BUILDING THE SENSE OF STRUCTURE THROUGH THE SUPPORT OF VISUALIZATION

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Abstract

In particular, Indonesian students enroll their first algebra class in secondary school. Generally speaking, most of the Indonesian classes will start with a very formal algebraic lesson which diminish the chance for the students to do an investigation to generalize an algebraic pattern. In one hand, generalization is the important aspect of algebra which distinguish it from its ancestor: arithmetic. This tendency leads to a number of obstacles encountered by the students in learning algebra (see Jupri, Drijvers & van den Heuvel-Panhuizen, 2014). Reflect to the global view of difficulties in learning algebra, the current research is conducted.The present study is aimed to design a local instructional theory which helps the students to build their sense of structure with the support of visualization to develop their generalization ability. Reflect to the aim of this study, design research is chosen as the appropriate approach. In addition, Pendidikan Matematika Realistik Indonesia (PMRI) which is adapted from Realistic Mathematics Education (RME), is applied as the guideline when designing the series of learning activity. The subject of the study are four fifth grade students of MIN 2 Palembang who are following fifth lessons of the pattern investigation activities. The data showed that the students were able to reveal the general structure of a pattern by the support of visualization. The sense of structure gives an opportunity for the students to do generalization based on their point of view toward the regularities on the pattern.

Keywords: algebra, structure, sense, generalization, PMRI, RME, design

INTRODUCTION

In Indonesia, algebra is a mandatory topic for secondary school mathematics. It is commonly started by the introduction of variable as an unknown and need to be solved using a rigor algorithm. Generally speaking, there is no relation between algebra and other topics in mathematics, especially arithmetic. The setting of algebra teaching and learning in Indonesia, leads to the students' limited understanding within the world of algebra (Jupri, Drijvers & van den Heuvel-Panhuizen, 2014). Instead of seeing algebra as a generalization of arithmetic, as a structure and regularities, or as a language of mathematics as is defined in Dekker & Dolk (2011), the students merely see algebra as a set of expressions where letters are involved (Booth, 1988). This is irony, since if we reflect to the historical aspect, arithmetic is a fundamental base of algebra.

The distance between algebra and arithmetic create an obstacle which is define by Carraher, Schlieman, Brizuel, & Earnes (2006) as the students' difficulties to shift from arithmetic to algebraic thinking. Algebraic thinking is the heart of learning algebra, it is defined as students' awareness towards the relation between the numbers (Kieran, 2004). The lack of students' establishment in algebraic thinking happened due to the fact that the students had develop a lot of arithmetical proficiency during primary school level, but all of sudden they have to jump into a formal world of algebra (Jupri et al., 2014). To deal with the aforementioned condition Jacobs, Franke, Carpenter, Levi & Battey (2007) strongly pointed to the need of a pre-algebraic activity which can support the students to bridge the gap.

Since 1989, The National Council of Teachers of Mathematics (NCTM) recommended the use of pattern in the pre-algebra related activity to support the students' development of algebraic thinking. Patterns investigation provides a rich learning environment for the students to learn arithmetic related problems while at the same time they start to think globally, to do generalization.

Even though this kind of pre-algebraic lesson is already used in several countries, this topic still needs to be studied further. There are two major reasons of conducting this study. First, pre-algebra activity is rarely implement in Indonesia. Hence, it is needed to develop a local instructional theory which appropriate with the Indonesian context. Second, doing generalization is not easy for the students (Lannin, 2005). The students might be able to continue the series of 1, 3, 5, 7, 9 … but it is need an insight to generalize that the series will only be continued by odd numbers (similar example can be seen in Kenney and Silver, 1997). Therefore, 357 will be in the series but 400 will not be there. This skill is called the sense of structure. To fostering the students' sense of structure, the pattern task should be integrated to the visual support, for instance using a geometrical representation (Herbert and Brown, 2000; Ma, 2007; Carraher, Martinez & Schliermann, 2007).

Reflect to the aforementioned condition, in the present study we employed the pattern activity using local context of Indonesia to give a chance for the students to develop their ability in doing generalization. To help the students in doing generalization, the learning activities will be supported with the visualization of the pattern in geometrical shape. In line with the aim of this study which is to design a local instructional theory in learning algebra using pattern, the research question addressed here is "*how visualization support the students' construction of structure sense*?"

THEORETICAL FRAMEWORK

School Algebra

Algebra is one of the topic in school mathematics which focuses on arithmetic generalization, problem solving, structure and relationship (Usiskin, 1988). According to Brawner (2012), teaching and learning algebra is aimed to prepare the students to be ready in learning advance mathematics in higher education. In past, algebra mostly related to higher educational system, which will only be introduced for secondary level students. But now, a number of studies proof that algebra is needed in earlier grades to promote the students' development of algebraic thinking. Learning algebra is not about remembering the algorithm to find the unknown value of an algebraic expression. Learning algebra is about looking for pattern, structure and regularities. In line with that, early algebraic lesson is aimed to give the students an experience to work with the structure of numbers which focus on the transition from the typical arithmetical problems to the algebraic problems.

Visual Support and Structure Sense: A Step towards Generalization

Lannin (2005) pointed out the advantage of using pattern as the center of early algebra activities is due to its "dynamical representation of variables" (p.233). An algebraic model of a pattern is the beginning of formula building, with attention to the investigation, identification and the relation between its structures algebraically (Drijvers, Dekker & Wijers, 2011). One strategy to use pattern activities is to use number patterns which are embodied in visual representations. Visual support of mathematical objects can be helpful to promote the students' construction of structure sense (Rivera, 2011). According to (Zazkis & Liljedahl, 2002) structure sense is a useful skill to develop personal references to do generalization while looking at a particular pattern.

Algebra in Indonesian Curriculum

Indonesian students start learning algebra in the first grade of lower secondary school. The text book initially discusses about variables and algebraic expressions, mostly linear equations with one variable. The focus of learning is on the procedures of solving linear equations. This causes the students to see a variable merely as the representation of an unknown, which means anytime they see that, they will be looking for a single number as an answer. In fact variables can be used not only as the representation of unknown, but also as the representation of a range of values, and. The second use of a variable is mostly forgotten in Indonesian classroom. Lee & Wheeler (1987) acknowledge it as the cause of students' inability in seeing algebra as a generalization of arithmetic.

Pendidikan Matematika Realistik Indonesia (PMRI)

As briefly mentioned in the beginning of this chapter, this study is aimed to design a series of learning activities in early algebra by using pattern activities. In order to support the students to learn in a meaningful way, our design will be based on the domain specific theory called Pendidikan Matematika Realistik Indonesia (PMRI) which is adopted from Realistic Mathematics Education (RME). Based on PMRI's point of view, mathematics is a human activity and therefore it should be connected to reality, close to people's experiences and has a contribution to the human being and civilization (van den Heuvel-Panhuizen, 2000; Zulkardi, 2002). Therefore in this study, we employed five principles of PMRI, including: (1) the use of familiar context, (2) the use of visual support as the vertical instrument to bridge the "real world" in students' mind with "abstract world" in mathematics, (3) students' own production, (4) interactivity between teacher and students or among the students themselves and (5) intertwinement with other learning strands.

METHOD

Research Method and the Position of the Present Study

In this study we will use arithmetic as a first step in a big leap to formal algebra. Pattern investigation will be used as the main activity of the study, due to its visual support for the students. Furthermore, the focus of the study will be on the development of algebraic thinking, especially in terms of the generalization ability. Reflect to the research question and the aim of this study, design research was deliberately chosen as the approach of the present study. During the study, we employed three steps in design research, which are: preliminary design, teaching experiment and retrospective analysis.

We designed five learning activities based on the tenets of PMRI which will be implemented to the fifth grade elementary school students. The first tenet which is the use of local context is implemented by adopting the story of dance formation which is going to be performed in a cultural event. Next, the visual support is given by the representation of certain arithmetical pattern into geometrical shape. The intertwinement aspect is applied by the collaboration between algebra, arithmetic and geometry topics in the given problems. Last, the socio-norms constructed during the lesson are contribute to establish the students' own production and interactivity principle. In this study, the students are asked to independently solve the problems with their pairs. The role of teacher is as the moderator of discussion who will only help by giving a scaffolding when the discussion gets stuck. The decision in solving the problems will be the responsibility of the students with their pairs.

This article is a part of a big study which aim is to design a learning activities which offer a new approach to how algebra should be learned in the Indonesian classroom. The overall study consist of two cycles of teaching experiment, pilot and the real teaching experiment. This article is merely focus on the result of the pilot experiment which is the first cycle of this study.

Research Subject

As mentioned previously, there are two cycles in the complete study. In each cycle, different group of students are involved. In the second cycle, the whole class of VA MIN2 Palembang and their mathematics teacher are participate in this study. In the first cycle the researcher became the teacher and we worked with a small group consists of four students of VB MIN 2 Palembang, namely Adi, Dela, Rafi and Kayla. Those students were selected based on the result of the pretest given to all students in their class and the discussion with the mathematics teacher. The four students in the first cycle represent different type of students: Dela is low achiever students, Kayla and Rafi are middle achiever, while Adi is high achiever students.

Data Collection and Data Analysis

The data from the pilot experiment phase will be collected through students' written works, field notes, audio and video tapping during pair and group discussion. This triangulation method in data collection is used to ensure the internal validity of the study.

The internal reliability of this study is guaranteed by the use of electronic devices used to record the whole lessons. In analyzing data, we employed a qualitative analysis since the aim of this study which is to get a deep information of how the visual support can help the students developing their sense of structure.

RESULT AND DISCUSSION

There are five activities designed on this study. Each meeting concerns in different aspects of pattern. The first meeting is about repeating pattern, the second meeting is about growing pattern with constant difference and the third meeting is about growing pattern with growing difference. In addition, the last two meetings focus on the representation of algebraic language used by the students. In this occasion, the focus is on the second lesson, in which the students start to construct their sense of structure by elaborating the given visual support of a growing pattern with constant difference.

The context used in the second lesson is Palembang Expo, a typical exhibition in Palembang. In line with that, MIN 2 Palembang were asked to participate in preparing a traditional dance which usually performed in the *V* formation. Start with the aforementioned context, the students were given the worksheet in which they can find the illustration of dancers in *V* formation (see Figure 1). Each dancer is represented by a dot.

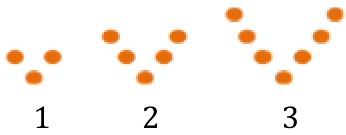


Figure 1: The *V* Formation

Find the Next

The first part of the task is to deal with the "next numbers". It is started by asking the students to draw the forth formation and find the number of dancers in the sixth formation.

The first pair used a so-called recursive formula by adding two from the previous formation. Their strategy can be observed in the following Figure 2 and Fragment 1.

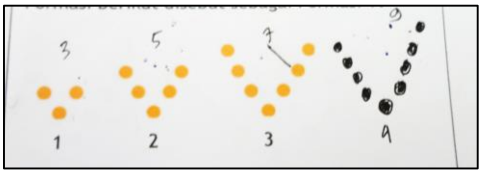


Figure 2: The First Pair's Strategy to Solve the First Task

Fragment 1: Adding Two

1 Researcher : What are you doing here?
2 Did you add by two?
3 Do you agree, Dela?
4 (*Dela nodded*)
5 Researcher : Doesn't it by one?
6 Kayla : No.

7	Researcher	:	Why?
8	Kayla	:	If you add by one you get even number.

The fragment of Kayla's and Dela's discussion showed that they started to think how to preserve the general structure of *V* formation. Hence, they added two dancers on top of the previous formation to get the current formation. They use this strategy to find the number of dancers in the sixth formation as well.

The second pair did it differently. They continued to draw until the fifth formation before they construct the general formula. They separated the part of the formation which "has pair" and the "single" one, as is can be seen in the Figure 3.

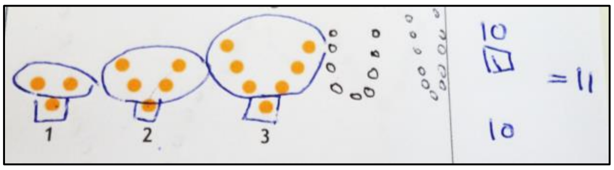


Figure 3: The Second Pair's Strategy to the First Task

Based on the Figure 3 we can observe that the students move from drawing to symbol representation by using numbers. They see the relation between the number of dancers which has pairs and the number of formation. Hence they conclude that in the fifth formation they will have ten dancers in the "paired" section and one dancer in the middle.

Observe the Pairs

The second task is to bridge the students to see the relation between the number of formation, the number of dancers which have the pairs and the total number of dancers. The questions are about to draw the *V* formation which has 17 dancers and to find how many pairs will be in the 45th formation.

To solve the third question, both pairs started to draw with the one dot which represent the dancer in the middle and continue to add two until they got 17 dancers.

The second pair got conflict when solving the fourth question. Rafi argue that there will be 22 pairs, because if there are 45 dancers and one in the middle the 44 dancers will be in 22 pairs. He got confused with the "number of pairs", "number of dancers" and "number of formation". Adi said that in the first formation there is one pair, in the second formation there are two pairs, etcetera. Hence in the 45th formation there will be 45 pairs. After a while, the researcher jump into discussion and brought a new conflict as can be seen in Fragment 2.

Fragment 2: Pairs in the Formation I

1	Researcher	:	How many pairs will be in the 45 formation?
2	Rafi	:	22 pairs
3	Researcher	:	How many pairs will be in the 22 formation?
4	Rafi	:	11 pairs
5	Researcher	:	How many pairs will be in the 11 formation?

6 Rafi : 5 pairs
7 Researcher : How many pairs will be in the 10 formation?
8 Rafi : 5 pairs
9 Eh, how come?

After that, Adi re-explained his idea about the number of pairs in each formation. Now, he added his explanation by pointing to the figure as is showed in the Fragment 3.

Fragment 3: Pairs in the Formation II

1 Adi : 1 pair (*pointed to the figure of the first formation*),
2 pairs (*pointed to the figure of the second formation*),
3 pairs (*pointed to the figure of the third formation*).
2 Rafi : Oh, yes! Right!

Looking for Generalization

The fifth problem is to explain the strategy find the number of dancers in the 100th formation. The first pair got lost here. They combined adding and doubling method, using the fifth formation as the reference to find the other number of dancers in other formation. Hence, they wrote that in the 10th formation there will be 22 dancers, the 15th formation will have 33 dancers and so on (see Figure 4).

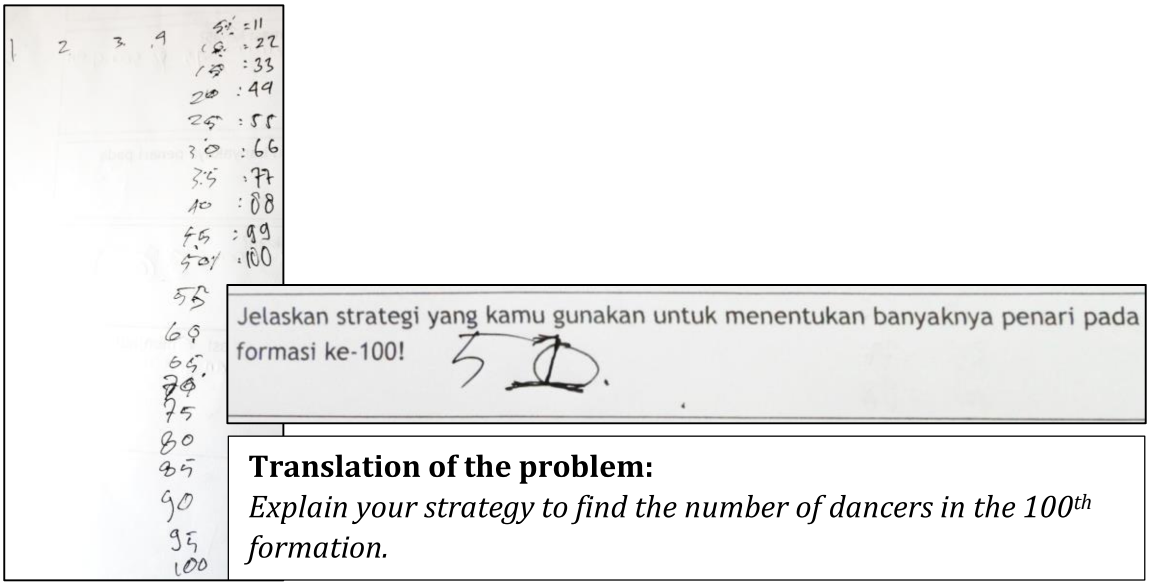


Figure 4: The First Pair's Strategy to Solve the Fifth Problem

When it come into the 50th formation, they wrote 100. All of sudden they got confused which was the row for the number of dancers and which for the number of formation. They mixed everything up and they concluded that in the 100th formation there will be 50 dancers (see the conclusion made in the right side of Figure 4). But then they remembered that there will be one person in the middle, and then they changed 50 into 51.

The second pair used the same strategy they used to solve the first problem. They found that there will be 200 dancers in the "paired" section and one in the middle. Their answer can be observed in the following Figure 5.

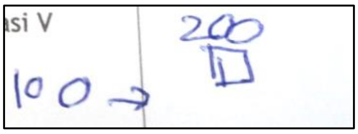


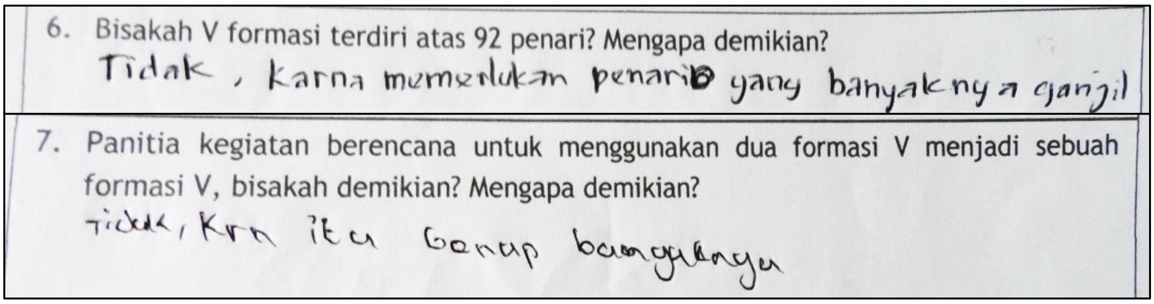
Figure 5: The Second Pair's Strategy to Solve the Fifth Problem

After both of pairs were done to discuss this problem, we conducted a group discussion. During group discussion, the students discussed the method they used to solve the problems. The second group explain their way of seeing the structure of *V* formation which help them to find the number of dancers in any formation, without the need of exhausted listing.

Could It Be a *V* Formation?

After the discussion, the students continued the task. They were asked to determine whether 92 dancers can perform a *V* formation. After that, they should explain whether the combination of any two *V* formations can make a *V* formation.

The second pair who have an insight about the structure of *V* formation from the very beginning, stated that the answer is "no" for both problems. They conclude that any *V* formation will need an odd numbers of dancers. Their answer can be observed in the Figure 6.


6. Bisakah V formasi terdiri atas 92 penari? Mengapa demikian?
Tidak , karna memerlukan penari yang banyaknya ganjil

7. Panitia kegiatan berencana untuk menggunakan dua formasi V menjadi sebuah formasi V, bisakah demikian? Mengapa demikian?
Tidak, Krn itu Genap bangunannya

Translation of students' answer:
6. No, because you need odd number of dancers.
7. No, because it will produce even numbers.

Figure 6: The Second Pair's Strategy to Solve the Last Tasks

The first pair who had a new understanding of how the structure of *V* formation use a picture to explain. Now, they concerned with the shape of *V* formation before they jump to number. They answered by pointed out the important of one person in the middle to perform a *V* formation. Furthermore they argue that with 92 dancers or if two *V* formations are combined, there will be no one dancer in the middle. Hence, it will not construct a *V* formation.

Fragment 4: Could it be a V?

1	Kayla	:	No, you can't. Because no one becomes alone.
2	Researcher	:	What do you mean by that?
3	Kayla	:	This all have their pairs, no one stands alone behind.
4	Researcher	:	Oh, you mean no one has no pair?
5	Kayla	:	Yes, it is *V* formation.

Based on the Fragment 4, we can see how Kayla considered the structure of *V* formation. Previously she and Dela always busy only with numbers (see Figure 4), but after got a new insight of how they can use the help of picture to work with mathematical world, she started to think about generalization: why she always need the odd numbers.

In the discussion, both pairs use the picture to explain their arguments. As observe from their answers, they try to relate the odd-even number properties with the shape of *V* formation. They conclude that V formation will never have even number as its number of dancers, because to create a V shape, there always one person in the end of the formation.

CONCLUSION

Based on the reflection toward the findings of this lesson, we conclude that visual representation can be a helpful model to be used by the students to develop their structure sense. Specifically, in this research we found that the students use the visual support as a model to read the pattern, to work with pattern and to restructure the structure of pattern based on how they see it. Different point of view may lead to different structure, as can be observed in Figure 2 and 3, how the focus students come up with different idea of how they should add the number of dancers in each formation. We also found that the students tend to use recursive formula when they are asked to find “the next” numbers. The strategy will develop a general formula when they are asked to find the “far away” next numbers.

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