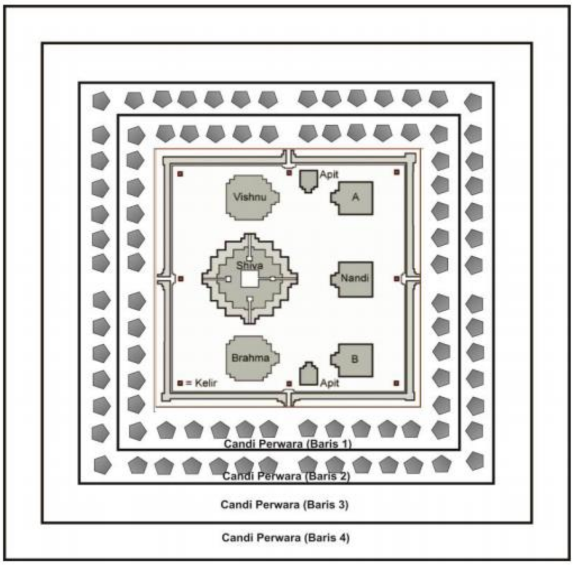
After revision

Task 2:

The concentric temple can also be seen at Prambanan temple. The component of the outer part of this temple is called 'Perwara temples' which are arranged in four rows facing three main temples: Vishnu, Shiva and Brahma.



The picture above shows the arrangement of Perwara temple which are already restored on row 1 and 2. The arrangment of the remaining Perwara temples which will be restored on the 3rd row and the 4th row follows the pattern of previous arrangements. What is the total number of perwara temples that still need to be restored ? Explain your answer.

Fig 6. Task 2 after revision

Prototype 2 was then tested on small group consisting of 12 students. The results of the small group show that the tasks has coefficient of high reliability of 0,838 but some tasks were empirically invalid. Therefore, we reviewed each item developed primarily on the invalid tasks for discarded, maintained with revisions, or retained without revision . This decision was based on the results of activities : (1) giving a questionnaire asking students 'opinions on the tasks, (2) examining the distribution of students' answers , and (3) interviewing two subjects of small group to invetigate whether the student were not able to solve the tasks in the absence of the aid scheme or as a matter of readability issues. The results of this evaluation resulted in a prototype 3 which was then used in a field trial test as many as 50 students in order to know the potential effect of the tasks as well as to invesitage the student's profile on performing mathematical literacy. The following table show a distribution of 12 tasks in prorotype 3 based on the domains of context, content, and process.

Table 3. Profile of Tasks

Unit of Context	Task	Content	Context	Process	Description
Concentric temple	1	Quantity	Societal	Interpret	Determine the best design of a temple representing a given photograph by finding the best appoximation of the number of small stupas on the temple
	2	Change and Relationship		Formulate	Find the number of temples needed to be restored based on the pattern given on the design on temple arrangement
Futsal Tournament	3	Uncertainty and data	Personal	Interpret	Find score obtained by a match of two futsal team based on the table consisting data about goals and the number of wins
	4	Uncertainty and data		Formulate	Find the number of drawn matches based on a procedure given by a textual information
Biopori	5	Space and shape	Scientific	Employ	Find the surface area of biopori which infiltrate water
	6	Space and shape		Formulate	Determine the maximum number of biopori which can be built based on a rectangular pattern given on the picture
	7	Change and relationship		Interpret	Determine a graph best representing the relationship between the diameter of a biopori hole and the maximum volume of waste that can be loaded by a biopori
Phone Assembly	8	Quantity	Societal	Interpret	Determine the validity of statement related to a concept of percentage
	9	Uncertainty and data		Formulate	Determine one of the two companies having the higher overall percentage of faulty assembled phones

Archery	10	Quantity	Societal	Employ	Find a score achieved by an archer based on the number of shots to some scoring zone on a archery board
	11	Uncertainty and data		Interpret	Determine an archer having the biggest probability to succesfully shot to the scoring zone having the biggest value
Making a net of Cuboid Cardboard	12	Space and Shape	Scientific	Interpret	Draw a net of cuboid box based on the cutting direction on the picture

Potential effects of the tasks

The purpose of the field test was to find out the potential effects of the tasks on students' mathematical literacy. After the students completed work on the tasks, we gave questionnaires to all students, and interviewed 6 students to obtain data about the potential effects. To discuss the potential effects, we compared the results of students' responses on the written questionnaire regarding the questions 1 (fundamental mathematical capabilities/FMC), questions 2 (students' interest and seriousness on the tasks), questions 3 (general impression on the tasks), and interviews.

The students' responses on the questionnaire is shown below.

Table 4. Students' Responses in Activating FMC

No	Activated FMC	Percentage of Students' Responses
1	Create mathematical models, such as creating mathematical equations, making the pattern sequence number, and the like (mathematising)	56%
2	Make written mathematical argumentation/calculation correctly (communication)	52%
3	Create model of representation and/or utilizing images, tables, graphs, and the like to help find the answer (representation)	64%
4	Choose and compare strategies for finding solutions (devising problem solving strategies)	62%
5	Make reasoning by linking the information with the existing experience (reasoning and argumentation)	80%
6	Use and manipulate formulas or specific mathematical procedure to obtain solutions (use of formal/symbolic language)	42%

Based on the table above, 6 of 7 FMCwere admitted to be used by more than 50% of students in solving the prototype 3. Among the FMC, reasoning and argumentation were recognized by most students (80%) in solving problems. This recognition is in line with some of the written comments in general impression, like as shown below.

student 1 : This problem requires a lot of logic and not fixated on formulas *student 2 : The tasks require more reasoning. It is different with what I used to solve, where the qonly ask me to use an available formula. it is good to train my thinking.*

Furthermore, the potential effect of the tasks is also seen from the extent to which students are interested and serious in solving the tasks. The data supporting students' responses on the tasks are given in the following table.

Table 5. Students' Responses on Prototype 3

How interesting are the tasks?	Percentage of Students' Response
I am interested and serious in working all the tasks	34%
I am only interested and serious in working certain tasks	64%
I am intereseted, but not serious in working the tasks	0%
I am not interested in the tasks at all	2%

As a proof how students activate their FMC into mathematical process: formulate, employ, interpret in solving the tasks, the following are examples of students' work on task 2.The aim of the task is to examine students' ability in observing empirically the pattern of arrangement of temple on certain rows. (see fig)

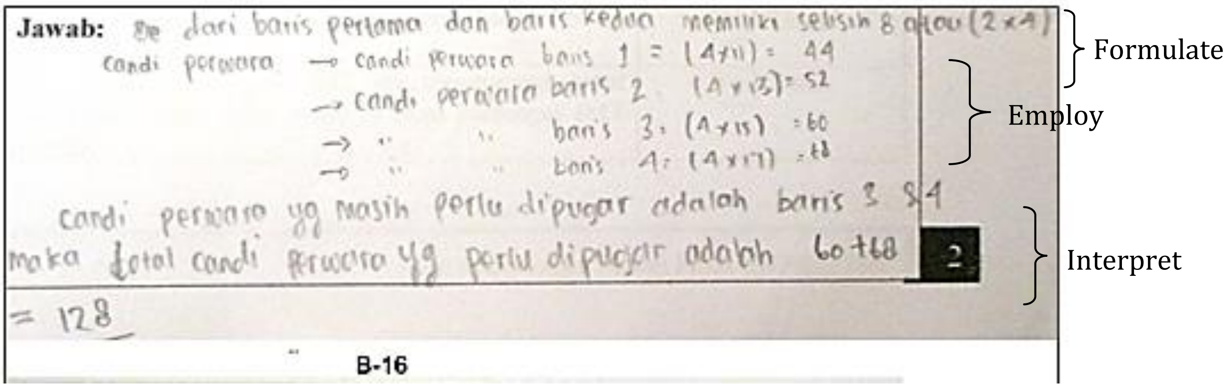


Fig 7. Student's work on task 2 (B-16)

Answer of student B-16 above shows activation on FMC in his work. The studentinvolved constructive *communications* by writing the result of inductive reasoning process to show that the difference between the number of temples on each two rows is 8 units. This idea can be seen as an interpretation which is based on the observation of the pictorial*representation*, namely the design of the temple arrangement. In terms of the process involved, it can be said that this activity is done as a manifestation of the process of *formulating*. Meanwhile, at the time of *employing,*the student execute*problem solving strategy* by listing the number of temples in each row then realized that there is a regularity of arrangement of temples on each side of the row. Therefore, it can be concluded that the number of temples on each side are same. This idea was used to write the number of temples on the 3rd row and 4th as (4x15) and (4x17) consecutively because there are 4 sides on each row. Meanwhile, another different idea was expressed by student 2 by using the arithmetic sequence formula in finding the solution. The following shows this idea.

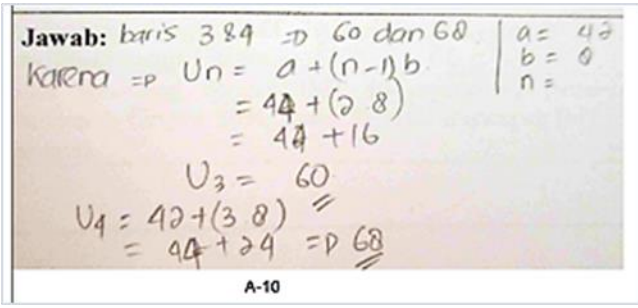


Fig 8. Student's work on task 2 (A-10)

In terms of FMC involved, student A-10 implementedher *strategy* to perform the *use of symbolic/simple algebraic operations* by linking the regularity of the form of the temple with a scheme that has been learned about the formula of arithmetic sequence. In *representing* solution, he was able to identify that the number of temples in the first row and the second row can be seen as the first and second terms in arithmetic sequence, in order to obtain the value of first term, a = 44 and different, b = 8. This activity continues to perform substitution value of n, which shows a sequence of rows into a formula that has been constructed to determine the number of temples in the 3rd row and 4th row. However, from the picture it appears that A-10 has not added up the number of temples on in the 3rd row and 4th. Here, we find incomplete steps performed by students in solving the task.

CONCLUSION

This study produced set of PISA-like mathematics tasks which are valid, practical. Based on the results, prototype 3 has potential effect to develop students' mathematical literacy, shown by their activation of FMC into each of mathematical processs: formulate, employ, and interpret, Other indications of this effect are also seen from their seriousness and interest when solving the tasks. Lastly, we suggest teachers and other practisioners to use the tasks from this study as tools in designing PISA problem based learning as well as design and evaluate tasks which satisfy the characteristics of mathematical literacy as mentioned in PISA framework.

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