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Southeast Asian Mathematics Education Journal – SEAMEJ

**Description of *South East Asian Mathematics Education Journal*
(SEAMEJ)**

South East Asia Mathematics Education Journal (SEAMEJ) is an academic journal devoted to reflect a variety of research ideas and methods in the field of mathematics education. *SEAMEJ* aims to stimulate discussions at all levels of mathematics education through significant and innovative research studies. The Journal welcomes articles highlighting empirical as well as theoretical research studies, which have a perspective wider than local or national interest. All contributions are peer reviewed.

SEAMEO QITEP in Mathematics aims to publish *SEAMEJ* twice a year, in June and December.

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Editorial

This is the third edition of the South East Asian Mathematics Education Journal (*SEAMEJ*) which is an academic journal devoted to publishing a variety of research studies and theoretical papers in the field of mathematics education. *SEAMEJ* seeks to stimulate discussion at all levels of the mathematics education community. *SEAMEJ* aims to eventually publish an edition twice a year, in June and December.

SEAMEJ is supported by the Southeast Asian Ministers of Education Organization (SEAMEO), Centre for Quality Improvement of Teachers and Education Personnel (QITEP) in Mathematics situated in Yogyakarta Indonesia. Launched on July 13, 2009, there are now three QITEP SEAMEO Centres for Quality Improvement of Teachers and Education Personnel in Indonesia. One centre is in Mathematics (Yogyakarta), one in Science (Bandung) and one in Languages (English - Jakarta).

The first edition was produced using revised papers from the first International Symposium of QITEP Mathematics in November 2011, where a number of paper presenters were approached to submit their reworked papers to this journal. In this issue we are proud to state, are papers that have been submitted by researchers from a number of countries. We hope that trend this will continue and swell as the journal becomes widely read and enable us to meet our aim of two editions in one year.

The second issue reflected the variety and richness of the Asian region with papers covering a wide range of issues and perspectives such as mathematics teaching in Shanghai China; a longitudinal study of Australian transition years and school student engagement; a further elaboration of bibliotherapy with a framework for use with pre-service teachers; a report on a professional learning workshop using a computer adaptive assessment program; and, the implications that brain research has for the teaching and learning of mathematics.

In this issue we present papers that were subjected to a blind review process by the International Review Panel and we are thankful to the panel for their continued support. While these papers come from a variety of countries and cover different aspects of teaching and learning, there seems to be a common theme of joyful and meaningful learning which is highlighted in the first two papers from Indonesian researchers.

As we are still refining our processes, we wish to apologise if we have made errors or omissions. We welcome feedback and suggestions for improvement, but most of all, we welcome paper contributions.

The Journal seeks articles highlighting empirical as well as theoretical research studies, particularly those that have a perspective wider than local or national interests. All contributions to *SEAMEJ* will be peer reviewed and we are indebted to those on the International Advisory Panel for their support.

Allan L. White
Wahyudi



Joyful And Meaningful Learning In Mathematics Classroom Through Internet Activities

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Abstract

This paper arises from the author experiences over the last eight years with regards to investigating how the Internet could be used as a tool for mathematics learning, mathematics teaching as well as for professional development. This paper illustrates three main categories of the potential of the Internet for learning: the Internet for information, the Internet for communication, the Internet for collaboration, followed by a description on how the use of the Internet could make learning mathematics interesting and meaningfully. Lastly, through the use of a case study and my experiences working with group of students using the Internet, I argue that the success of integrating the Internet into mathematics teaching and learning depends very much on teachers' knowledge and their philosophical beliefs about learning and teaching, learners, mathematics, and technology.

Keywords: Internet for communication, Internet for collaboration, Internet for learning, meaningful learning, teacher knowledge, beliefs.

Introduction

The genesis of my investigation about Internet for learning came firstly from my personal experiences. I found that the Internet had been a significant medium to achieve my objectives, namely to improve my qualities as a lecturer, a researcher, and a facilitator for mathematics teachers. This was primarily because in a situation where I lacked learning resources (e.g., current books and journals), the Internet enabled me to find many valuable resources, and even allowed me to communicate with several experts in other countries (e.g., sharing teaching and learning experiences, asking for online references). It has been meaningful and joyful learning experiences for me. This has motivated me to encourage others to take advantages from the accessibility of the Internet (see e.g., Ernawati & Patahuddin, 2009; Patahuddin, 2007; Patahuddin & Rokhim, 2009; Patahuddin, Rokhmah, & Nur, 2010; Rokhim & Patahuddin, 2010; Rokhmah, Patahuddin, & Nur, 2010). This paper focuses on the potential of the Internet as a tool for meaningful and joyful mathematics learning.

What Is The Internet And Its Potential?

The Internet is not one place or one company. It is a descriptive term for a web of thousands of interconnected broad-band and narrow-band telephone, satellite, and wireless networks built on existing and planned communication technology. This infrastructure is a network of networks, reaching out and connecting separate islands of computer, telephone, and cable resources into a seamless web (Gattiker, 2001; p. 3).

The citation above shows us the sophistication of current technology which has become part of people's lives. It can be categorised into three main functions: the Internet for

information, the Internet for communication, the Internet for collaboration. All these can be linked to mathematics education and has the potential advantages and limitations of the Internet in enhancing mathematics teaching and learning.

The Internet for Information

A wide range of information can be found through the Internet such as information from online books, atlases, newspapers, journals, magazines, dictionaries, radio or television programs, and so on. The Internet, supported by multimedia and good networks, can have multiple functions, for instance, serving as a radio, as a television, as a video, as well as a virtual visiting speaker. The Internet has opened broad opportunities for more communities to access information relatively instantly, regionally, nationally, and internationally. The Internet has also become increasingly familiar to more and more people due to its user-friendliness. Unfortunately, because nobody owns the Internet, anybody is able to post reliable, but also non-reliable information via the Internet. Through the Internet, users can be provided, not only with up-to-date information, but also out-of-date information. Internet users must also be aware of its instability, as information that is available now can be unavailable at any time in the future.

The literature has identified the Internet as a source of information, highlighting potential advantages and disadvantages for educational purposes. For example, Glavac (2004) described three useful websites for teachers and students, that may even be useful for parents, that contain many different educational topics on different subjects, including mathematics. One of the websites has been created by teachers for teachers. Engelbrecht and Harding (2005), in their attempt to classify mathematical sites on the Internet for undergraduate students, have also provided several examples of sites for mathematical enrichment, mathematical visualisation, sites that provide mathematical exercises for practice, quizzes, Olympiad questions, as well as information about full-online courses from a range of universities. Byerly and Brodie (2004) have provided a list of almost thirty mathematical websites, providing a brief description of their contents. Ameis and Ebenezer (2000) have written a book entitled “Mathematics on the Internet, A resource for K-12 Teachers”, which includes practical information about using the Internet, a discussion on how mathematics learning can be facilitated via the Internet, and which also provides links to mathematical teaching resources and sites for professional development.

The Internet for Communication

The Internet increasingly has become popular as a means of communication at a regional, national, and global level. There are different ways that communication is enabled through the Internet, including via electronic mail (e-mail), mailing lists, online discussions or conferences, bulletin boards, chat rooms, Facebook, MySpace, Friendster, Twitter, Skype,

and so on. The Internet as a communication tool is distinct from other forms of communication, such as the telephone, telegram, air mail, or face-to-face communication as users are able to send or receive messages relatively instantly and are able to send messages simultaneously to recipients around the globe. Recipients of messages can also benefit from this flexibility because they are able to check their e-mail or participate in discussion forums anytime, anywhere. All these features have opened opportunities to build up learning communities among Internet users from all over the world. However, these positive features can be outweighed by negative features, such as making users vulnerable to receiving harmful messages from unknown senders.

Engelbrecht and Harding (2005) have described sites (e.g., Math Forum) which enable users to share ideas, ask mathematics questions or get answers to mathematic problems, and access links to mathematical resources. Newell, Wilsman, Langenfeld, and McIntosh (2002) described how mathematics teachers from distant areas are communicating with other teachers instantly via the Internet instead of waiting for face-to-face communication at conferences. Further, Frid (2002) described how students benefited from communicating mathematical ideas with other students from different schools in different countries.

The Internet for Collaboration

Collaboration usually involves more than one person, or a group, working together to achieve particular goals. It requires communication between members and usually requires all members to be present at a site at some point. Lack of communication among members will inhibit the effectiveness of collaboration. With the Internet, communication has become increasingly easier, cheaper and even more effective. By using the Internet, people at a distance are able to undertake a project together in a virtual site. Thus, the Internet has clearly opened many opportunities to enable people from all over the world to collaborate.

Chausis (2002), a technology trainer, presents her ideas about tele-collaboration, that is by using the Internet tools and online resources to work together with people in different places. She introduces several Listserv resources, websites that provide opportunities for teachers to interact with other teachers via the Internet, and provide more specific websites to support collaboration between teachers for classroom activities such as TeacherLine, NickNacks Telecollaboration, Teacher-to-Teacher Collaboration, and so on. In the Teacher-to-Teacher Collaboration website, a great number of websites can be found that provide opportunities for teacher professional development.

Clark (2000) highlights the potential of the Internet as an aid to collaboration and reveals several methods to facilitate and stimulate collaboration in online environments. He argues that collaboration is a solution of a problem caused by disadvantages of using the Internet for communication; it is “the loss of student interaction and friendships” (p.8). According to Clark, collaborative skills can be fostered by using e-mails, class bulletin

boards, online conferences, gated conferences and so on, as well as by involving participants in discussions, group projects, and group papers.

Generally speaking, it has been widely accepted that the Internet has advantages and disadvantages as a source of information, as a medium of communication and as a site for collaboration. The rapid growth of the Internet should motivate researchers or educators to explore ways to optimise the use of the Internet for teaching and learning.

Internet Activities for Joyful and Meaningful Mathematics Learning

Several important questions are how Internet technology can assist teaching and learning to enable students to learn mathematics joyfully and meaningfully? What mathematical websites that are feasible to use for this purpose? This section, is trying to answer those questions.

The Internet can assist in supporting individual learning needs as well as assist learners in developing skills and knowledge. The Internet provides opportunities for students to engage with more dynamic forms of mathematics with multi-representations of mathematics concepts than those traditionally displayed in textbooks (see for example in <http://illuminations.nctm.org>; <http://www.beenleigss.eq.edu.au>). The role of knowing different representation of a particular concept in mathematics is to enhance conceptual understanding (Alagic, 2003; Crawford & Evelyn, 2003) and this means potentially meaningful for learners (Wiske, Sick, & Wirsig, 2001). As a result, teachers need to be aware of the influence these different representations have on the way students decode information. Crawford & Evelyn (2003, p. 172) argues that:

Digital manipulatives can be appropriately and successfully integrated into a mathematical learning environment through the use of web-based materials. [this use] provides an interactive environment with immediate feedback to explore indepth mathematical theories that would be difficult to stimulate with concrete models. Additionally, younger students are able to “see” (conceptualize) concepts that would normally be regulated to indepth abstract mathematical principles.

Moor and Zaskis (2000) found that “the interactive aspect of the Internet holds the attention of the student much longer than a regular page of information such as is found in a textbook (p. 103). They also found that words expressed by 25 of 36 students to describe their experiences in using the Web, are such as “fun”, “exciting”, “entertaining”, “interesting”. Loong’s study (2012) also “points to the potential of the Internet to motivate students. Interactive web objects that animate or can be virtually manipulated, and provide feedback to students seem to engage and motivate students” (p. 358). This finding indicates that students found using the Web to learn mathematics motivational. In other words, this is potentially joyful for learners.

The Internet could be used for a range of classroom activities, including direct access to learning objects and manipulatives, exploring investigations by students, comparing and

communicating ideas with others, finding research on mathematical ideas, exploring lesson ideas, and also for accessing to community resources and excursion activities. In the following paragraphs, I briefly describe several websites in terms of what they look like, the mathematical concepts involved and how these activities engaged the students joyfully and meaningfully.

Fido Puzzle

The Fido puzzle (Figure 1) is an interactive game where the computer will “read the user’s mind” (see <http://www.digicc.com/fido/>). In this game, the Internet users need to click through screens that instruct them, first to write down a 3 or 4 digit numbers, then to jumble the digits to make another number. Then they are instructed to subtract the smaller number from the larger one. Next, they are asked to draw a circle around one of the digits in their answer without letting the computer “see” the answer. Finally the Internet users are instructed to jumble the answer and type the number into the computer, excluding the number they circled. The computer then gives them the number they circled.

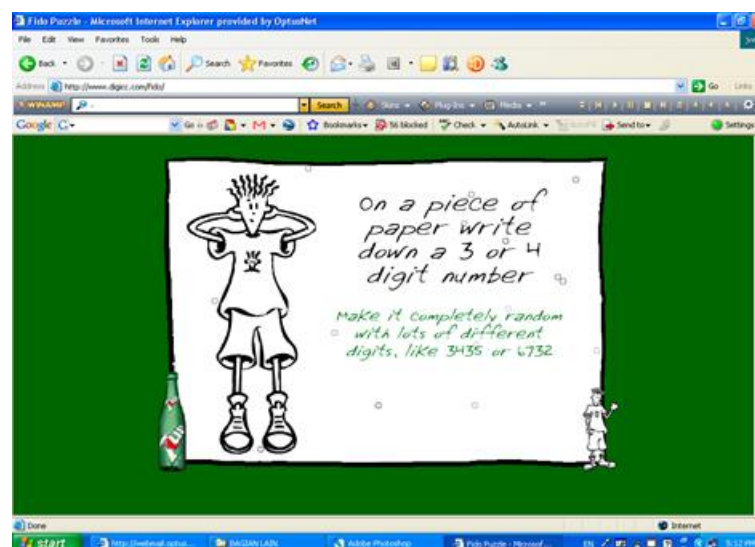


Figure 1. A page from Fido Puzzle website

In my study in one classroom in Australia, through the observation, I saw two pairs of students very excited about how the computer could know the number they were hiding. A pair of students was thrilled when they found that the computer was wrong and they reported to the teacher proudly that they could win. In fact, they did the wrong computation. They went back to check their computation after the teacher said that the computer was never wrong. At another time, I saw a student being helped by his peer. With the Fido Puzzle, it appeared that some students were learning a lot about computation in an engaging and motivating virtual game situation that required them to reflect upon their own thinking and calculation strategies. This Internet activity seemed to foster a dynamic learning of several students in the computer groups.

Geometry

The geometry website (<http://illuminations.nctm.org/>) in figure below, which is provided by National Council of Teachers of Mathematics (NCTM, 2005), allows students to explore the properties of various geometric solids, such as the number of faces, edges, and vertices. The teacher can also formulate his/her own questions to suit the students' age and ability. This website was used by students in one classroom I studied in Australia through rotational activities and a group of students in Surabaya. This website was used to assist students to look at the features of different geometrical shapes and finding relationship between their components.

My experiences observing or working with students using this website, I found that students could easily navigate this website. It was a tool for students to observe different solid shapes since it allowed students to turn around the solid, to colour the faces of three dimensional figures. It appears to me that this “manipulatives website” is potential to develop students' mathematical thinking compared to the uses of physical models and the printed visual model. I also found from my observation in an Australian class and Indonesian groups of students that the students appeared to be on task and engaged in their learning joyfully and meaningfully using this geometric website.

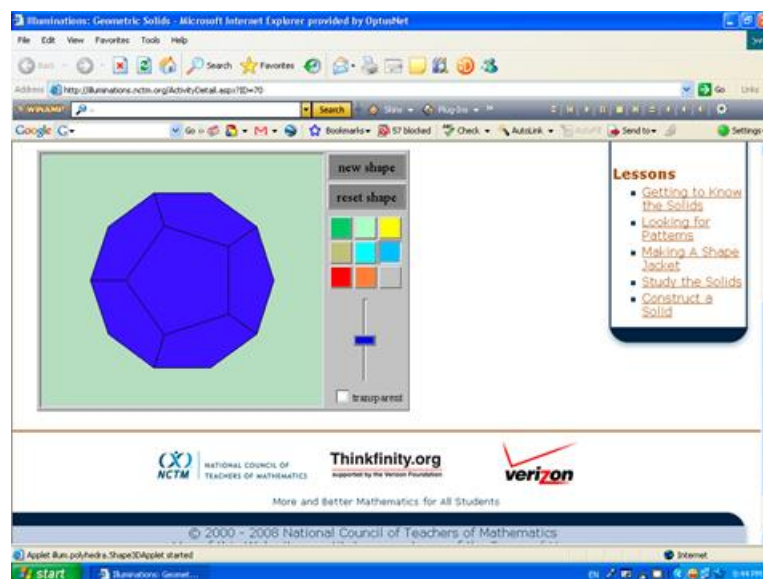


Figure 2. A page from Geometry website

Learning about time

There are many websites about “Time”. One that I identified was called “Stop the clock” (Figure 3). The picture in the screen showed five clocks set in analogue time and five boxes set in digital time. The instruction asked the user to drag the five digital times to the correct analogue times and then press “STOP THE CLOCK” to check the answer and to show how long the user took. I found there were three levels of difficulty in this exercise. In Level

1, the times were set at either the hour or the half hour. In Level 2, the times were set at either half hour or at quarter hourly intervals. In Level 3, the times were in five minute intervals.



Figure 3. A page from a website “Stop the clock”

I also found other games such as “Set the Clock” (Figure 4) in which the user has to click the arrow to demonstrate on a clock face the hour given in words; a matching game (Figure 5) where the user has to match the time on the clock with digital time or words; and a game requiring deep concentration in which the user has to match the hidden times that appear when cards are flipped by clicking.

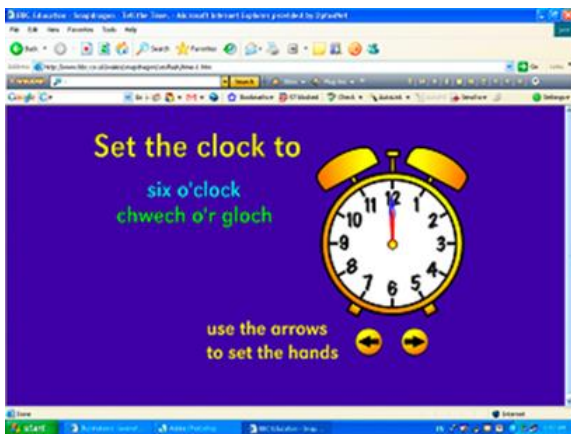


Figure 4. A page from a website “Set the clock” Figure 5. A page of matching game

My field notes described what occurred when I offered the clock websites to Grade 2 students in Australia.

I worked with four students using the websites on two computers. The four students worked in pairs. I facilitated them to do Level 1; the students worked out the problem and in just several minutes most of these students could match the digital clock and the analogue clock very quickly: many of them said “it’s getting easy.” They asked, “Can we go to the next level?” I let them click Level 2. With some discussion amongst the four

students, they eventually solved the problems quickly. Several minutes after that, students commented that Level 2 was getting easier. They agreed to go to Level 3. With a little help from me, they could do the tasks at this level very well. Eventually, students asked permission to do the matching game.

Some other students came to see this learning and asked for a chance to do the computer activity. They looked curious and interested and one commented on these websites by saying “Cooooooool.” One of the students touched the mouse, indicating that he wanted to have a turn in doing the Internet activities. While I facilitated the students with the website for about 20 minutes, their teacher continued his teaching about addition strategies. [27/04/06]

During this Internet activity, the students were very enthusiastic. They engaged in discussion and worked out the problem in pairs. They seemed to be challenged by the different levels and variety of the websites. I could see students were capable of higher level tasks than their teacher indicated when he checked the websites.

Geography and Computations skills

Another online game I found in which students could practise mathematics at different levels is shown in figure 6. This could be used to give students an opportunity to learn mathematics while playing games, or that this website could also be appropriate for students for extension work. Some other websites will be presented in the symposium.



Figure 6. A page of “around the world in 60 second”

Through the Internet, we could also find websites for online teaching resources which were already categorised into the syllabus strands. As a result, we can have access to “ready made strands”. For example:

http://www.beenleigss.eq.edu.au/requested_sites/mathsbyoutcome/index.html (Figure 7) and <http://nlvm.usu.edu/en/nav/vlibrary.html> (Figure 8)



Figure 7. A page of Beenleigh mathematical website



Figure 8. A page of NLVM website

Learning From A Case Of Ann: Joyful And Meaningful Learning

The most noticeable thing about Ann's classroom was its crowded feel. There was little space for students or the teacher to move. The classroom was very open with no doors, and many people pass the classroom via the verandah. Voices from other classrooms could be heard and people who pass the classroom can be seen by the students. The tables in the classroom were arranged in three long strips with desks facing each other. Four other desks were on the verandah and were usually used for small group teaching.

In Ann's classroom with 26 students (who came from low socioeconomic status families), there were 6 computers (including one very old computer that cannot save any work), but only 2 computers were connected to the Internet. The computers were placed in one corner of the classroom, where they could be monitored by the teacher while she was helping students in other groups. Two of the six computers and one public printer were placed outside the classroom on the enclosed verandah/walkway.

The school did not have ICT staff to support the use of computers in the classroom. In one visit, I saw a student reporting a problem with a computer and Ann tried to help. Since Ann could not solve the problem at the time, she eventually turned off the computer. This problem was unresolved the next day. In a conversation with Ann, she told me that she often solved the technical problems in the classroom by herself. She added that this challenge might factor why other teachers in her school rarely used computers in their classroom for teaching.

Interviewing and observing Ann has provided evidence about how Ann encourages new ways of learning by navigating the Internet. Many students were easily finding websites since Ann had created a list of educational websites in the computer Bookmark. Using this Bookmark, Ann's students could directly use the Internet and become engaged in their own learning. Ann also encouraged and welcomed students' initiatives for using the Internet. During the classroom observations, student learning with the Internet was rarely controlled by the teacher, as happened with students in the non-computer group. However, Ann pointed out that the observed way students interacted with the Internet was not how it was at the beginning of the year, and she stated that "all students required specific assistance in using the Internet in class." Ann further explained how

Ann's experiences using the Internet in her teaching provided her with evidence that the use of the Internet impacted positively upon students' learning. She stated that students were more engaged in learning, they were more active in asking questions and discussing ideas, they collaborated with each other, students who normally did not engage became engaged in lessons that included the Internet, students who had extra needs had the opportunity to extend their mathematics learning, and students with learning difficulties had greater opportunity to engage with the concepts at greater depth and in a range of modes.

Some of these impacts were observed. It was quite often that students in the computer group were enthusiastic in learning. Many students were seen positively interacting with the Internet, for example, by using their hands to count, there was shaking of heads, responding

orally when solving problems, asking questions to peers or to the teacher at various times. In mathematical rotations, compared to other groups, the computer group were seen to focus on tasks quicker than non-computer groups and during the transition period in rotational activities, the movement from the non-computer group to computer group was more organised and quicker than the movement from computer group to non-computer group.

Three main reasons for using the Internet in Ann's mathematics teaching were identified, namely (1) to achieve her mathematics teaching objectives that mathematics is everywhere, a great tool to solve daily problems as well as mathematics can support cooperation/collaboration, (2) to facilitate student' learning, and (3) the belief of the important for students to have a good understanding and skill in using ICT.

Ann explained that Internet activities, for example, problem solving, math research, virtual hands-on activities, will help students gain the three objectives above. Furthermore, aligning with her philosophical beliefs about teaching and learning mathematics and individual learners (as examined in Patahuddin & Dole, 2006), she views the Internet as assisting her to act as a facilitator, particularly in engaging learners in their own learning. As she stated:

I often find that I can engage them into some appropriate math learning related to what I am teaching, using the Internet.Whereas normally, being the facilitator without the Internet is quite hard because you are there and you've got a whole class in front of you, and you can't often have a focus or enough focus ... to keep the children discussing. Whereas, I find the Internet very good for sustaining kids' discussions and focus.

Also, as a facilitator, she has found it easier to cater to diversity, and to build mathematical knowledge and processes for students, as described in the following quotations.

...you can offer extensions through the Internet. If you have children that are extra gifted who want to go into something a little further than the rest of the class... On the other side, I have used it a lot for those students who can't necessarily engage in the math that I am teaching so often there are Internet activities or resources that will engage them.

Even though Ann stated many positive things about the impact of the use of technology on students' learning, and some were justified by my observations, I often think that this was particularly determined by Ann's ability in facilitating learning. The reason was I also had opportunity to observe Ann's teaching mathematics without computers. For example, students identified mathematics in a newspaper and Ann challenged them with questions to make sense of the mathematics. Students worked in groups investigating mathematical concepts using a monitor box. Students discussed what mathematics they could find through investigating the box. Some students measured the surface of the box and one student went into the box to measure the inside part. Ann's questions led them to a lively discussion between teacher and students and conversation between students and students. I also observed Ann implement several games to focus students' attention, such as, asking

every student to stand up and be ready to catch the ball from Ann and answering Ann's questions. This was to develop the fluency in counting and to explain students' strategies in counting.

The observations on different learning scenarios (with and without the Internet) posed questions about how this dynamic learning contributed to the effective use of the Internet for learning. Focusing my observations on to the teaching with the Internet was problematic since the learning with the Internet could not be isolated from the other scenarios because students worked either individually or in pairs at the computers. However, over my extended day visits to Ann's classroom, I saw a very engaged class, where Ann asked children to elaborate their thinking at every opportunity. From discussions with Ann, and the myriad examples of incorporating the Internet into her teaching that she described, together with my observations, it appeared that Ann was a competent teacher who made the most possible use of any resources that would enhance students' learning.

Concluding Comments

The use of the Internet in this digital era seems appropriate and relevant because mathematics curriculum also highlight the importance of the use of communicating technologies in teaching mathematics, encourage context links in real life (computers and the Internet are very much part of life now, and many real life contexts are easily accessible on line), emphasis on thinking, reasoning and working mathematically, and build on conceptual knowledge. Those aspects link directly to the educational potential of the Internet and its capacities as a source of information, a means for communication, and as a site for collaboration.

With the Internet, many new ways of learning are enabled. With the Fido puzzle, students interacted with the Internet, learning about computation. With the geometric sites, students explored virtually the properties of geometric solids. With the Clock websites, students have chance to learn in different level and in different format such as matching game, recognising ways of writing the time in different ways. Those are possible if teacher's role in teaching mathematics is not one of being a transmitter or a centre of learning, but as a facilitator.

Through using the Internet, the teacher could engage students in learning and to cater to the various needs of different students. One thing to remember that the Internet cannot replace the role of the teacher as facilitator, as teachers must set up the task, pose questions, provide appropriate websites, and give feedback.

Due to its obvious potential for enhancing educational improvement (Dogruera, Eyyamb, & Menevisab, 2011), it seems clear that Internet availability and access should be optimised. Many studies (e.g., Cuban, Kirkpatrick, & Peck, 2001; Gibson & Oberg, 2004),

however, have raised the issue that the increasing amount of Internet accessibility for teachers does not necessarily indicate the effective use of the Internet by teachers.

From my study and my experiences working with teachers and learners, I argue that the Internet has potential as a medium for learning mathematics in a richer and meaningful way. However, this is determined by many factors including teachers' knowledge and their philosophical beliefs about learning and teaching. It also depends on their beliefs and expectations of learners, teacher's personal situation as well as their own knowledge and beliefs about mathematics and technology. In addition, technology does not necessarily mean student engagement. Internet use could enhance student motivation, but also the fact that it does not always lead to student engagement, motivation and learning.

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Designing Joyful and Meaningful New School Mathematics Using Indonesian Realistic Mathematics Education

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Abstract

A new issue in education in Indonesia is the change of curriculum in 2013. This paper presents what are the changes in content, media, method, and evaluation of mathematics subject at the school levels. As informed by the minister of education and culture in the national newspaper, the changes in the primary school mathematics will connect and integrate with science. This might increase the understanding of students in both the concept of mathematics and the application of mathematics in their daily lives that is also related to science. Hence, mathematics teachers need a way to design meaningful learning materials to integrate the two subjects. It is called thematic-integrated approach or one intertwined among strands or subjects. This article will discuss these both approaches. The former is mentioned in the new curriculum while the latter is one of five characteristics of Indonesian Realistic Mathematics Education (PMRI). Then the current development of PMRI after 12 years of implementation and dissemination in Indonesia will be discussed. During this discussion it will reveal how to design and implement joyful and meaningful mathematics learning materials by using the Indonesia context or culture.

Keywords: Realistic Mathematics Education, PMRI, joyful and meaningful school mathematics

Issues on Mathematics in Curriculum 2013

The Indonesian Minister of Education and Culture, Muhammad Nuh, stated in 2012 that the curriculum was going to be changed next year (Kompas, 2012). Some of the reasons why the curriculum was to be changed were linked to the results of PISA and TIMSS which were very low, the lack of ICT used in the classroom, and too many subjects in the curriculum.

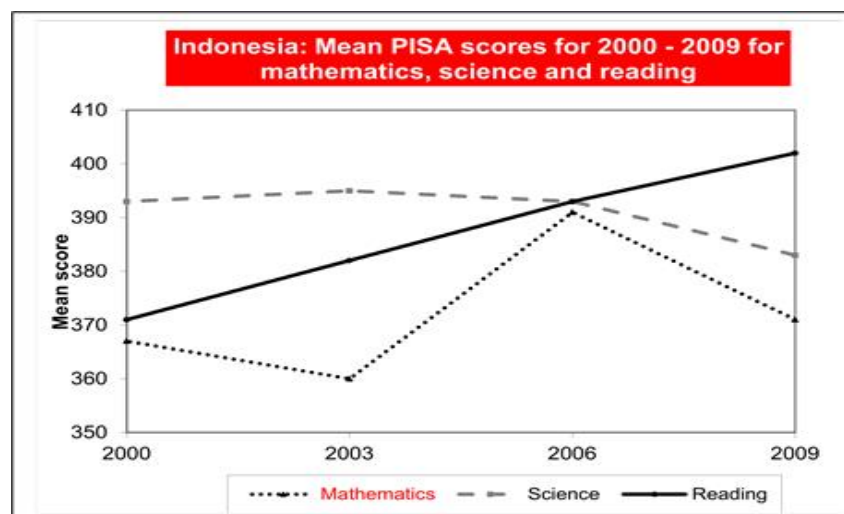


Figure 1. PISA results of Indonesia from year 2000 to year 2009 (Stacey, 2010)

Learning from the last PISA results (Stacey, 2011), Indonesia was ranked 61 out of 65 countries. From figure 1, one can see that the results for mathematics is sinking down or is unstable compared to the results of reading. In general, Indonesian students are only able to solve PISA problems at the levels 1-4, but almost none of students are able to solve problems at levels 4-6. Hence, the new curriculum is really focusing on the competences that were evaluated in PISA such as reasoning, problem solving, argumentation, modelling and communication.

Many subjects including mathematics will be changed at all school levels. The new curriculum for mathematics is analyzed based on the three main components of curriculum that is contents and competences, instructional media and method and assessment. These components are explained below.

Changes in the Three School Levels

The development of ICT is supported by the development of mathematics in the field of number theory, algebra, analysis, probability and discrete mathematics. Hence, students have to understand better in mathematics while young in order to create new technology in the future. Mathematics subjects need to be learned by students at all levels.

In the primary school (SD) some mathematics topics will connect and integrate with some topics of science. This might increase the understanding of students of both the concept of mathematics and the context or application of mathematics in their daily live that is also related to science. Yet, how do teachers develop or design such instructional materials that integrate mathematics and science? Hence, mathematics teachers need a way to design contextual and meaningful learning materials. It is called a thematic-integrated approach or is intertwined among strands or subjects.

At Junior high school level (SMP), it was also mentioned in the newspaper that information communication technology (ICT) will be integrated in all subjects including mathematics. ICT or the internet will be used as a tutor or a tool in helping students to learn mathematics.

In the senior high school (SMA), mathematics will be categorized into three levels namely: mathematics A; mathematics B; and, mathematics C.

New Content and Goals

New knowledge and skills need to be mastered through mathematics such as logical thinking, critical thinking, consistency, alternatives, innovation, creativity and teamwork. These competences are needed in getting, managing and using information for a better life in the competitive world. In addition, the knowledge and skills of problem solving and communication are also important (MoEC, 2012).

Use of context to make interesting and meaningful mathematics in the new curriculum (MoEC, 2012; Zulkardi, 2012), it is stated in the new curriculum that each mathematics lesson should use a contextual problem [*Dalam setiap kesempatan, pembelajaran matematika hendaknya dimulai dengan pengenalan masalah yang sesuai dengan situasi* (contextual problem)]. This is one of the positive changes in the new curriculum. Lee (2012) stated that the use of context in mathematics problems has been used in China for over 1500 years. By using context, moreover, he mentioned that the problem should be easy to understand, easy to use and easy to transfer to another problem with a different context. In Realistic Mathematics Education, Freudenthal (1991) stated that the use of context can help students in exploring mathematics and progressing their mathematical thinking. In PMRI, the contextual problems have been used to make mathematics more interesting and meaningful for more than a decade (Zulkardi, 2002; Sembiring, Hoogland & Dolk, 2010). In brief, contextual problems can play a role in making joyful and meaningful mathematics.

Example of Meaningful Contextual Problems

The following is an example of student materials. The context is Finding the Price of an Item. In this section you will use the strategy of exchanging to solve problems involving money.

Question 1

Without knowing the prices of a pair of glasses or a calculator, you can determine which item is more expensive. Explain how.

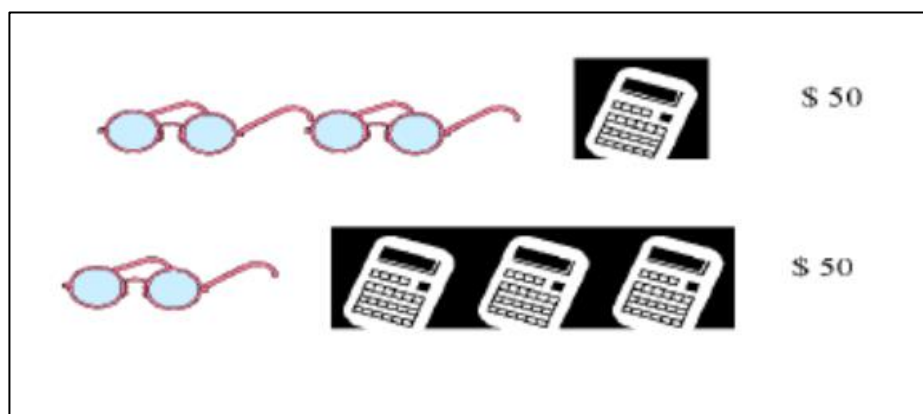


Figure 2. Find the Price: Calculator and pair of glasses

How many calculators can you buy for \$50? What is the price of one pair of glasses? Explain your reasoning.

Question 2

Which more expensive, a cap or an umbrella? How much more expensive is it? Use the pictures below to make a new combination of umbrellas and caps. Write down the cost of combination.

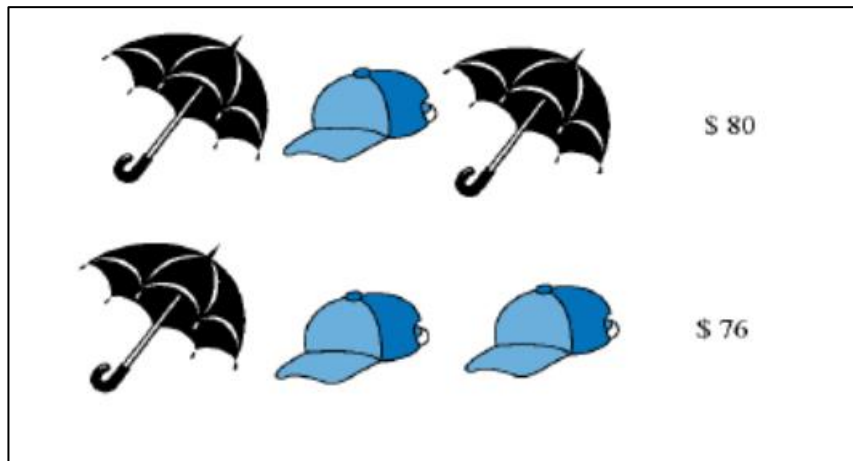


Figure 3. Find the Price: Cap and Umbrella

Make a group of only caps or only umbrellas then find its price. What is the price of one umbrella? What is the price of one cap?

From the contextual problems above, students are guided to use informal methods or an exchanging strategy before the use of a formal algebraic strategy such as the elimination method or substitution method. In fact, when this activity is offered to the pre-service teachers (student teachers) most of them use the formal one.

New Goals of Learning Mathematics

The following are the goals of school mathematics in the draft of the new curriculum (MoEC, 2012).

Goal 1: Understanding mathematics concepts by explaining and using them in problem solving. To make it easy for teachers, it is suggested they use context and manipulatives in instructional process.

Goal 2: Use of patterns, reasoning and generalization based on the availability of data. The goal is to guide students learn how to reason.

Example: 1. If two angles of a triangle are 60° and 100° then the third angle is?

Example: 2. How to read and predict data.

A TV reporter presented data about robbery in the graph from 1998 to 1999 saying it has increased by a big number. Is this a correct statement? Give your reasoning.

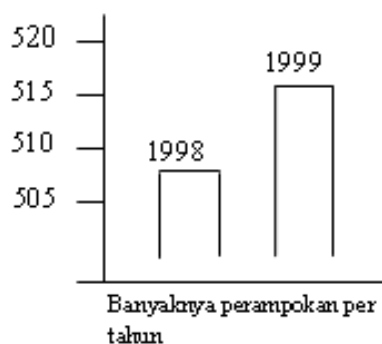


Figure 4. Robbery data 1998 - 1999

Goal 3: Using reasoning in manipulating and analyzing the steps in the problem solving either in a direct mathematics context or indirect mathematics context (such as real life, science and technology). The steps are: understanding the problem; developing a mathematics model; manipulating the model; and use the results to solve the real life problem.

Goal 4: Able to communicate ideas, reasoning and justify, or proof in mathematics using full sentences, symbols, tables, diagrams or other media to clarify solution of the problem.

Goal 5: Having a good attitude towards mathematics and its uses in the daily life such as curiosity, attention, and motivation to learn mathematics, and confidence in solving problems. Here an interesting and meaningful instructional mathematics plays an important role in supporting students to be more creative, interactive and joyful.

Goal 6: Having good attitude and habit that matches with the values in mathematics and its learning such as consistency, democracy, self-confidence, openness, discipline and honesty.

New Instructional Media and Method

In order to achieve the new goals, successful media and methods have to be used. For instance, to increase the effectiveness of the instructional process, it is suggested that schools use simple manipulatives and ICT such as the computer and internet. More information on the use of ICT in mathematics can be found at <http://p4mri.net>, a learning environment for learning PMRI on the web.

As far as the teaching and learning methods are concerned, teachers are suggested to design instructional sequences that foster the creativity of students. In other words, the instructional sequence is designed in such a way to stimulate students to develop their ideas in creative and innovative ways. Some techniques are proposed such as teachers should: (1) use interesting and meaningful contexts that guide their learning from informal to formal mathematics; (2) give students chances to develop their contributions and productions; (3) use their students contribution and products; (4) give students more time; and (5) ask questions that guide students to think in a creative way such as: “why”, “how”, “what if” (MoEC,

2012). In addition, teachers are also suggested to use a problem solving approach and a constructivist learning theory during the instructional process.

New Assessment

There are two main components of assessment in the new curriculum: focus on problem solving; and use of open problems. Open problems means problems that have multiple solutions or multiple strategies. Students are asked to communicate their solutions or strategies in a systematic and logical way. Actually these two have been stated and used in the *Kurikulum Tingkat Satuan Pendidikan (KTSP)* or current school level curriculum, yet not too many used either in the mathematics textbooks or in the national examinations. These components match with the domain of problem solving especially for higher-order thinking in mathematics.

The following example is a problem that was used in the Mathematics Literacy Contest (KLM). The problem is called a PISA-typed problem. Basic Competence: *Memecahkan masalah yang berkaitan dengan perbandingan dan skala.*

Problem: Guess how tall is the name label *Jembatan Barito*. Explain your reasoning.

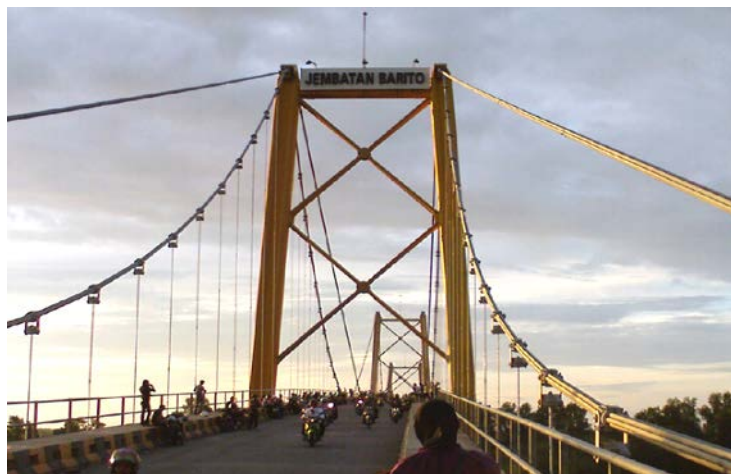


Figure 5. The Barito Bridge - <http://www.google.co.id/>

An alternative solution is by guessing how tall a man is who is standing on the bridge and then use it in guessing the height of the bridge. This problem has multiple strategies and solutions. The important issue is both the accuracy of the answer and the quality of the reasoning. Of course, since this is a guessing problem then the answer has a range from 18 - 22 metres.

RME or PMRI

Realistic mathematics education (RME) is an instructional or teaching and learning theory in mathematics education that was originally developed in the Netherlands. It stresses the idea that mathematics is a human activity. It is called a global philosophy of RME. RME was introduced and developed by Freudenthal in the Netherlands (Freudenthal, 1991).

Three Principles of RME

Guided reinvention and didactical phenomenology.

Because mathematics in the RME theory is a human activity, guided reinvention can be described where teacher gives students a chance to understand and do the mathematics process by themselves and discover the mathematics needed. This principle can be inspired by using a procedure informally. This effort will be reached if teaching and learning processes use real life context that are related to the mathematics concept.

Progressive mathematization.

The situation that contains the phenomenon that can be used as an application area in the teaching and learning mathematics should begin from a real situation before moving to the formal mathematics. Two kinds of mathematization should be used as references in teaching and learning mathematics from concrete to abstract (formal).

Self-developed models.

The role of self-developed models is as a bridge for students from concrete to abstract or informal to formal.

The Characteristics of RME

RME of PMRI has five characteristics: (1) use real-life contexts as a starting point for learning; (2) use models as a bridge between abstract and real, that help students learn mathematics at different levels of abstractions; (3) use student's own production or strategy as a result of their doing mathematics; (4) interaction is essential for learning mathematics between teacher and students, students and students; and (5) connection among the strands, to other disciplines, and to meaningful problems in the real world. PMRI is RME but using Indonesian context and culture.

The Development of PMRI in Indonesia

PMRI was initiated in Indonesia since 2001 in 12 Sekolah Dasar (Primary schools), 4 Madrasah Ibtidayah Negeri (MIN – Islamic primary schools), in collaboration with 4 LPTKs (teacher training universities): Universitas Pendidikan Indonesia (UPI), Universitas Negeri Yogyakarta (UNY), Universitas Sanata Darma (USD), and Universitas Negeri Surabaya (UNESA). This activity was conducted by the PMRI team with a small budget founded by Dikti (Directorate for Higher Education) and the Dutch government. Up to 2012, 23 LPTKs were involved, with each LPTK working with a number of schools either SD/MIN or SMP/MTs (Junior secondary schools/ Islamic junior secondary schools).

Many activities (see also Sembiring & Zulkardi, 2012) have been conducted by the PMRI team as follows: (1) PMRI textbooks writing; (2) PMRI magazine; (3) Designing

International Master program on Mathematics Education (IMPoME) in collaboration with Unsri, Unesa and Utrecht University since 2009; (4) Giving training for PGSD lecturers; (5) Training of Character building via mathematics; (6) PMRI in the international forum; (7) PPPPTK and SEAMEO Regional Centre for QITEP in Mathematics; (8) Design research task force; (9) designing web in using Information Communication and Technology; (ICT) as dissemination tool; and (10) Mathematics literacy contest and workshop PISA for mathematics teachers since 2010.

The Intersections between New Curriculum and PMRI

It has been discussed earlier that some new components in the new curriculum are similar to the characteristics of PMRI. First, in the content and the use of context or theme is an important point for both, as is how to integrate the strands of mathematics and how to integrate mathematics and science. One of the characteristics of PMRI is intertwining of the strands in mathematics with other subject matter. Moreover, an approach that is proposed in the new curriculum is a thematic integrated approach while in PMRI it is called design research. They seek to achieve similar goals such as: to produce good instructional learning materials that use of context; or the use of themes as a starting point of learning mathematics.

New competences that are related to assessment are also stressed in the new curriculum that is problem solving, reasoning, communication and modelling. Open-ended problems are also focused upon in the new curriculum. These competences are used in the three levels of assessment in PMRI (Zulkardi, 2002).

In brief, there are similarities between PMRI characteristics and some components of the new curriculum. Therefore, one can say that PMRI is now institutionalized in the curriculum 2013. This is an indicator of sustainability of PMRI for the future (Sembiring, Hadi, Zulkardi, & Hoogland, 2010).

Concluding Remarks

The Indonesian curriculum 2013 makes extensive use of thematic or contextual problems. Our experiences show that context can lead to a joyful and meaningful mathematics learning of concepts. Yet, it is not an easy task to find a good context that is relevant for all mathematics concepts. Also, a mathematics teacher has to design instructional materials that can guide students in learning the concepts of mathematics from informal to the formal mathematics. In order to do so, PMRI, its standard and design research method can be used (Hadi, Zulkardi, & Hoogland, 2010).

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Enjoying Maximising Opportunities For Learning Mathematics

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Abstract

In this keynote we will examine and participate in mathematics teaching and learning practices that contribute to strong learning focussed relationships and enjoyment of mathematics learning. Pedagogies, learning experiences, and caring teacher behaviours that include and extend beyond traditional mathematics teaching practices will be presented as examples of how academic relationships can be fostered towards all students making strong mathematics learning gains. The use of contexts that students find realistic, meaningful, and engaging will be discussed. A culturally responsive mathematics education model that encompasses cognitive, social, physical, and spiritual dimensions will be used to consider themes from the keynote session. Examples of research-practice links will be discussed.

Keywords: culturally responsive mathematics teaching, learning-focussed relationships, mathematics learning activities

Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.
Cling to the main vine, not the loose one.

Introduction

Teaching mathematics in ways that enthuse learners, give them confidence in their mathematical ability, and assist them in seeing real world applications of their learning are all important for effective learning. Teaching in ways that cater for, and respond to, students' ways of being and knowing is important for engagement, maximising pleasure in learning, and for transfer of learning to students' own real world contexts. In this presentation we will consider how teachers can maximise students' engagement in, and passion for, mathematics learning. Reflecting on the proverb above, we will be exploring the 'main vines' of mathematics teaching and learning. To do this we will consider a range of ways to respond to students' cultures, including using pedagogies, protocols, models, and realistic and meaningful contexts to promote mathematics learning. Partnerships between teachers, teacher educators, and researchers are known to inform and promote developments in our field. I will draw from a range of research and writing projects undertaken in New Zealand that collectively involved researchers, teachers, student teachers, and school students.

Making opportunities for strong mathematical classroom investigations and discussions is important for enjoying mathematics teaching and learning. In exploring the themes of this talk we will consider how to create environments that are conducive to students taking an active part in mathematical investigations and discussions. We will explore a range of factors and mathematics learning activities that help create such environments and open up productive mathematical discussions.

Culture And Mathematics

There are many ways of thinking about ‘culture’. Culture refers to everything that helps define who we are – the ways we do things, how we act, how we behave, how we interpret events and interactions and how we respond to these, how we celebrate, and how we ask and answer questions (Banks, 2006; Gay, 2010). What does responding to culture mean in mathematics teaching and learning? We are all part of many cultural groups – our professional group (teachers, researchers, teacher educators), our community groups (linked to our religions, hobbies, sports, pastimes), groups determined by where we live, and those of our heritage cultures, often linked to our ethnicity. Our students similarly are part of many cultural groups – school culture, social networks, heritage cultures, religions, sports – that impact on their ways of being and thinking as well as on what they bring with them to their learning. In this talk, I use the word ‘culture’ broadly to refer to everything that makes students who they are. Which aspects of culture should we attend to in our teaching? Which can we attend to? And, how can we attend to these in ways that acknowledge, empower, enhance comfort, and most importantly, maximise students’ mathematics learning? Are the same or different strategies suitable for attending to culture within initial teacher education and professional development?

The international literature offers many ideas to help answer these questions. There are various factors to consider: managing effective teacher-student relationships (Bishop, Berryman, Tiakiwai & Richardson, 2003; Ladson-Billings, 1994); listening to students’ perspectives about their learning (Macfarlane, 2004); and linking learning to students’ experiences and interests (Gay, 2010; Kanu, 2011; Presmeg, 2007). Teacher education needs to help aspiring teachers learn to teach in culturally responsive ways and realise what knowledge they need to acquire to do so (Nieto & Bode, 2008; Villegas & Lucas, 2002). New Zealand education policy requires that teachers attend to students’ cultural heritages in their teaching. There is a particularly strong focus on ensuring schools and classrooms enable Māori students to experience and enjoy academic success as Māori (e.g., Ministry of Education, 2007, 2008, 2011). In our team’s research and practice we have sought to find ways to satisfy these policies and their predecessors.

There is a range of factors that contribute to creating classroom environments that attend to students’ cultures in mathematics teaching and learning. We will touch on four inter-related areas: using realistic contexts; using pedagogies, protocols, and languages consistent with students’ heritage cultures; attending holistically to students’ needs; and ensuring effective teacher-student relationships are in place. Mathematics classroom examples will be used to illustrate each. I will use examples from Aotearoa New Zealand. Please consider these in relation to ideologies and practices suitable for the teachers, learners, and researchers you work with.

Realistic and Meaningful Contexts

We can reflect cultures in our teaching through using realistic and meaningful contexts for our learning tasks (e.g., Harvey & Averill, 2012). In New Zealand the term ‘ako’ means teaching and learning – the one word indicates the reciprocal nature of teaching-learning situations. Every time we teach, we learn – we use informal diagnostic assessment to work out suitable next learning steps, we find out about students’ misconceptions and which concepts they find simplest and which more complicated. When we teach using realistic contexts we can also learn a lot about our students as people. Given the right classroom environment, the contexts can open up opportunities for students (and teachers) to share their knowledge, show their passions and expertise, and bring their own personality and humour to their learning.

Cultural knowledge, sensitivity, care, and respect are very important in using contextual problems as every culture has different dos and don’ts, things that make others feel comfortable or uncomfortable. It is very hard for people to learn well when they feel offended or uncomfortable. For example, internationally, food is often used to help students develop fraction concepts. However, in some New Zealand communities it can be offensive to use food as a classroom material, so it is safest for New Zealand teachers to use alternatives to food. When there are many cultural groups in a classroom it can be a big challenge for teachers to know enough about all of their students to always teach sensitively.

Connectionist learning theories (Askew, 2010) hold that students learn new ideas by being able to connect them with things they already know. Realistic mathematics education (Gravemeijer, 1994) also emphasises the importance of using realist contexts to help learning and to assist students to see the relevance and usefulness of the learning in their everyday lives (e.g., Sembiring, Hadi & Dolk, 2008). Teachers need to know their students well to use contexts safely, particularly contexts linked to heritage cultures (Gay, 2010). However, teaching using contexts can be powerful for learning about our students while promoting mathematical thinking and learning. For example, setting up graphs that use contexts students know about can help them identify personally with the concepts and share their own experiences (e.g., Figure 1). Students can be asked to identify people they know that could be represented by the points marked on the graphs, and to explain why the points represent those people well. Their answers will provide good diagnostic information about their understanding of how graphs work as well as some insights into their own experiences. A next step can be to provide axes with no labels and ask students to choose a context for the graph and labels, to include two or three points, and describe who or what the points could represent. The graphs can be made outdoors using chalk or tape on the playground with students standing inside the graph at the places that best represent them. Students can share

information they know about and bring their own ideas and humour to the activity. These things help them engage with the class, the teacher, and with mathematical ideas.

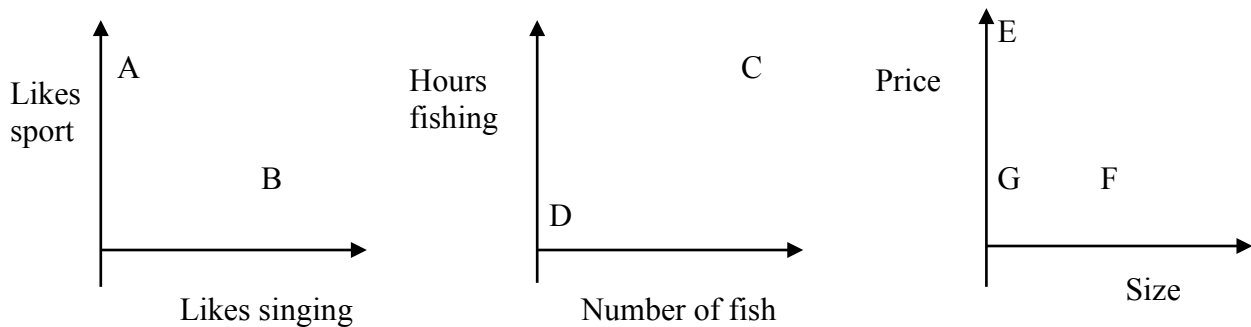


Figure 1. Realistic contexts for developing understanding of graphing

There are many ways of linking the contexts of mathematics learning activities to students' prior experiences and knowledge, while opening up opportunities for acknowledging students' cultural heritages. Some examples of realistic contexts that can open up mathematical and contextual discussions include:

- identifying geometrical features, shapes, properties, and transformations that can be found in flags of different countries (Figure 2) or simple jewellery (Figure 3); and
- collecting statistical information from students and comparing theirs with those of other groups, such as other classes in the school or, if the data are available, students nationally (e.g., Censusatschool, <http://www.censusatschool.org.nz/>).

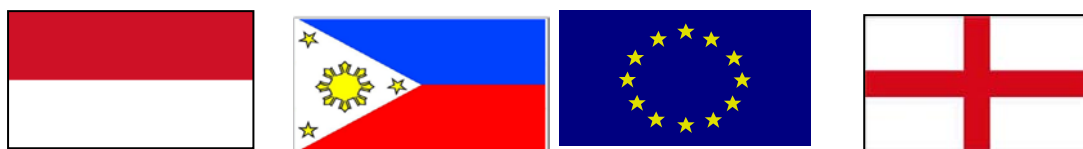


Figure 2. Flags as a context for discussing geometrical features, properties and transformations



Figure 3. Samoan jewellery as a context for geometrical transformations (photographs from Averill, Phillips and French, 2003).

Pedagogies, Protocols, and Languages Consistent With Students' Heritage Cultures

We can reflect cultures in our teaching through using pedagogies and languages consistent with those traditionally and currently used in the cultural groups of our students (Averill et al., 2009). Cultures use a range of pedagogies such as teaching using proverbs, stories, legends, songs, games, modelling, and working together/participation (e.g., Hemara, 2000). There are many ways of incorporating pedagogies and languages consistent with our students' cultures into our mathematics teaching.

This keynote opened with a *whakatauki* – a proverb – a pedagogy consistent with many cultures for setting the scene and explaining the purpose of what follows. There are many legends that we can use to introduce mathematical concepts (such as time, distance, mass, estimation, counting, patterns...), and excellent story and picture books that can provide ways to link students' learning to their reading (e.g., Perger, 2010). Songs are also a very useful tool for aiding memory and for having students share an experience that is fun, and that encourages them to breathe, think, and participate. In our teacher education lectures, we often use songs or chants to help students remember terms and rules and to set up and explore number patterns (Figures 4 and 5). We also try to know about and use protocols that enable all students to feel comfortable to participate in learning. This could be by starting the lecture with a prayer and making time to ensure we all have introduced ourselves fully and appropriately when we first meet.

1	3
1 2 1	3 5 3
1 2 3 2 1	3 5 7 5 3
1 2 3 4 3 2 1	3 5 7 9 7 5 3
1 2 3 4 5 4 3 2 1	3 5 7 9 11 9 7 5 3

Figure 4. Number Pattern Action Chant

Whanui, whanui	Luas, luas	Wide, wide
Whaiti, whaiti	Sempit, sempit	Narrow, narrow
Teitei, poto, iti	Tinggi, pendek, kecil	High, short, small
Teitei, poto, iti	Tinggi, pendek, kecil	High, short, small
Tawhito, tawhito	Kuno, kuno	Ancient, ancient

Figure 5. Action Song of Measurement Terms

Using games to develop mathematical understanding is another way of using pedagogy consistent with that used in many cultural groups. Many games have mathematical elements (e.g., Mancala) and many depend on decisions about relative likelihood (e.g., Beetle). Games are fun and engaging and can enable children to be themselves. Probability lends itself to investigating using games. Dice games are possibilities as are other games

familiar to many children, such as ‘Rock, Scissors, Paper’. Children can be asked to predict outcomes, relative likelihoods, then can play the game and systematically record their results to compare the results of their game with their predictions. Theoretical probabilities can be used to analyse the games and to compare with their experimental results.

Many cultural groups use ways of modelling, working together, and mentoring – experts and more experienced people guiding those with less experience. In Aotearoa New Zealand, *tuakana-teina* is a pedagogy that is sometimes described as an older sibling mentoring their younger brother or sister. Many New Zealand teachers use some form of student-student mentoring that parallels the *tuakana-teina* concept.

We have now briefly considered mathematics teaching that uses realistic contexts and pedagogies, protocols, and language consistent with students’ heritage cultures. Next we move to investigating culturally responsive classroom practice using a culturally-based holistic model of health and wellbeing. To examine the model, we will return to some of the activity examples we have discussed so far.

Attending To Our Students’ Needs, the Whare Tapa Wha:

A Model For Health And Wellbeing

In my thesis study, I wanted to find out how teachers make strong teacher-student academic relationships. I observed 100 lessons in three multicultural city schools to record and analyse how teachers made and kept effective relationships (Averill, 2012). I analysed the data using Durie’s (1998) model of health and wellbeing. The model uses a four-sided house as a metaphor; the four sides are *te taha hinengaro* (related to thinking), *te taha whānau* (the social side), *te taha wairua* (the spiritual side), and *te taha tinana* (relating to our physical selves). My study looked at how teachers created strong teacher-student relationships in their mathematics teaching in relation to each of these four aspects of health and wellbeing. We will look at each in turn.

Teacher practices that developed cognitive aspects of health and wellbeing (*te taha hinengaro*) included teachers reinforcing firm boundaries, setting high (and attainable) expectations and making sure students were aware of these, sharing the learning purpose, and maintaining a strong focus on learning. Teachers kept students engaged by challenging their thinking, involving them in decision making, varying the lesson activities, and having a sense of urgency for completing activities. Practices such as co-constructing learning, differentiated learning, taking prior learning into account, and feeding back and feeding forward were also important for responding to students’ cognitive needs.

Teachers developed social aspects of students’ health and wellbeing (*te taha whānau*) through nurturing students’ sense of classroom community, for example by encouraging students to be responsible for everyone’s learning, themselves, and others. Teachers used inclusive language (e.g., we’ll look at..., let’s try...), showed they felt students’ learning was

important, and incorporated activities that encouraged class community (e.g., through students working together and teachers taking part in learning tasks themselves). Teachers used activities that encouraged students' sharing of their own knowledge and personalities.

Students' emotional and spiritual health and wellbeing (*te taha wairua*) was promoted by teachers showing respect for students, providing timely assistance, and encouraging student ownership of their learning. Teachers interacted one-to-one many times with many students every lesson, set work of appropriate challenge, and provided repeated opportunities for students to feel academic success and satisfaction. Teachers attended to students' emotional and psychological needs, for example by discussing their work in quiet one-to-one discussions and using specific praise and encouragement. They were consistent, explained their decision making, sought and used students' responses to promote learning, and showed they liked their students and enjoyed working with them.

Academic teacher-student relationships were strengthened by teachers attending to students' physical wellbeing (*te taha tinana*) through ensuring the classroom was well lit, had fresh air and sufficient heat, and showing concern for their students' health. They made opportunities for students to move around inside and outside of the room to carry out learning activities, for example through students writing questions and responses on the board, working outside, with others, and with equipment. Linking mathematics learning to other curriculum areas such as physical education, drama and dance, and science can be one way of incorporating movement in mathematics lessons. For example, links with science learning are possible when measuring the horsepower needed for people moving up steps at different speeds. Horsepower can be calculated using:

$$\text{HP} = \frac{\text{mass (kg)} \times (\text{height moved}) \text{ (in m)}}{\text{time (seconds)}}$$

Some teachers used classroom protocols and routines that students expected and were comfortable with, such as greeting and farewelling each student, starting each lesson with a problem, game or prayer, and having consistent ways of giving feedback on work and homework. Each of these protocols contributed to several of the dimensions of wellbeing. The study indicated that all of the teacher practices mentioned above helped develop teacher-student relationships and helped students enjoy their learning, teachers enjoy their work, and for all to enjoy one another's company, all very important ways to maximise students' learning.

Now we will return to the mathematics learning activities discussed above to see how using them can impact on the four dimensions of students' health and wellbeing. Many of the activities involve mathematical purpose, some social aspects, some physical movement, and various ways for students to share of themselves in their learning (Table 1).

Table 1.

Examples of links between mathematics learning activities and the whare tapa wha.

	<i>Taha hinengaro</i>	<i>Taha whānau</i>	<i>Taha wairua</i>	<i>Taha tinana</i>
Number chant	Skip counting, Number patterns, algebra	All chanting together	Performance, having fun, having success	Physical movement
Graphs with contexts	Graphing, axes, plotting points, explaining features of graphs	Learning about one another	Acknowledging individuals' knowledge and experiences	Making graphs outside
Flags and jewellery	Noticing and discussing geometrical features and transformations	Learning about one another's heritage groups and nations	Acknowledging individuals' knowledge and experiences	Making own flags and jewellery to given mathematical conditions
Measurement action song	Measurement terminology	All chanting together	Performance, having fun, having success, acknowledging languages	Physical movement
Horsepower activity	Measurement, algebra, statistics	Group work, teacher taking part	Finding something out about themselves	Physical movement
Probability game	Experimental probability, predictions, modifying game to make it more fair	Group work, teacher taking part	Having fun, being successful, using a pedagogy consistent with heritage cultures	Physical movement involved with playing a hand game

Table 1 shows examples of links between the activities we have investigated in this keynote and the four dimensions of the *whare tapa wha*. How the teacher uses the activity with their own class will affect which dimensions are present and how strongly they are reflected. The activities we have used are all potentially very productive for students' mathematics learning. They are also fun. Students enjoying their work are more likely to be relaxed, comfortable, engaged, and achieving. Enjoying their work contributes to students' mathematical identity, their confidence in their mathematical ability, and their motivation to learn more mathematics. How teachers use the activities is very important in maximising how

they can contribute to mathematics lessons that attend to the four dimensions of students' health and wellbeing.

Strong Academic Teacher-Student Relationships

Strong teacher-student relationships are very important for effective teaching and learning (Averill, 2012; Gay, 2010). They are particularly important when teaching using contexts and cultural elements that are important to students. They help students feel comfortable to be themselves, ask questions, and offer their ideas. They help increase the chance that teachers will know students well enough to use suitable contexts with care and respect. Above all, respect for others, care for learning, and care for individuals are essential for developing strong teacher-student relationships. Using learning activities that include realistic and meaningful contexts, incorporate a range of pedagogies, and attend to students' academic, social, emotional, and physical needs are most likely to create strong relationships and effective learning.

Conclusion

Mathematics, teaching mathematics, and learning mathematics are all human endeavours. People enjoy having fun, being curious, being able to explore, investigate, discuss, problem solve, and, with suitable support, to work things out for themselves. We like to know the purpose of our activities and to share decisions about, and responsibility for, our learning. Traditional skills and practice type mathematics teaching and resources may not sufficiently capitalise on these human characteristics. Re-examining mathematics teaching and learning in relation to ensuring pleasure, awareness of the uses of mathematics, and confidence-building, whilst maintaining a keen focus on effective learning and achievement, is timely. Doing so in ways that maximise our own pleasure in our teaching is important for our own learning, developing our field, and developing strong learning-focussed teacher-student relationships. All of these are valuable ingredients towards maximising students' mathematics achievement. Further research is needed to investigate the human elements of teaching and learning mathematics in relation to specific learning contexts, and how these can be emphasised towards maximising students' engagement with mathematics and their achievement.

Implications of this work for teachers include teacher behaviours that promote effective relationships and that go beyond traditional teaching practices. Teachers can develop strong academic relationships by attending to the specific and holistic learning needs of their students through:

- incorporating pedagogies, protocols, and languages of the heritage ethnicities of their students

- using realistic and meaningful contexts of mathematical learning activities
- using learning tasks that involve movement and fun
- expecting and promoting strong academic progress
- prioritising one-to-one teacher-student interactions
- showing respect for students and their learning
- incorporating collaborative learning tasks
- making opportunities for sharing personal identities, and
- ensuring the learning environment is physically and emotionally comfortable.

Learning mathematics is a human activity and attending to all of these needs enables mathematics teaching and learning to be effective and enjoyed by all. In these ways we can test the local vines of mathematics teaching and learning, cling to those that are strongest and strengthen those that are loose – to make all of our students learn mathematics joyfully and well.

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A Model Eliciting Framework For Integrating Mathematics And Robotics Learning

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Abstract

Robotics is taught in many Australian ICT classrooms, in both primary and secondary schools. Robotics activities, including those developed using the LEGO Mindstorms NXT technology, are mathematics-rich and provide a fertile ground for learners to develop and extend their mathematical thinking. However, this context for learning mathematics is often under-exploited. In this paper a variant of the model construction sequence (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003) is proposed, with the purpose of explicitly integrating robotics and mathematics teaching and learning. Lesh et al.'s model construction sequence and the model eliciting activities it embeds were initially researched in primary mathematics classrooms and more recently in university engineering courses. The model construction sequence involves learners working collaboratively upon product-focussed tasks, through which they develop and expose their conceptual understanding. The integrating model proposed in this paper has been used to design and analyse a sequence of activities in an Australian Year 4 classroom. In that sequence more traditional classroom learning was complemented by the programming of LEGO-based robots to 'act out' the addition and subtraction of simple fractions (tenths) on a number-line. The framework was found to be useful for planning the sequence of learning and, more importantly, provided the participating teacher with the ability to critically reflect upon robotics technology as a tool to scaffold the learning of mathematics.

Keywords: mathematics, robotics, model eliciting, number line

Introduction

There has been a variety of teaching and research activities at the intersection of mathematics education and robotics (or more broadly, technology education). The integration of mathematics and robotics curricula is the focus an ongoing research project titled 'ELLA – Enhanced Learning through LEGO Activities'. In this paper, the first iteration of the ELLA project is reported upon, in which a class of Year 4 students in a typical Australian primary school were exposed to LEGO robotics and the technology was used to engage and scaffold the students' learning of fraction concepts, in particular tenths. What is novel to this research is the adaptation of the model construction sequence (Lesh et al., 2003) as the basis for designing and reflecting upon two parallel but complementary sequences of classroom activities: the regular mathematics lessons specified in a school curriculum and a sequence of robotics-based activities that were designed specifically to augment the regular mathematics lessons.

In the following sections, a brief review of pertinent literature that outlines the nexus of theory regarding mathematical modelling as a classroom pedagogy, robotics as a vehicle for learning and the analysis and description of mathematical knowledge and understanding is

presented. This leads to the proposition of the conceptual framework, and in particular the proposition of the integrating model eliciting framework, underpinning this work. The frameworks application to the design and analysis of classroom activities related to the learning of fraction concepts is presented. Finally, some comments regarding the success of this first iteration and some implications for future iterations of the project are provided.

Literature Review

The ELLA project is aiming to develop pedagogical theory and practice regarding the use of robotics in the learning of school-level mathematics. This includes the development of theory or principles regarding learning activities and resources and exemplars of such theory activities. Central to this project has been the proposition of pedagogically driven conceptual framework that integrates three related conceptual bases: (1) mathematical modelling (Lesh et al., 2003; Lesh & English, 2005); (2) constructionism (Papert, 1980, 1991); and (3) the genetic decomposition-based description and analysis of learners' knowledge and understanding (Nutchey, 2011a, 2011b). Each of these bases are now discussed in turn, from which the conceptual framework for the ELLA project is synthesised.

Mathematical Modelling

Bell (1993), amongst others, describes a model of mathematical problem-solving used by real-world mathematicians that can be thought of in three distinct and cyclical stages: (1) Mathematising; (2) Manipulating; and (3) Interpreting. Problem solving proficiency involves learners working with complex and unfamiliar situations and developing non-routine, or innovative, solutions and should be integral to a contemporary mathematics classroom. This is consistent with the view of Bell (1993), who, whilst recognising the importance of learning about problem solving and learning mathematics for problem solving (i.e., a more traditional applications approach), also stressed that increasing importance should be placed upon learning via problem solving. That is, learners should be challenged by real problems, ones for which there is no clear or obvious path to their solution. It is through such problem solving that a shift from “mathematics as computation” towards “mathematics as “conceptualisation, description and explanation” (Lesh & English, 2005, p. 1) will be achieved and so develop within students the more complex and futures oriented view of mathematics which is needed for work in the 21st Century (Lesh & English, 2005).

In more recent years, the term ‘problem solving’ is often been replaced by the term ‘models and modelling perspective’ or more simply ‘mathematical modelling’. Galbraith (2011) provided a survey of the usage of this term, and has identified six different meanings or groupings of mathematics education research and practice. Galbraith suggested that of these (i.e., using real problems as a preliminary basis for abstraction, emergent modelling, modelling as curve fitting, and word problems) do not adequately address the need of

developing conceptualisation, description and evaluation abilities. The fifth meaning associated with mathematical modelling is described by Galbraith as modelling as a vehicle. Galbraith adopts the term vehicle from the work of Julie (2002) to describe those cases in which a mathematical modelling activity is used with the “prime purpose of eliciting and consolidating new mathematical ideas” (Galbraith, 2011, p. 286). The sixth meaning of mathematical modelling described by Galbraith takes a very much process-oriented view, or modelling as content (Julie, 2002), which serves the “prime purpose of helping students to access and use their existing store of mathematical knowledge” (Galbraith, 2011, p. 286). In both these latter meanings, the mathematical model is conceived of as some conceptual structure that is constructed or used by the learner to make sense of experience.

Galbraith identifies the model eliciting work of Lesh and associates (e.g., Lesh et al., 2003) as a pedagogical tool that can be used to develop students’ mathematical modelling (or problem solving) abilities, from both modelling as vehicle and modelling as real-world problem solving viewpoints. A sequence of model eliciting activities (MEAs) provides a way for learners to make explicit their developing ideas with regards to the problem at hand and their developing mathematical models. A significant aspect of Lesh et al.’s work is the belief that representational fluency – being able to flexibly express a mathematical concept in a variety of representational forms – is a significant factor that contributes to deep understanding. In terms of Bell’s (1993) principles, this relates to the connectedness of the learner’s developing organisation of mathematical ideas. Whilst model eliciting has a strong product focus (i.e., groups of learners working collaboratively on the solution of some problem), it is through such activities that the learners’ conceptual understanding is revealed.

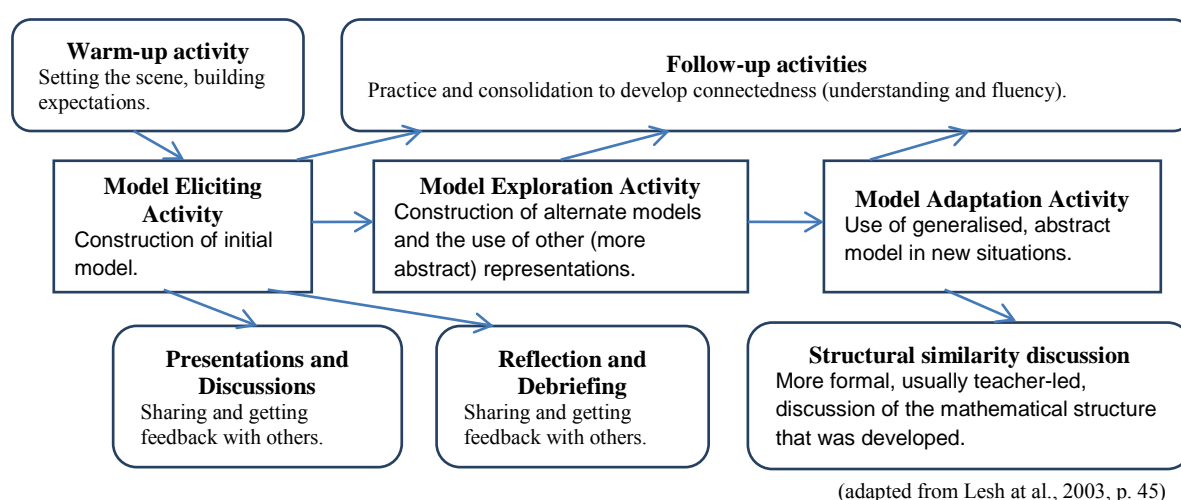


Figure 1. Lesh et al.'s (2003) model construction sequence

As suggested by the cycle of mathematisation (Bell, 1993), problem solving is rarely a simple one-pass process: multiple cycles of mathematical work often occur as learners (or any mathematician) either refine a solution or explore and compare a variety of solutions. This cyclical nature is accommodated by Lesh et al. (2003), who suggest that a single MEA should

not be used in isolation. Instead, a sequence of model construction activities should be used (in combination with other classroom activities including practice activities aimed at developing or consolidating computational fluency). Such a sequence is described in Figure 1, which shows three model construction activities: the model eliciting activity, the model exploration activity and the model adaptation activity.

The model construction sequence thus embodies both modelling as vehicle and modelling as real-world problem solving meanings. In the sequence, the earlier model eliciting activity could be aimed at creating a new mathematical idea (i.e., the model). Then, further modelling activities (referred to as model exploration activities by Lesh et al.) provide opportunities for the learner to represent the mathematical ideas in different ways and to realise its connections to other mathematical ideas. Later in the sequence, the modelling activit(ies) aim to adapt the model to new situations that are different to those in which the model was originally developed. Thus, the model construction sequence provides a framework for explicitly applying, discussing and revealing how the mathematical concepts and processes constructed during the modelling activities relate to the overall structure or organisation of mathematics.

Constructionism and LEGO® Robotics

The seminal work of Seymour Papert and his constructionist theory (e.g., Papert, 1980, 1991) has been the basis of several educational technologies, including LOGO Microworlds, the SCRATCH visual programming environment for creating animations and games and the LEGO Mindstorms NXT robotics technology. Central to the constructionist theory is the premise that the use of technology to construct solutions to problems encourages peer-to-peer discussion and allows the teacher to observe the thinking that is going on as the ‘product’ is being designed, built and evaluated. That is, technology is a vehicle for learning which reveals the learners’ thinking process(es) that lead to the product. When considered from a mathematics learning viewpoint, LEGO robotics provides learners opportunities to socially construct meaning related to concepts including: distance and angle measurement; time; whole, decimal and fractional number notation; geometric shapes and reasoning; rate, ratio and proportional relationships and the associated reasoning required to solve equations; and the development and management of multi-step solutions.

Graphical Representation of Knowledge and Understanding

Nutchev (2011a, 2011b) has proposed genetic decompositions as a graphical way in which to describe both the structure of mathematical knowledge shared in a community and the understanding that an individual has of that knowledge. This work is a synthesis of Popper’s (1978) three-world conceptualisation of knowledge and Piaget’s notion of reflective abstraction (Dubinsky, 1991; Piaget, 2001). Genetic decompositions identify the set of

knowledge objects (the problems, concepts and representations of the domain) as well as the reflective abstraction-based knowledge associations. Together, these constructs can be used to weave together a complex description of the knowledge in some mathematical domain. For the reader's reference, a summary of the graphical language's constructs used to create genetic decompositions is presented as an appendix. Additionally, Nutchey has conceptualised understanding in terms of the sequence of experiences that a learner has as they explore the structure of shared mathematical knowledge. Quite literally, the implication for instruction is that learning experiences should be designed such that the learner has the opportunity to encounter all connections in the organisation of mathematical ideas, such that they too construct the meaningful associations which link together problems, concepts and representations.

Conceptual Framework

Mathematical modelling necessitates a context that is accessible and from which learners can develop intuitive through to formal understanding. It is the premise of this research project that LEGO-based robotics provide such a context in which mathematical modelling can be conducted. Moreover, LEGO-based robotics activities can be carefully designed so that learner's are scaffolded and directed in their exploration the structure of shared mathematical knowledge, either to elicit new mathematical ideas or to consolidate existing mathematical ideas. To organise learning activities to achieve such a scaffolding, a variant of Lesh et al.'s model construction sequence has been proposed and is illustrated in Figure 2. This so called integrating model eliciting framework explicitly sequences regular mathematics lessons with ELLA activities in a zig-zag fashion, such that the more typical mathematics lessons provide the opportunity to prelude and/or follow up the mathematical content of the activities in the ELLA model construction sequence (and vice versa). As with Lesh et al.'s original model, the sequence of ELLA modelling activities progressively expand upon the mathematical model used by the students, firstly eliciting the model, then exploring its use and finally adapting it for new uses. As a model of both curriculum integration and research design, the middle activity design section refers to not only the design of LEGO-based activities to complement the regular mathematics lessons, but also the ongoing research-teacher dialog that features in the ELLA project. Using the proposed integrating model eliciting framework it is the goal of the ELLA project to explore how the use of LEGO-based mathematics model eliciting might provide contribute to the more innovative and futures-oriented interpretation of "mathematics as conceptualisation, description and explanation" (Lesh & English, 2005, p. 1).

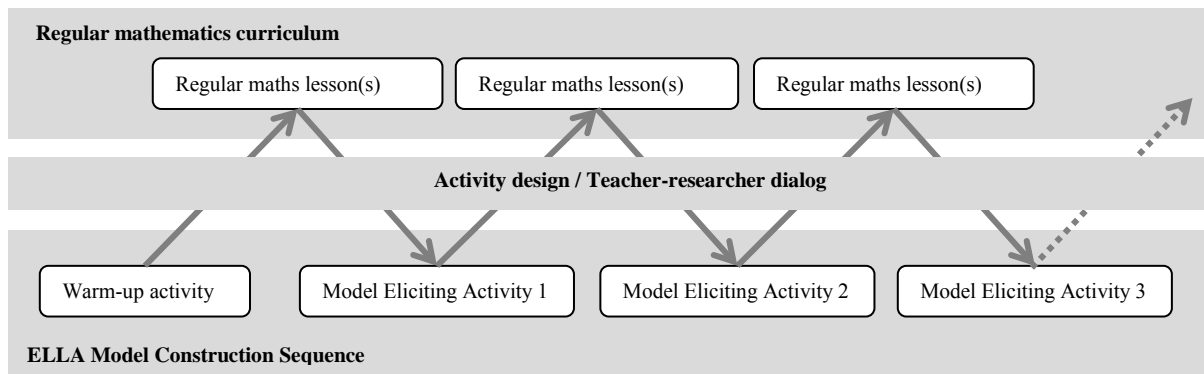


Figure 2. Proposed integrating model eliciting framework

Methodology

Complementing the design-based problem solving approach to learning mathematics, the ELLA project has adopted a three-tiered teaching experiment methodology in a similar way to Lesh et al.'s own work in the study of model eliciting (Lesh & Kelly, 2000). The ELLA project has considered three tiers: the researcher's developing understanding of LEGO®-based modelling activities; the teacher-participants' use of LEGO®-based modelling activities to inform evidence-based teaching design; the student-participants' developing knowledge and understanding of mathematics, including their proficiencies and content knowledge. Each tier of the teaching experiment can be considered as an iterative, longitudinal study: At each cycle of study, the developing theory used to explain the observations at each level is refined and fed into the next cycle of research.

In this paper, the first iteration of ELLA is reported upon. The participants in this iteration were a class of 26 mixed ability Grade 4 students (aged 9-10 years) in a typical Australian primary school. Their teacher also participated in the iteration; whilst having no substantial prior experience with LEGO robotics, she actively contributed to the design and delivery of the ELLA learning activities, as well as teaching the regular mathematics lessons. This teacher input is central to the ELLA project and the proposed integrating model eliciting framework: the LEGO activities were designed to align to the regular mathematics lessons and to match the specific learning needs of the student participants. Data gathered during the iteration included: student generated artefacts from the robotics sessions (e.g., completed worksheets), teaching plans associated with the regular mathematics lessons (i.e., mathematics teaching and learning that occurred away from the robotics sessions) and field notes of the researcher that reflected upon the robotics activities and the discussions that occurred with the teacher both before and after the ELLA sessions. This data was then analysed to identify the mathematical ideas encountered by the learners in both the regular mathematics and ELLA activities. This analysis was guided by Nutchey's (2011b) knowledge modelling technique and genetic decompositions for both regular mathematics lessons and ELLA activities were created. These genetic decompositions served as a basis for comparing

the regular mathematics and ELLA activities, with the view to identifying potential opportunities for improving the teaching and learning of fraction knowledge in an integrated fashion using the proposed model eliciting framework.

Data and Analysis

The iteration of the ELLA project teaching experiment reported upon in this paper was conducted over a month long period in which students participated in four ELLA sessions. Each week the students (working in small groups of 2-3) participated in 70 minute sessions in which they worked on completing different ELLA activities that complemented their regular mathematics curriculum. This particularisation of the integrating model eliciting framework to this iteration is illustrated in Figure 3. The focal mathematics concepts developed during this period included: the construction of fractions using various concrete materials; expression of the fractions using both common fraction (mixed and improper) and decimal notation; the use of a number-line model to represent the relative size, position and equivalence of fractions; and counting by fractions and associated rudimentary addition and subtraction strategies for fractions, in particular tenths;.

In the following sub-sections, the mathematical content explored in the regular mathematics lessons and the activities of the ELLA model construction sequence are summarised and their contribution to the participating learners' development of understanding is discussed. The mathematical ideas experienced (and so potentially understood by the students) have been analysed and described using the genetic decomposition technique.

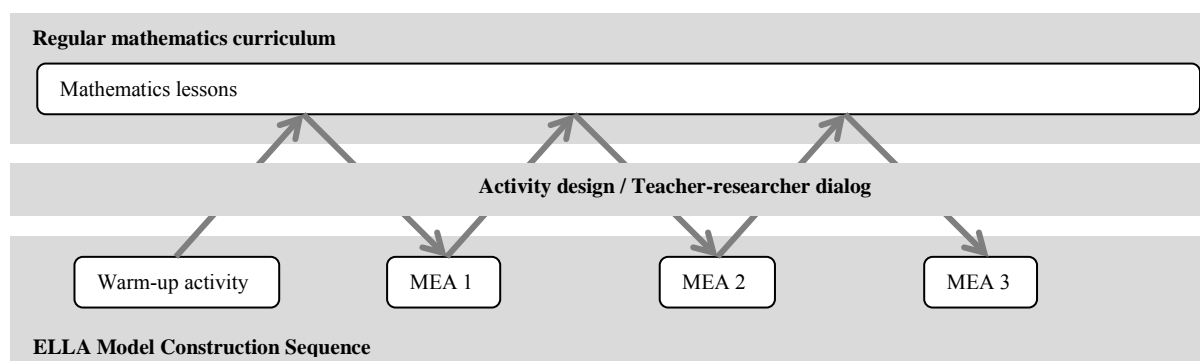


Figure 3. Particular instance of the integrating model eliciting framework

Regular Mathematics Lessons

In the new Australian Curriculum: Mathematics (ACARA, 2012), Year 4 students are expected to study fraction related ideas including: investigating equivalent fractions used in context; count by quarters, halves and thirds, including mixed numbers, and be able to locate these numbers on a number line; extend the place value system into tenths and hundredths; and make connections between decimal and common fraction notation for fractional numbers.

Many of these ideas were developed, often at an introductory level, during the lessons related to this study. During the regular mathematics lessons, the teacher followed a curriculum designed by the state-level public school organisation; the teacher was free to adopt or adapt the materials provided in this curriculum to suit the specific needs of the students, and for the most part the sequence of instruction specified in this state-level curriculum was adhered to.

Lesson sequence

The regular lesson sequence (approximately eight 60 minute sessions) began by using an area model-based approach to the construction of fractions: The children performed activities such as folding a piece of paper to get halves, and then refolding to get quarters and eighths. The equal area of each part was stressed to highlight that the partitioning of a whole into fractional parts requires each part to be of the same size. In addition to the concrete representation of fraction, the symbols (e.g., $\frac{1}{2}$) and the literal names (e.g., one half) were also introduced and linked to each of the fractions created. A similar approach using length model-based paper strips was used to also construct half, quarter and eighth sized paper strips. To reinforce the idea that the size of a fraction is not absolute but instead proportional to the size of the whole, different sized ‘whole’ pieces of paper and paper strips were used. In a related way, it was discussed that as the number of parts increased, the size of those parts decreased. Terminology such as denominator (the name of the part) and numerator (how many of the parts) were also used when discussing the fraction activities. Once halving strategies were soundly developed, alternate partitioning strategies, such as by thirds and by fifths, were introduced to develop a range of fractions including sixths, tenths and fifteenths.

Counting by fractions was then introduced. Students began by counting in halves, using both improper fractions (e.g., 5 halves or $\frac{5}{2}$) and also mixed numbers (e.g., two wholes and one half or $2\frac{1}{2}$). To scaffold this, the number line was used to locate each of the two forms of fractional number and so identify their equivalence.

The relative size of fractions was also considered; the paper strip fractions constructed using various folding strategies (partitioning to halves, thirds fifths and combinations thereof, and all beginning with the same sized whole) were compared with each other, both to identify relative size and also to identify fractions that were equivalent (e.g., $\frac{1}{5}$ is equal to $\frac{2}{10}$).

With the previously developed understanding of partitioning, fraction equivalence and fractional numbers bigger than one, students then returned to a more detailed study of tenths. The students counted in tenths, represented number using improper and mixed number forms and used the renaming strategy to convert between improper and mixed number forms. Number expanders were used to scaffold the students’ use of improper and mixed number forms. Decimal notation was then introduced as an alternative symbolic representation. Just as students previously counted using common fraction representations of tenths, students then used decimal fraction representation to count along number lines in tenths, which was further

extended to counting in other patterns, such as two tenths (0.2) and five tenths (0.5). This counting by tenths provided a basis for rudimentary addition and subtraction by tenths using a count all-like strategy (similar to that they would have encountered when first learning whole number addition).

Genetic decomposition of lesson sequence knowledge

A genetic decomposition that summarises the organisation of mathematical knowledge experienced by the students during the regular lesson sequence is shown in Figure 4. This genetic decomposition is only partial: it identifies the problems (tasks) encountered during the lesson sequence and the coordination of mathematical concepts used to solve these problems – the genetic decomposition does not describe the various representations used by the students to express the problems or concepts.

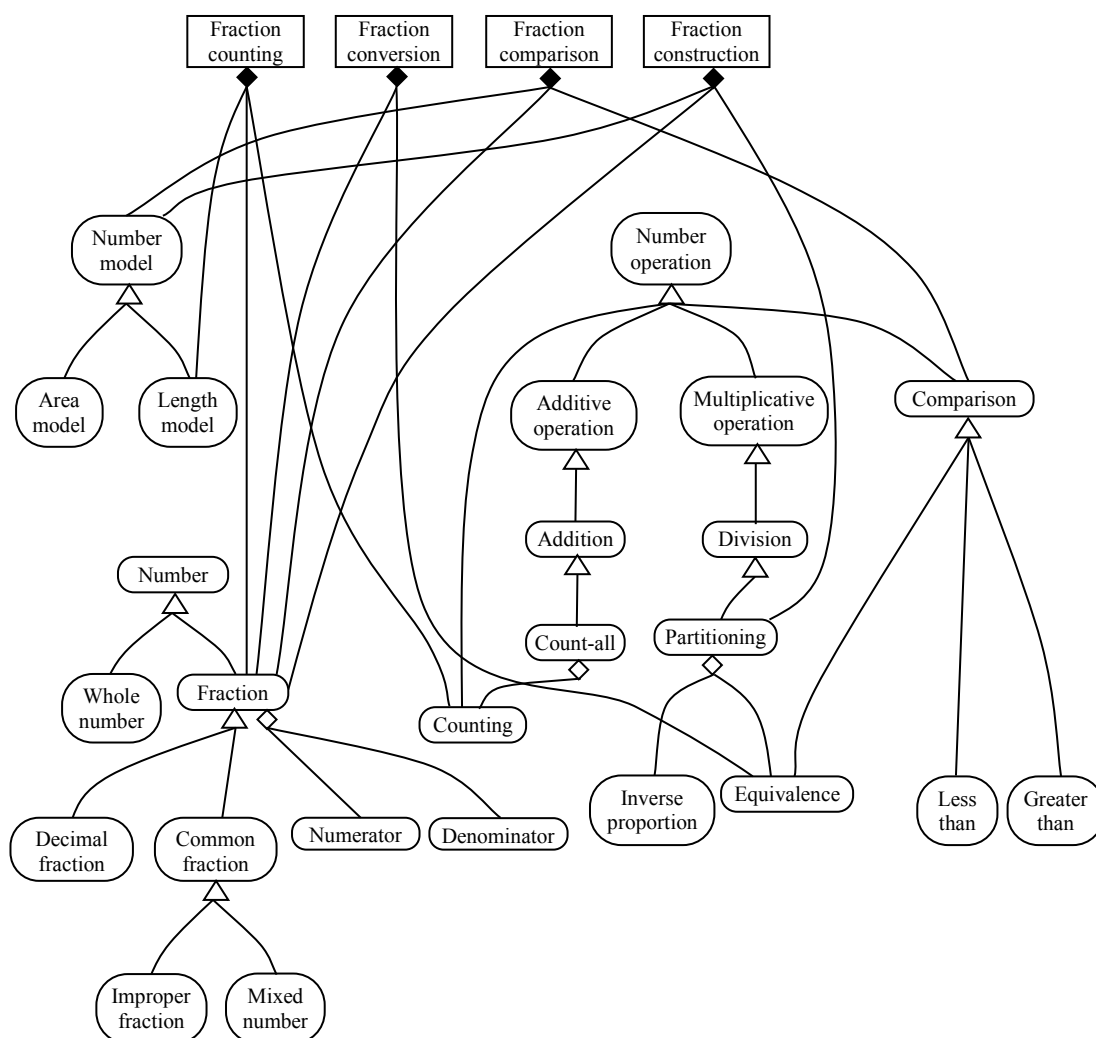


Figure 4 Genetic decomposition of the knowledge covered in the regular mathematics lessons

LEGO-Based Model Construction Sequence

In the LEGO-based model construction sequence, the ultimate aim was to use a simple three-wheeled robot, called the Tribot, and program it to ‘act out’ relatively simple addition and subtraction problems using tenth operands.

Warm-up activity

In the warm-up activity (aimed at familiarising, engaging and enthusing students to participate in the ELLA activities), each small group followed a set of picture-based instructions to construct the Tribot’. This warm-up activity not only familiarised the students with the LEGO NXT technology, but provided the teacher and researcher opportunity to observe group work skills and identify students who might potential struggle to make sense of the technology. By the end of this session, all groups had successfully constructed their robot.

MEA 1

In the first MEA, the students were introduced to programming the robot, and were challenged to program the robot to travel a prescribed distance in a straight line. The bigger aim of this MEA was to scaffold the students’ connection of the robot’s straight line movement to their prior experience of the number line model – that is, to elicit the number line model in this new setting.

Four marks were put on the floor in a straight line and (unknown to the students) these marks were spaced a distance corresponding to exactly 10 wheel rotations of the robot. The students were then challenged to determine how many wheel rotations were required to move their robot exactly from one mark to the next. They quickly established that ten wheel rotations were required. That is, they applied the process of quotitioning-like division to establish how many wheel-rotation sized parts were in the whole. A whole class discussion then ensued to realise the number-line nature of the marks on the floor, with various positions along the number line being identified (e.g., 1 whole, 1 whole and 5 tenths, 2 wholes, 3 wholes, one half) and also what the equivalent tenth fractions were (e.g., 10 tenths, 15 tenths, 20 tenths, 30 tenths, 5 tenths)

MEA 2

In the second MEA the number-line model used to describe the robot’s movement was explored in more detail. Also, the students were introduced to multi-step programming (i.e., programming a sequence of robot moves) so that they would be able to make the robot traverse the number-line in a series of jumps.

After reviewing the previous MEA and the notion that one wheel rotation was equivalent to one tenth of the distance along the number line, the students were given a worksheet prepared by the teacher. The first question was quite direct: it gave explicit instruction regarding where the robot was to start, and how many and what direction the

Tribot's wheels should rotate, i.e., "Start at 0 on the number line, move forward 15 rotations, reverse 3 rotations, move forward again another 8 rotations, where on the number line do you end up?" The next question was similar in structure but, instead of specifying the instructions in number of wheel rotations, the Tribot's movement was specified in the movement of tenths on the number line, i.e., "Your starting point is at 2. Reverse back 0.6, move forward 1.2, reverse again 0.6. What is your end point?". The final questions of the worksheet were more open ended, giving the students start and end point and asking them to determine suitable instructions e.g., "If I start at 1.5 on the number line and I want to get to 2.5 can you provide three instructions to get there?"

MEA 3

In the final MEA of the sequence, a larger number line (i.e., 10 wholes) was constructed. Each group was given a set of 10 question cards. The questions ranged in difficulty and covered tenth-based addition, subtraction and equivalence. The answer for each question formed the argument to a 10-step program the students wrote to traverse the large number line (i.e., on each question card, the students were instructed to program the robot to move either forward or backwards by the unknown amount). On the worksheet provided to the students, a 0-10 number line was provided and students were asked to not only write the 10-step program but also to predict/calculate where their robot would finish. Thus, this MEA provided an opportunity for the students to adapt their number-line model of the robot's movement to a somewhat different context.

Genetic decomposition of ELLA model construction sequence knowledge

A genetic decomposition that summarises the organisation of mathematical knowledge experienced by the students during the ELLA activities is shown in Figure 5. It shows how the students actively engaged in problems requiring them to construct, compare, convert, add and subtract fractions. In each of these tasks, the students drew on a similar set of concepts (compared to their regular mathematics lesson activities), except that instead of partitioning, quotitioning was the process by which the fractional positions along the number line were constructed. Also, because the instructions for programming the robot were often specified in terms of the addition and subtraction problems, new strategies such as count-on and count-back were evidenced in the students solving of these problems. As with the analysis of the regular mathematics lesson sequence, this genetic decomposition does not describe the various representations used to express the mathematical problems and concepts.

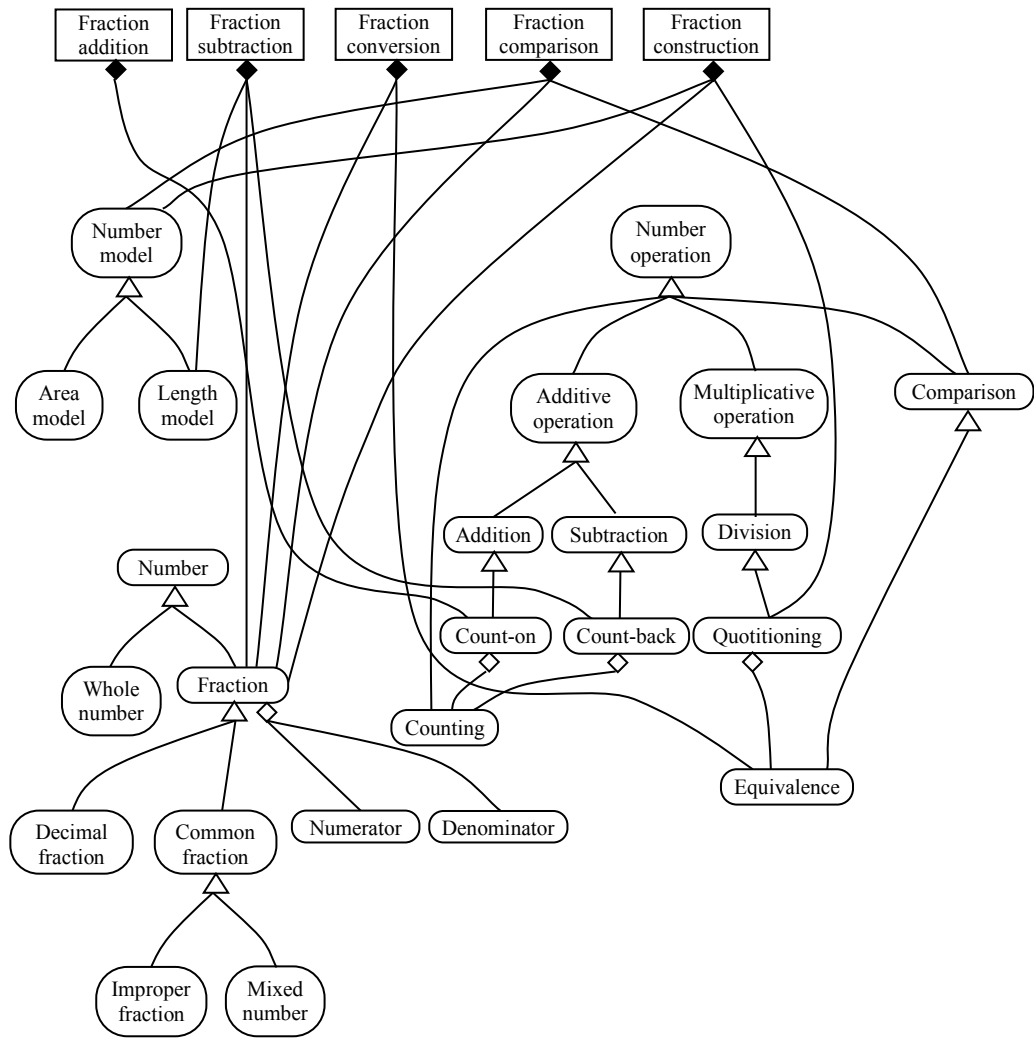


Figure 5. Genetic decomposition of the knowledge covered in the LEGO-based model construction sequence

Implications and Conclusion

The ELLA project is aiming to develop theory and exemplary practices of how LEGO-based activities, in particular those using robotics, can be used in an integrated manner to develop students’ understanding of mathematics. The first iteration of this project has demonstrated the viability of the proposed integrating model eliciting framework as a pedagogical approach to bring together mathematics and technology/ICT curricula.

Upon reflection and discussion, both the researcher and teacher involved in this iteration feel that the zig-zag nature of the integrating framework has allowed the robotics activities to be carefully and strategically designed and inserted into the sequence of activities that have contributed to the learners’ mathematics understanding. The regular lesson and ELLA activities have been complementary, both in the sense that one has provided opportunity to consolidate the learning of another and also in the sense that one has provided opportunity to introduce new concepts not covered in the other. A particular example of this

is the notion of quotitioning which is an important form of multiplicative (division) thinking and which was not covered in the regular lesson sequence.

The constructionist theory underpinning the LEGO robotics technology has been very evident during the ELLA sessions: students were actively engaged in discussing the robot's movement, both through verbal language and also through the use of mathematical representations such as the number line to predict and/or troubleshoot the robot's movement. The use of the number line model, both on paper and as the 'track' along which the robot moved was very telling of the students' thinking with regards to fractions. Some students' use of the number line model indicated their beginning understanding of fraction counting, addition and subtraction, since they resorted to marking every movement by a tenth on the number line (or their robot only ever moved 1 wheel rotation at a time). Other students, with a more sophisticated understanding of fraction addition and subtraction, were able to show the robot's movements as jumps on the number line and their robot moved in similar ways. Such feedback to the teacher is invaluable in ascertaining learner understanding and to tailor instruction to leverage current understanding and address misconception.

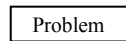
Whilst this first iteration of the ELLA project has been successful, it has identified opportunities for further enhancement of the research methodology and the ELLA activities. The integrating model eliciting framework has been presented as complementary and tightly connecting regular mathematics lessons and robotics activities. From a research methodology as well as classroom practice perspective this could be further strengthened. Greater detail regarding the student's activity in their regular mathematics lessons would allow for much better design of LEGO activities that are timely and relevant to the students' learning. This is as much an issue of research technique as anything else. One way in which this could be improved is more accurate record keeping by the teacher as to what activities (and conceptual challenges they caused) were conducted during the mathematics lessons. Also, this requires an ability to quickly design and prepare/adapt LEGO activities that are of immediate relevance. In a related way, more detailed analysis of the mathematical knowledge and anticipated learning needs to occur before instruction, because this will better inform the design of both the regular classroom activities and the ELLA activities. For example, it was during the post-sequence data analysis that the difference between division strategies (partitioning vs. quotitioning) was identified. Similarly, a greater alignment between the pre-addition (counting) activities of the regular mathematics lessons and the programming of the robot could have been achieved by simply including programming tasks such as making the robot act out fraction counting patterns.

The research activity presented in this paper has proposed a novel adaptation of the model eliciting framework that might allow for a closer integration of mathematics and ICT curricula, with the view to using robotics as a resource to more effectively teach mathematics. Whilst only in its infancy, the data gathered thus far has shown how even young students are

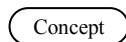
able to use the technology, and that through its use the students application and elicitation of mathematical concepts has been strong. Importantly, the nature of the ELLA tasks has necessitated students to discuss their developing understanding of the embedded mathematical concepts, which has not only contributed to their deeper understanding but has given the teacher (and researcher) valuable insight into the students' mathematical thinking.

Appendix –The Graphical Language

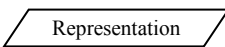
Knowledge objects – the three different types of knowledge objects are used to identify the different mathematical ideas in some domain of knowledge



The **problem** object describes a problem, or task, that might be encountered when engaging in mathematical activity

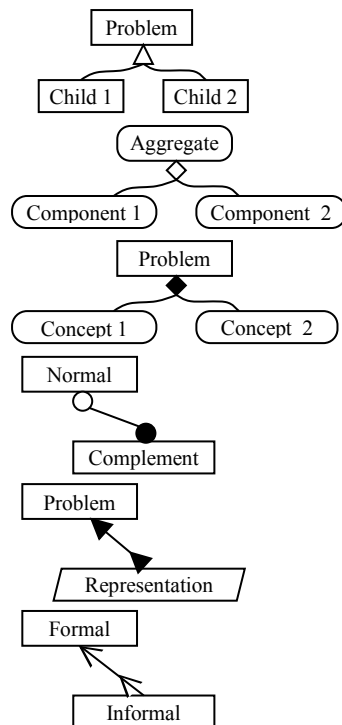


The **concept** object describes some process, concept or principle used when conducting mathematical activity



The **representation** object describes some way in which a mathematical problem or concept might be expressed.

Knowledge associations – drawing on Piaget's notion of reflective abstraction, the different associations describe the cognitive mechanisms by which the organisation of knowledge is formed.



Inheritance describes a super-ordinate relationship between an abstract parent and one or more children, and can be applied to problems, concepts and representation objects.

Aggregation describes an aggregate object composed of one or more component objects, and can be applied to problem, concept and representation objects.

Solution describes a problem being solved using a coordination of one or more concepts.

Inversion describes one object (the normal object) having an inverse (the complement object). This association can be applied to problems and concepts.

Expression describes the expression of either a problem or concept using a representation object.

Formalisation describes one representation (the formal representation) to be more abstract than another (the informal representation).

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Mathematics Education Research Food For Thought With Flavours From Asia

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Abstract

This paper will dip into the wonderfully rich, internationally flavoured, mathematics education research smorgasbord. In more recent times the focus has concentrated upon the cook (the teacher) and the skills needed (pedagogical knowledge) to produce delightfully pleasing brain food to nourish the growth of students. However, if this focus ignores the other essential ingredients (context, students, parents, and policies) then the results do not meet expectations. These expectations are also many and varied and include academic standards and achievement, engagement and deep understanding, exciting and enjoyable teaching and learning, relevant and meaningful learning, and the development of thinking and reasoning skills. Mathematics education research is a source of many recipes for success, and where the various countries provide their own cultural flavours. This paper will attempt to provide a taste of a range of samples drawn from research in order to satisfy an assortment of expectations while providing food for further thought.

Keywords: mathematics pedagogical repertoires, global, cognitive challenge, brain research

Global Movements in Food and Mathematics Education

Food is essential for humanity to survive and it is truly amazing to consider the enormous variety of food and ingredients that are used across the globe. The variety of food eaten usually reflects aspects of the climate and culture, and this also applies to the cooking styles and the combination of ingredients. As the world has become a global community through the advances in technology and communication, this has also lead to the different food styles and recipes spreading throughout the world. In my own suburb of Sydney Australia, which is near a large international university, I counted twenty-three restaurants offering food dishes from other countries. Currently my television stations offer many shows involving travel cooking by some Australian celebrity cook going into another country. For example ‘*Luke Nguyen’s Greater Mekong*’ or ‘*My Sri Lanka with Peter Kuruvita*’, are only two of many shows that include game format cooking shows as well such as ‘*Master Chef, Junior Chef*’ and so on. The reason for focussing on food and cooking is to make a comparison. When we compare the spread of food and cooking styles with the spread of mathematics education styles of teaching and learning, it raises some interesting similarities and differences.

If we consider the global food movement, while we often compare different dishes or recipes, we usually do not get too excited at trying to rank which is best nor do we decide to adopt a recipe from another country and throw away our own well-loved dishes. Instead, we get excited about the rich diversity that these new dishes offer, which according to our tastes, we incorporate certain ingredients or cooking styles into our own to improve or provide

greater variety to our own dishes. Imagine the shock at home if we decided to totally adopt the Japanese cooking style and diet and discard our own.

Why is it that the mathematics education movement seems to do the opposite of what I have described about the global food movement? Why do education authorities and government bodies expend so much energy in ranking countries using international comparison examinations such as TIMSS and PISA? Especially since research has shown these tests to be invalid when comparing countries. In Australia, the current government has introduced a National Assessment Plan in Literacy and Numeracy (NAPLAN) in response to a perceived drop in performance. The teaching community has complained that the testing narrows the curriculum and interferes with the learning cycles of school. The teachers want the many millions of dollars directed to programs that improve student outcomes, not just measure them. Surprisingly the Australian government seemed to be ignorant of the fact that the drop in performance had more to do with changes in the demographics of the sample chosen than changes in the general achievement performance of the children. So it seems that these tests can cause harm when they are not used in a measured and considered fashion.

However, these tests can be very useful, and teachers have adapted and used items in their teaching and the video material of classrooms in different countries has been extremely informative. Watching an Indonesian, German or a Malaysian teacher in the classroom is very interesting and informative and different aspects can be modified and adopted into Australian classrooms in the same manner as watching an Indonesian cooking show on television.

So it is how the international comparison test items and results are used that is the problem and ranking countries is both problematic and unfair.

International studies of mathematics achievement have profound influence on mathematics education worldwide in the past 15 years. Results of studies such as TIMSS and PISA have dominated the agenda of discussion in the mathematics education community as well as among policy makers. Much attention however has been paid on the ranking of countries in the league tables generated from such studies, often without due consideration of the nature of these studies, as well as the contextual factors that affect the performance of students from different countries (Leung, 2012, p. 34).

I find it very heartening to observe that the Southeast Asian Ministers of Education Organisation (SEAMEO) through their regional centres of excellence such as the Regional Centre for Education in Science and Mathematics (RECSAM) in Penang, Malaysia, and the Regional Centre for Quality Improvement of Teachers and Educational Personnel (QITEP) in Yogyakarta Indonesia are developing their own examinations and it is hoped they will learn from the mistakes of the other studies.

Consider one more illustration concerning the global movement of food compared to mathematics teaching and learning. A Japanese recipe for *sushi* would be adapted in Australian home kitchens using local rice, fish, sauces, and although it would be similar to the Japanese dish, a Japanese person eating it would notice differences. Compare this with the

spread of Japanese Lesson Study. Japanese Lesson Study spread throughout the world and particularly across the Asia Pacific region and it has had a global influence upon the teaching of mathematics. The spread of Japanese Lesson Study has received support through the growth of information communication technologies and the ease of international travel. For example, the World Association of Lesson Studies (WALS: <http://www.worldals.org/>) was formed and this promoted Lesson Study at many levels from systems to individual schools across a range of countries. Another project, one which was supported by the Asian Pacific Economic Cooperation (APEC: http://hrd.apecwiki.org/index.php/Lesson_Study#Lesson_Study_inMathematics), was designed to encourage the spread of Lesson Study across the region. There were admirable projects but the problem was in how this spread was adopted, with some countries adapting it to their educational, cultural and social contexts and others trying to make an exact copy.

Nowadays, most packaged food contains a list of ingredients and warnings about possible food allergies. Regarding Japanese Lesson Study, there were warnings from Japanese scholars against trying to copy exactly their model to other countries.

When thinking about the global issue of improving the quality of education, Lesson Study, which has a dialectical relationship with the theories and practices employed on the front lines of education, must be versatile enough to be applicable beyond the Japanese context" (Baba, 2007, p.6).

Yet in spite of these warning there were attempts to copy exactly the Japanese Lesson Study model into other systems, and some attempts even imported Japanese teachers but the results were unimpressive. It seemed that those countries that used a 'glocal approach were more successful (White & Lim, 2008).

The term 'glocalisation' was constructed to explain the process whereby the global and the local influences interpenetrate each other, creating a hybrid model. This hybrid contains sources of global trends adapted and blended with local conditions and options. In other words, global trends are contextualised by the specifics of the local settings. In terms of food, foreign recipes are modified to cater for local tastes and ingredients. In a global world there is increasing pressure to use and adapt successful mathematics teaching and learning practices from other countries. However, the dangers of adapting these practices require consideration of both the practicality of technical implementation and the extent to which the beliefs underlying the adapted practice resonate with local cultural values.

To summarise this section, a brief comparison between global food movement and the teaching and learning of school mathematics revealed that both were experiencing similar global influences through greater exposure and communication due to improvements in technology but differences were evident in the reactions to these influences. In the rest of the paper, the food metaphor will be used to further highlight some of the current issues facing the teaching and learning of mathematics.

While food and exercise are needed for a healthy body, the brain requires stimulation, challenge and exercise if it is to stay healthy. As a result of brain research, we no longer regard the brain as a fixed organ. Research has shown that the brain can reorganise itself. Neuroplasticity was the term given to the remarkable ways in which the brain adapts and transforms itself as a result of a change in stimuli. Brain researchers showed that:

Children are not always stuck with mental abilities they are born with; that the damaged brain can often reorganise itself so that when one part fails, another can often substitute; ... One of these scientists even showed that thinking, learning, and acting can turn our genes on and off, thus shaping our brain anatomy and our behaviour (Doidge, 2008, p. xv).

In the next section, the paper will consider if the correct brain food is being delivered by some common teaching strategies, or whether students' brains are being starved through lack of stimulation, challenge or exercise.

Are You Starving Your Students?

For many school students, mathematics is more like eating Brussels sprouts or broccoli; they know it is healthy and good for them but no one wants to eat it. Why is this true for mathematics when for cooking a good cook can easily modify the recipe and methods to overcome the taste of the unloved ingredients? Why can't many mathematics teachers do the same? Prominent researchers have criticised the brain food being dished up in some mathematics classrooms and pre-teacher education courses:

There is an urgent need to change school mathematics and mathematics teacher education in fundamental ways... Old patterns and methods are so deeply entrenched in many schools and teacher education institutions, and particularly in the minds of teachers, lecturers and students, that there is an urgent need to problematise existing practice and equip and empower practitioners to achieve change. (Clements, 2003, p. 638)

While many school mathematics teachers work very hard and are very dedicated, and many of their classroom strategies are nourishing for young minds, nevertheless Clements was deeply disturbed by what he regarded as serious examples of cognitive undernourishment. He made use of Brousseau's (1984) didactical contract construct (Clements, 2004) to illustrate the issue and a deeper treatment of this can be found elsewhere (White, 2011). What follows is just a brief summary.

Brousseau (1984) drew attention to how a teacher's intervention could reduce a student's role to answering a series of relatively simple questions, by 'emptying' the task of much of its cognitive challenge for the students. Cognitively challenging questions were removed from the menu and replaced by bite-size portions (I will call it baby food). The seduction of behaviourism that promotes mastery learning and 'breaking a long journey into small steps' was evident. When teachers adopt this style of cognitively emptying the task of challenge and break it into a number of smaller steps, in an attempt to help students tackle

higher-level mathematics tasks, they deny their students the opportunity to formulate and apply strategies of their own (Clements, 2004). If this becomes the predominant classroom teaching strategy then the students are starved through being fed mathematics emptied of any goodness and cognitive challenge. They are served small portions that are easily chewed and digested and totally leave out the mathematical solids that would challenge the brain. Examine the following dialogue.

Teacher: Add one half and two thirds

Student: Cannot

Teacher: Ok multiply 3 and 2

Student: 6

Teacher: Good write that down at the bottom of a fraction, now what is 1 by 3?

Student: 3

Teacher: Good write that on top of that fraction to make a half, and what is 2 by 2?

Student: 4

Teacher: Good write that on top of that fraction to make two thirds, now add 3 and 4

Student: 7

Teacher: So write 7 over 6, now write it as a mixed number

Student: One and one sixth

Teacher: Very good, understand?

Student: Yes.

Skemp (1976) using his distinction between instrumental and relational understanding, would classify the student's understanding here as instrumental. While the teacher's intentions are kind and helpful, good intentions are not enough. The teacher also makes an assumption that if the student answered each step, then the student had learnt what had just been taught, and the student could construct the whole from the parts thus the student should be able to add two fifths and one third following the same procedure. Of course not every student could and the retention by the students was very poor. The strategy of cognitive emptying has been shown by a large number of research studies to produce poor results and students' were often unable to apply this learning to other novel problems. Imagine serving baby food to teenage school students, they would probably spit it back at us. Perhaps that is what is happening in mathematics lessons which serve 'chopped up muck'.

Brain research provides an alternative and helps explain the poor results of this teaching strategy and why mathematics teachers are often frustrated when students are able to solve problems or use a procedure correctly one day but cannot remember how to do the same things on the next day because the brain treats it as new. The reasons are:

Students may diligently follow the teacher's instructions to memorize facts or perform a sequence of tasks repeatedly, and may even get the correct answers. But if they have not found meaning by the end of the learning episode, there is little likelihood of long-term storage (Sousa, 2008, p. 56).

So in the dialogue presented earlier, unless the students has formed some meaning from the teachers instructions, then the teachers instructions will not be remembered. For

longer retention, the teacher needed to help the student with the concepts of fractions and addition of fractions rather than just with the procedures to get the answer. Conceptual change remains one of the most essential outcomes of learning. It is an intentional and constructive effort to bring about deep understanding. The fact that some teachers have to do so much revision and student cramming before the end of year examination points to the methods of teaching and the lack of help to students trying to make meaning. Instead the usual outcome of this cramming process is to blame the poor results on the intelligence of the students.

We will examine another poor example of teaching before looking at some good and successful ones, and examine their ingredients of success. There are many examples reported in research of strategies that reduce the cognitive challenge and high level thinking of the students by stifling discussion and over emphasising drill and practice.

It has been stated that drill and practice are the rice dishes of Asian mathematics classes. While it is common for many Asian meals to have rice as one dish, it would be highly unusual to have a meal of only plain rice and no other dishes of vegetables, fish, or meat. A single plain rice dish diet would quickly become boring and lead to poor health. Some mathematics teaching strategies are used like a diet of single rice and result in bored students who are sick of mathematics. For example, after viewing the TIMMS videotapes of Australian Year 8 mathematics classes, a widely respected mathematics educator, Alistair McIntosh was moved to make the following comment:

What overall picture [do the lessons] give of a typical Year 8 Australian lesson? The teacher talks a lot, the students mainly reply with very few words, most of the time the students work using only pencil and paper, on a repetitive set of low level problems, mostly presented via the board or textbooks or worksheets, discussion of solutions is mainly limited to giving the right answer or going through the one procedure taught. There is little or no opportunity for students to explain their thinking, to have a choice of solution methods or to realise that alternative solution methods are possible; and very few connections are drawn out between mathematical ideas, facts and procedures (McIntosh, 2003, p. 107)

Where is the intellectual challenge? Where is the brain food? Where is cognitive stimulation? How are students supposed to develop thinking skills and construct meaning in such an environment? This seems to be a classic case of cognitive starvation, and it would not be surprising if these students were disengaged with mathematics and gave up study of the subject at the first opportunity.

Luckily not all teaching strategies lead to cognitive starvation and some actually promote the development of thinking by increasing the level of cognitive challenge. There is a crucial distinction between a strategy that empties a mathematical problem of challenge and a strategy that gradually increases the level of challenge presented to the students by differentiating the curriculum and catering for individual needs.

A classical education of the nineteenth and early twentieth involved learning other languages which strengthened auditory memory, precise handwriting that helped strengthen motor capacities which added speed and fluency to reading, and an emphasis on correct speech and pronunciation. As a leading brain researcher states:

... for hundreds of years educators did seem to sense that children's brains had to be built up through exercises of increasing difficulty that strengthened brain functions (Doidge, 2008, p. 41).

Amazingly, cognitive exercises (brain food) have been designed by brain researchers and trialled that have improved memory, problem solving abilities, and language skills in mature aged subjects and children, reversing the aging process by twenty to thirty years in some adults. These exercises have also been used with autistic children with amazing effects on their language skills and autistic behavioural traits.

For example, *Fast For Word* is a brain training program consisting of seven brain exercises. It has had remarkable success with language-impaired and learning-impaired children including autistic children. The program which is a series of plasticity based techniques

... have helped hundreds of thousands. *Fast For Word* is disguised as a children's game. What is amazing about it is how quickly the change occurs. In some cases people who have had a lifetime of cognitive difficulties get better after only thirty to sixth hours of treatment (Doidge, 2008, p. 47).

So in considering the question of cognitive starvation, it has emerged that the need for challenge, construction of meaning and rewards are important for nourishment of the brain. An easy reaction is to blame the cook or teacher and this is often the path taken by authorities and sometimes researchers but maybe authorities and researchers are also part of the problem. The next section will briefly consider this proposition.

Stop Blaming the Cook and Fix the Kitchen

If a restaurant has a cook that is producing poor meals then the cook is either fired or is mentored by an experienced chef and sometimes sent for further training. It may also involve changing the kitchen if it is part of the problem.

A good deal of the early research into mathematics pedagogy was based upon a variety of assumptions. A common one was that teachers developed characteristic patterns of behaviour depending upon their classroom situation. These patterns may have been conscious and deliberately chosen, or have arisen from experience and may have been largely unreflected and unconscious. Such teaching patterns were often designated as orientations, styles or modes of teaching.

Many researchers represented teaching styles upon a continuum with their theoretical opposites as end points. Some researchers used more than one continuum. The aim of this type of research was either to present an overall structure for current teaching practice

according to the researcher's theoretical presentation, or to categorise teachers according to a particular theoretical orientation. The assumption was that all teachers could be located somewhere along the continuum.

For example, Brady (1985) investigated this area of research and concluded that all the models could be placed upon a series of continuums with the end-points being: (a) teacher centred intentions and behaviour; and (b) child centred intentions and behaviour. A major criticism of Brady's models was that the classification of teaching behaviour or intention by assigning them to a position upon the continuum did little to explain the totality of teacher classroom behaviour.

Another modern example of this type of research can be found in the following study which stated that:

To identify the extremes of teachers' beliefs and to facilitate categorisation of responses, an artificial continuum of teaching and learning was used. At one end of this continuum, mathematics is seen as a fixed body of facts to be delivered by teachers and internalised by students. Referred to as a *traditional* teaching approach, this perspective is associated with individual student work, rehearsal of routine questions, and reliance on textbooks or worksheets. This view may be accompanied by a belief that problem solving is an *end* and that problems should be presented to students after they have mastered basic facts and skills. At the other end of the continuum, termed a *contemporary* teaching approach, mathematics is seen as a dynamic subject to be explored and investigated. Classroom practices associated with this perspective usually involve group work and the use of non-routine questions that promote mathematical thinking, and the development of problem-solving skills. This teaching approach may be accompanied by a belief that problem solving is a *means* to learning mathematics (Anderson, Sullivan & White, 2004, p. 40).

In this example there is an unstated but automatic assumption that student centred pedagogy (or problem solving) is always best. Yet this is too simplistic and ignores the cases of student centred pedagogy existing in a classroom that is intellectually undemanding or empty of cognitive challenge. Using a food analogy, the equivalent is to categorise all humans on a continuum of 'hamburger eaters' and 'not hamburger eaters'. All people must fit along this continuum. Thus some people may gorge themselves on fast foods whereas others are the consumers of healthier alternatives. How useful is this continuum in reflecting what people really eat? How useful is it helping people to change?

Being labelled as 'teacher centred' comes with a significant amount of shame. It usually carries the stigma of being a teacher who does not care for their students. Just as the label 'burger eaters' may attract the stigma of 'unhealthy slobs'. This sort of labelling hardly contributes to an enlightening debate. It is time for research to stop calling teachers names and to help them to develop cognitively challenging classrooms. The classroom teacher who uses the cognitive emptying/elicitation/exposition strategy does so because he or she believes that this is the best way of assisting student learning. It is these beliefs that must be exposed and challenged before the classroom teacher is likely to change strategies.

This section has briefly considered how teaching strategies can starve the brain damaging learning outcomes and how they can be designed to exercise and develop the brain. The brain is a muscle and the more it is used the better it works. Teaching strategies that deliver brain food in the form of stimulation, exercise and challenge are needed in today's mathematics classrooms.

Is there only one best way of teaching or cooking or are there many ways? The next section will briefly consider this proposition.

A Collection of Recipes or Pedagogical Repertoires

In 1952 Cunday and Rollett stated that "*Mathematics is often regarded as the bread and butter of science. If the butter is omitted, the result is indigestion, loss of appetite, or both*" (cited in Ollerton, 2003, p. 57). Does the lack of one ingredient destroy the final dish? A good chef knows that the success of the final dish depends upon the way the individual ingredients work as a whole. For the teacher it is the same realisation that everything is connected.

There is a need to dismiss the assumption that mathematics classroom teachers use only one pedagogical style within their classroom or for that matter in any one lesson. The context of the mathematic classrooms and the pedagogical strategies employed within the classrooms are more complex than a mere choice between two extremes. Major longitudinal research coming from Singapore has revealed that experienced and successful teachers continually vary their approach where they 'weave' a number of different pedagogical strategies into their classroom (Luke, 2005). This seems to resonate with the popular expression: variety is the spice of life.

Thus the Singaporean teachers present a healthy diet to the class by varying the ingredients according to the conditions. A mathematics teacher doesn't adopt one approach and stick to it, but rather adjusts the pedagogy to the multitude of variables that influence the classroom context in order to create conditions favourable to student learning including cognitive challenge and meaning construction.

Creating such conditions requires the teacher to recognize when it is useful to intervene, when to be strongly didactic, when to offer hints and advice and when to stand back and strategically decide not to 'interfere'. Seeking to offer students choice in their learning inevitably means finding problems that encourage students to make decisions about how to proceed and about what approaches and what resources they might use in order to work towards a solution (Ollerton, 2003, p. 98).

Thus the teachers must be assisted by researchers and authorities to develop a rich repertoire of teaching strategies and challenging tasks in order to deliver brain food in the form of stimulation, exercise, challenge and meaning. It is the function of symposiums such

as this one to assist in the transformation of research into practice by working towards joyful and meaningful mathematics.

When food is prepared, the way it is eaten has important outcomes. A person can shovel the food down as quickly as possible or the person can concentrate upon the food and savour the tastes and sensations. While both actions may sustain the body, it is the second that will long be remembered and treasured. Does the same apply to mathematics education? This proposal will be briefly considered in the next section.

Engagement and Intensity

The fast food industry has long understood the value of engagement and uses many promotional strategies to attract people to their products such as product identification (some unique symbol or character), free give away prizes (usually toys to engage the pester power of children), many advertisements that are colourful, lively and target peoples desires for happiness, freedom and having a good time, and finally they use bright, clean, lively premises staffed with happy smiling young faces. So, because of these marketing strategies the attraction is strong, but engagement is something much deeper.

Engagement in mathematics is important in mathematics because it is argued that decreased engagement can have negative effects on students' lives (Sullivan, Mousley, & Zevenbergen, 2005). One of my colleagues (Attard, 2010, 2011a, b, 2012) conducted a longitudinal case study investigating the problem of lowered engagement with mathematics using the students' perspectives of the factors that influenced their engagement during the middle years of schooling within an Australian setting. The study spanned three school years. The study used a multi-dimensional view of engagement which occurs with the meeting of all three facets; cognitive, operative and affective, that leads to students valuing and enjoying school mathematics and seeing connections between school mathematics and their own lives beyond the classroom (Fair Go Team, NSW Department of Education & Training, 2006; Munns & Martin, 2005).

One finding that is worth repeating is in regard to having fun in mathematics classrooms. The data clearly revealed the aspects of lessons that made them fun were not always based on games but involved relevance to the students' lives, an element of challenge built into the tasks, and the ability for students to see the mathematics as useful within practical situations. As in the first section of this paper, students need cognitive challenge, to be able to construct meaning through the relevance and applicability of the mathematics. Lessons that promoted affective, operative and cognitive engagement with mathematics were regarded as fun. It is within this context that brain research can also add some interesting research findings.

Brain research has found that the speed at which we think is also plastic. It is possible to train the brain to fire brain neurons more quickly in response to stimuli. The essence of the training lay in paying close attention.

Merzenich discovered that paying close attention is essential to long-term plastic change. In numerous experiments he found that lasting changes occurred *only* when his monkeys paid close attention. When animals performed tasks automatically, without paying attention... the change did not last. We often praise “the ability to multitask.” While you can learn when you divide your attention, divided attention doesn’t lead to abiding change in your brain (Doidge, 2008, p. 68).

So it seems that the intensity of engagement is the key for stimulating the control centre to produce acetylcholine (helps concentration) and dopamine (pleasure).

That’s why learning a new language in old age is so good for improving and maintaining the memory generally. Because it requires intense focus, studying a new language turns on the control system for plasticity and keeps it in good shape for laying down sharp memories of all kinds (Doidge, 2008, p. 87).

So the classical education of the 1850s to the 1900s involving the learning of other languages and the demands upon students to pay close attention to their hand writing, speech and pronunciation all played a part in developing the capacities of the brain.

Now during a 40 minute lesson, brain research has found that there are optimal times when students are more open to paying close attention. We tend to remember best what comes first and second best what comes last and this is known as the *primacy-recency effect*. In other words there are windows of time for teachers when there is a learning opportunity that is best for new material because the students are more predisposed to pay close attention. In the figure below, these windows for learning and retaining new material are shown.

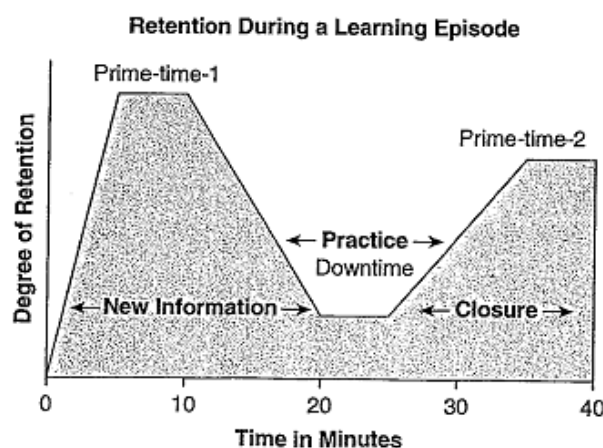


Figure 1. New information can be presented in prime time 1, closure in prime time 2 and practice is appropriate in the downtime. (Sousa, 2008, p. 61).

Just as there seems to be times when people are hungrier for food than at other times, so it seems that there are times when it is best to challenge students with new material and other times when they would be best doing drill and practice.

Now on the surface the importance of intensity may sound like it is supporting the practice of cramming for an examination. Yet students will tell you that cramming for an examination usually results in what has been memorised is lost within a week or two afterwards. This seems a contradiction.

Firstly, the intensity principle is referring to the learning of new material, which I suspect for some crammers may also be true. Secondly, for retention the new material would need a period of practice. There is a distinction made in the literature between *massed practice* and *distributed practice* (Sousa, 2008). Cramming is an example of massed practice where material is crammed into the working memory, but will be quickly dropped or forgotten without sustained practice. The material has no further meaning or need for long-term retention. Distributed practice on the other hand is sustained practice over time resulting in long-term storage.

Now all education systems formulate ways of giving feedback to the students in order to communicate both the desired system outcomes and the student's progress towards meeting these outcomes. In the next section, this will be briefly explored.

Student Feedback

When dealing with food, a cook is quickly informed about the customer's satisfaction with the meal. Walking out without paying the bill sends a strong message. Good cooks adjust their cooking to suit the tastes of their customers. Parents quickly become attuned to the various likes and dislikes of their children. Usually the feedback is directed at the food and not at the cook, but positive feedback encourages the cook to do better.

Early behaviourism studies used stimulus response experiments and relied on rewards and punishments to change behaviour. This was also adopted into teaching strategies and involved using differing feedback strategies to promote behaviour modification, usually with an emphasis on punishments such as hitting with a cane or keeping students in after school. While behaviourism was unable to satisfactorily explain why this seemed to be mostly effective, brain research is able to deepen our understanding of the processes involved.

The, *Fast For Word* brain training program mentioned earlier consists of seven brain exercises that were developed from the latest brain research. It has had remarkable success with language-impaired and learning-impaired children including autistic children. A reward is a crucial feature of the program because each time the child receives a reward the brain secretes neurotransmitters such as dopamine and acetylcholine which helps consolidate the brain changes the child has made. Dopamine reinforces the reward while acetylcholine helps concentration and sharpens the memory.

The program which is a series of plasticity based techniques has

... helped hundreds of thousands. *Fast ForWord* is disguised as a children's game. What is amazing about it is how quickly the change occurs. In some cases people who have had a lifetime of cognitive difficulties get better after only thirty to sixth hours of treatment (Doidge, 2008, p. 47).

Research into technology and the use of digital games has focussed on extending the thinking of students, particularly in literacy and numeracy and upon using levels and rewards as a means of providing feedback to the student. For example, Lowrie (2005) worked with eight-year old students using the Pokemon environment and found that the children worked well beyond the experiences being provided in the standard school curriculum in terms of spatial representation and visualisation. This work highlighted the possibilities of the digital games environment for enhancing mathematical learning and understandings that were beyond the realms of standard pencil-and-paper representations.

This is the last section of my smorgasbord of the wonderfully rich, internationally flavoured, mathematics education research. It is time to bring all the sections together.

Conclusion

The sign of a good meal is that the consumer is left with the desire to have more, to repeat the experience at another time. I hope this paper has had the same effect.

Brain research is still in its infancy regarding applications to teaching and learning but what has already arisen has added to our understanding. We now have a better understanding of the effect of education upon students and the realisation that the brain requires nourishment, because the saying, use it or lose it applies to every student in every mathematics classroom.

... post-mortem examinations have shown that education increases the number of branches among neurons. An increased number of branches drives the neurons farther apart, leading us to an increase in the volume and thickness of the brain. The idea that the brain is like a muscle that grows with exercise is not just a metaphor (Doidge, 2008, p. 43).

Cognitively well-nourished mathematics students are the result of a rich and varied diet of challenging, meaningful and rewarding classroom experiences. Recipes that are unhealthy should be removed. Using too much salt is just as unhealthy as using cognitive emptying recipes in the classroom. Teachers must resist becoming mere packet warmers by some educational administrators and systems and seek to adapt international programs rather than just copy.

So I conclude upon a hopeful note. This symposium fills me with hope. It is evident in the desire and passion of the mathematics teacher chefs who give freely of their time to attend. And it is through this collective power via the Regional Centre for Quality Improvement of Teachers and Educational Personnel (QITEP) in Yogyakarta that I feel the mathematics education profession will demand more than just 'microwave warming'

solutions that are on offer from other countries. It is my hope that this symposium will assist all participants in transforming research into practice by integrating the best of global mathematics education practices with their own successful teaching and learning practices to produce joyful and meaningful mathematics.

Post Script

An assumption of this paper is that mathematics provokes hunger, passion and desire in teachers and students. Hopefully the staffrooms of mathematics teachers are kitchens of delight staffed by passionate experts that inspire the students. What follows is a very unique but interesting and delightful understanding of the number system.

The famous Danish writer, Peter Høeg is one student who was inspired by his mathematics teachers. In one of his now famous novels he has the lead female character named Smilla trying to explain her 'sense of snow' using mathematics to a man living below her apartment. It fits loosely with the theme of this paper as the man is doing the cooking (making it a fantasy tale for some female readers?). The extracts from the novel are shown in italics (Høeg, 1992, pp. 121-122) and I have added some remarks that appear in normal script.

To set the scene, the male (who is a government operative) lives in a ground floor apartment and has been watching this attractive girl from Greenland (who has been making some uncomfortable enquiries involving the government) pass by his door each day. Finally, he has invited her in for dinner, and while he cooks, they chat and she tells him of her love of ice, snow and mathematics. Her description may challenge your own view of the number system, at the very least I hope it will cause you to look at numbers in a different way.

It begins with the Smilla, saying:

Do you know what the foundation of mathematics is?' ... 'The foundation of mathematics is numbers. If anyone asked me what makes me truly happy, I would say: numbers. Snow and ice and numbers. And do you know why?

How to answer such a question? How could he respond without making a fool of himself? So,

he splits the claws with a nutcracker and pulls out the meat with curved tweezers.

Undaunted by his silence she continues,

Because the number system is like human life. First you have the natural numbers. The ones that are whole and positive. The numbers of a small child. But human consciousness expands. The child discovers a sense of longing, and do you know what the mathematical expression is for longing?

Obviously he is a strong silent type because

he adds sour cream and several drops of orange juice to the soup. She continues, The negative numbers. The formalization of the feeling that you are missing something. And human consciousness expands and grows even more, and the child discovers the in between spaces.

Between stones, between pieces of moss on the stones, between people. And between numbers. And do you know what that leads to? It leads to fractions. Whole numbers plus fractions produce rational numbers. And human consciousness doesn't stop there. It wants to go beyond reason. It adds an operation as absurd as the extraction of roots. And produces irrational numbers.

Having decided that silence is his best strategy,

he warms French bread in the oven and fills the pepper mill. She continues It's a form of madness. Because the irrational numbers are infinite. They can't be written down. They force human consciousness out beyond the limits. And by adding irrational numbers to rational numbers, you get real numbers.

She notices that he seems to be listening (although still saying nothing), so she continues,

It doesn't stop. It never stops. Because now, on the spot, we expand the real numbers with imaginary square roots of negative numbers. These are the numbers we can't picture, numbers that normal human consciousness cannot comprehend. And when we add the imaginary numbers to the real numbers, we have the complex number system. The first number system in which it's possible to explain satisfactorily the crystal formation of ice. It's like a vast, open landscape. The horizons. That is Greenland, and that's what I can't be without!

There is a long silent pause and now what does a typical male do in such a situation? He is confronted with a beautiful woman bearing her soul, exposing her innermost feelings to him. He cannot remain silent and do nothing so what is he to do?

'Smilla,' he says, 'can I kiss you?'

Disappointed at this ending my students suggested an alternative suggestion

'Ok, Smilla, your number is up!'

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