INVENTED STRATEGIES FOR PARTITIVE DIVISION ON FRACTIONS USING DURATION CONTEXT

Sitti Busyrah Muchsin[1], Yusuf Hartono[2], Ratu Ilma Indra Putri[3]
Sriwijaya University, Indonesia[1,2,3]
sittibusyrah@yahoo.co.id[1], yusufhartono@yahoo.com[2], ratuilma@yahoo.com[3]

Abstract

This study aims to produce a learning trajectory in the division of fraction. The Indonesia Realistic Mathematics Education (PMRI) that affiliated with thematic integrative learning curriculum 2013 was selected as the research approach. Design research that consists of three stages; preliminary design, teaching experiments, and retrospective analysis was chosen to achieve the research objectives by designed Hypothetical Learning Trajectory (HLT). The HLT was tested on 34 students of class V SDN 179 Palembang. The results showed that the given designed activities can stimulate informal knowledge and it provided the information about students' prerequisite knowledge as a bridge in understanding division of fractions. Completing symmetrical patterns from the given partial part of the patterns can stimulate students to understand the concept of reciprocal or multiplicative inverse. Moreover, by partitioning the area and the set of objects, students' models and representations on the partitive problems are observable. Furthermore, students' strategies have developed gradually to a more formal mathematics. In which the students used the area models as a model of partition situation. The more formal reasoning such as ratio, repeated addition, and multiplication are the model for, that bring the students to the rules of division of fractions what so called invert and multiply algorithm.

Keywords: PMRI, division of fractions, partitive problems

INTRODUCTION

Division of fractions is an important concept mastered by students as a basis for advanced algebraic topic. Nevertheless, the reality shows that many fifth grade students in Indonesia have difficulty for understanding the division of fractions (Epon, 2012). Some difficulties caused division is the most complex of the mathematical operations and fractions are the most complicated numbers to deal with in arithmetic (Ma, 2010). In addition, lack of understanding and meaningfulness of the division of fractions caused the presentation material separated with the context of student life. Learning tends to a mechanistic which students were directly given formula and also determined for memorizing and operating on the routine problem. It is sometimes giving rise to an error when the children did not seriously remember the step of algorithm procedures (Tirosh, 2000; Freiman&Volkov, 2004; Roux, 2004; Epon et al, 2012).

In Indonesia, algorithm of the division of fractions often used is "invert and multiply" algorithm, which inverts divisor fraction then multiply to dividend fraction. However, students do not know and understand the meaning behind the procedures and algorithms. Van de Walle (2010) also expressed that the division of fractions by fractions through the "invert and multiply" algorithm is a mysterious thing for elementary students. In addition to that the results of the study also mentioned even that the teachers did not understand or were confused when encountered the same situation, in the other words, they did not understand the fractions algorithm (Ma, 1999; Philip, 2000; Coslett, 2009; Chen, 2010).

According to Streefland (1991) realistic context was to be the source of concept formation. In line with this, Zulkardi and Putri (2006) stated that context was the first step in learning mathematics. Many studies reveal that the context or the problems relates to share equally in partitive division model which helps students to understand the concept of division fraction (Streefland, 1991; Empson, 2010; Fosnot&Dolk, 2002). Therefore, in helping students to understand the concept of division of fraction and make sense of the algorithm, a learning sequences is designed and developed properly based on the principle of PendidikanMAtematikaMatematika Indonesia (PMRI). The activities were at first embedded in situation involved daily life, which is to solve partitive problem on duration context. In this paper, we present the strategies of students to interpret and model the partitive problem. Those strategies support the understanding of the concept of division of fractions. The aim of the present study is to know how far the students interpret and model the partitiveproblem to have own strategies. Within a design research, weformulated the research question for this study as "How do the students using their own strategies to solve partitive division problem on fractions in duration context?"

THEORETICAL BACKGROUND

Partitive Interpretation of Division

As well as the integer division, division of fractions is also divided into two types of division, namely measurement division and partitive division (Zaleta, 2006; Van de Walle 2010; Gregg & Gregg 2007; Tirosh 2000). In the measurement division, the total and the size of each group are well known that they lead us to find the number of groups. So nicely that the question for this division is "how many__ are in __". While in the partitive division, the total and the number of groups are known then we must find the size of each group or we ask "how much for one".
However, Ma (2010) described that:

Partitive model of division by integers is revised when fractions are introduced... With a whole number divisor, the condition is that "several times the unit is known", but with a fractional divisor the condition is that "a fraction of the unit is known". Therefore, conceptually, these two approaches are identical. (pp 64 – 65)

Thus, we can conclude that partitive model is to find the number that represents a unit when either a certain amount or fractional part of the unit is known. Connect to algorithm that used in division of fractions operations, partitive division model is related to the "invert and multiply" algorithm (Gregg & Gregg, 2010; Van de Walle, 2010). In line with this, Fosnot and Dolk (2002) stated that with partitive problems, the algorithm of "invert and multiply" as usually a strategy is very apparent.

Pendidikan Matematika Realistik Indonesia (PMRI)

PendidikanMatematikaRealistik Indonesia (PMRI) is adopted approach of Realistic Mathematics Education (RME). PMRI emphasizes the significance of the concept meaningfully rather than memorization procedure or algorithm. By PMRI approach, mathematics concepts are presented through the context or "real" situation. It is as a bridge to connect students from real level to formal mathematics (Zulkardi, 2002). Gravemeijer (1994) described that there are three principles in RME, namely:

a. Guided reinventionand progressive mathematizing

Informally through the strategies in solving problems, students will reinvent the properties or the mathematical concepts that already exist and then will be brought to mathematics formal.

b. Didactical phenomenology

Didactical phenomenology is students learn the concepts, principles, or other material related to the mathematical based on the contextual issues that have a variety of possible solutions, or at least the problems that can be imagined by students as a real problem.

c. Self-developed models

This activity serves as a bridge of the students' knowledge from real situation to abstract, or from informal to formal mathematics. Students create or use a model to solve problems with a process of generalization and formalization.

Emergent Modeling

According to Gravemeijer (1994), there are four levels in RME, i.e. situational level, referential level, general level, and formal level. Those levels are steps in modeling the given problem and encourage students to have own strategies to solve the problem.

The level of the situation; where domain-specific, situational knowledge and strategies are used within the context of the situation (Gravemeijer, 1994). The context in this study is partitive problems in duration context, which student were asked to find the time using to paste paper on the wall. Then, in referential level, models and strategies refer to the situation, which is sketched in the problem. In represent the fraction, students tend to manipulate the geometrical shape such as square and rectangle (Sharp, 2002; Bulgar, 2009).These are as *models of* situation which students represent fraction of area of wall in geometrical shape.

Furthermore, general level; where a mathematical focuses on strategies dominates the references to the context (Gravemeijer, 1994). In general level, students have own strategies based on other mathematical concept to solve the given problem. Those strategies bridge them to find invert and multiply algorithm which used as formal level in division of fractions.

METHOD

Participants

This study was implemented to thirty-four fifth grade students SDN 179 Palembang, project school that have been involved in Realistic Mathematics Education Indonesia since 2010, through two cycles: pilot experiment in a small group (6 students) and teaching experiment in the classroom (28 students).

Research Design

This present study is a part of a research project on designing the instructional the concept of division of fractions through partitive model (see in Muchsin*et al.,* 2013). Researcher used design research, which consist of three parts; preliminary design, teaching experiment, and retrospective analysis. In preliminary design, there were three

instruction activities developed in Hypothetical Learning Trajectory (HLT). It would be implemented in the classroom. Then, data collected to see how the design affects the efficiency in learning. However, the present study only focuses on the last one of the three instructional activities that conducted in a second cycle of an explanatory teaching experiment, namely, paste the wallpaper. In this activity students were encouraged tosolve problem of partitive division of fractionsgradually, from division of fractions by unit fractions to the division of non-unit fractions by non-unit fractions. Given problem werepartitive division problems with context time duration. Therefore, from a series of division of fractions that have been mentioned, which is used in the context of each problem will be same, namely to calculate how much time that takes for each room to paste the wallpaper.
Problems that were given as follows:

1) A handyman will paste wallpaper in an art gallery. He takes$\frac{1}{4}$hour to paste wallpaper on $\frac{1}{2}$ of the wall room. How much time does it take to put up the wallpaper such that it will cover the one room?(Problem 1)
2) A handyman will paste wallpaper in an art gallery. He takes$\frac{3}{4}$hour to paste wallpaper on $\frac{1}{3}$ of the wall room. How much time does it take to put up the wallpaper such that it will cover the one room?(Problem 2)
3) A handyman will paste wallpaper in an art gallery. He takes$\frac{3}{4}$hour to paste wallpaper on $\frac{2}{5}$ of the wall room. How much time does it take to put up the wallpaper such that it will cover the one room?(Problem 3)

DATA COLLECTION AND DATA ANALYSIS

The learning activities were videotaped by two video recorders, one video recorder capture the whole classroom activity, and the other focus on target group. The video is segmented into clips based on sequences of observed interactions, negotiations and activities that appeared relevant to each didactical episode in the activity (Van Nes& Van Eerde, 2010; Andrews, 2004; Powell, Francisco, & Maher, 2003). During the learning activity, we also made some notes based on some important moments. All students' works were cross-interpreted to avoid subjectivity in interpretation in retrospective analysis stage. Together with the teacher, we discussed why students' strategiesare in such way. To gain more insight on students' modeling and interpretation, researcher conducted unstructured interview with some students. The interview was also aimed to clarify students' thinking and interpretation.

RESULT AND DISCUSSION

Teaching Experiment

The results in teaching experiment showed that students solve partitive problem mentioned above with a variety of strategies including:

a) Summing Then Multiplying Strategy

In solving the problem 1 (division fraction by fraction), firstly students model the fraction of wall ($\frac{1}{2}$) by sketch rectangular shape. Then, they fillthe time in part of wall which was known. Next filled the time for remaining wall based on a part of wall that has known before. Then add up the total time in a wall of the intact.Students completed the first problem by way ofsumming up all the time on the parts of the wallto cover an entire wall. To simplify, students see that repeated addition is equal to multiplication. Thus, they multiply the time $\frac{1}{4}$ hour to 2 because there are two parts $\frac{1}{2}$ in whole wall.

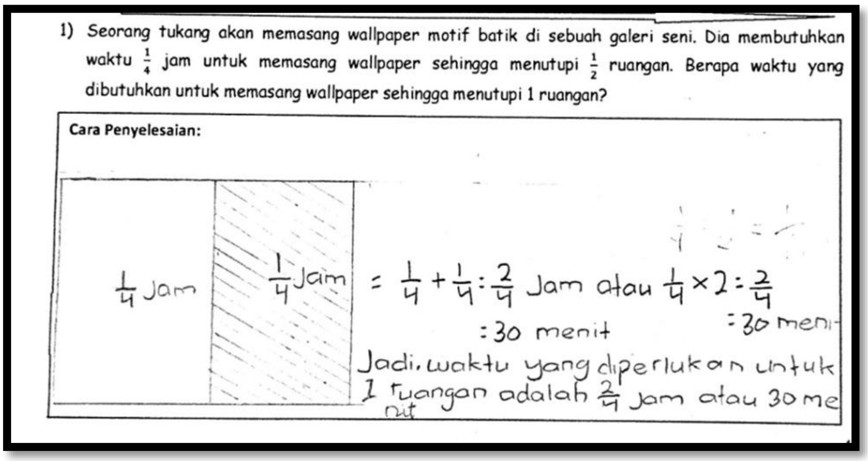


Figure 1. Summing Then MultiplyingStrategy

b) Converting Hour to Minutes Strategy

Because the context of a given problem is time duration, then there are several groups converted time $\frac{1}{2}$ hour into 30 minutes. After that, they multiplied it by the number of wallsections in their modeling.

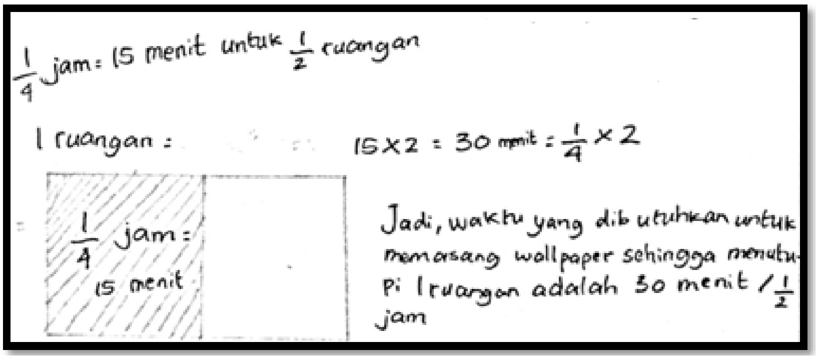


Figure 2. Converting Hour to Minutes Strategy

Researcher tried to gain students' opinion behind their strategies. Dialogue between researcher and students is presented below:

Researcher	:how much time that used in every $\frac{1}{2}$ of wall or room is?
Aprilia	: 15 minutes
Belda	: $\frac{1}{4}$ hour
Researcher	: If for one room?
Aprilia	: 30 minutes
Researcher	: Where did you get 30 minutes?
Aprilia	: 15 x 2
Researcher	: Why multiplied two?
Aprilia & Belda	: Because there are two parts (pointing the barmodels made)
Researcher	: 30 minutes?
Aprilia	: Yah, half hour

c) Comparing Strategy

There was a group in class completed the worksheet by using the concept of comparison. For example, in problem 2, this group modeled both fraction either divisor or dividend in bar model as the in figure 3. Then students thought that to make fraction $\frac{1}{3}$ to 1, it would be multiplied by 3. Therefore, the time for each $\frac{1}{3}$ part of the wall, $\frac{3}{4}$ hour, is also multiplied by 3. Such that, students earned time for wallpaper installation of the whole wall is $\frac{3}{4} \times 3 = \frac{9}{4}$ hours.

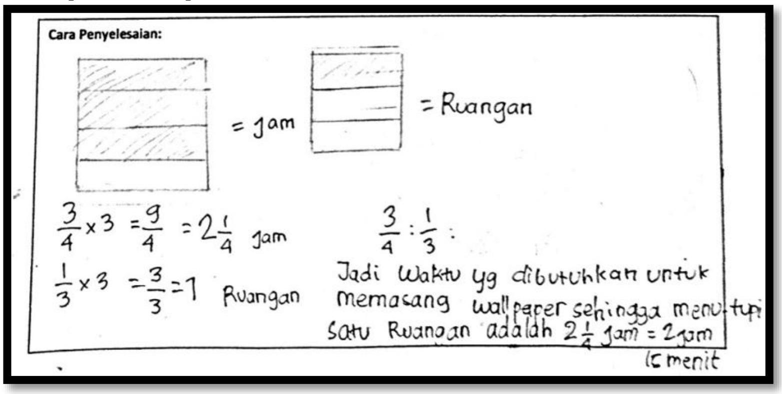


Figure 3. Comparing Strategy

d) Finding for Every Unit Partition Strategy

One of researcher conjectures in HLT came after investigating the discussion in group. Teacher guided the students to find how much time was needed in each section or unit partition. For example, in problem 3, students were directed to find how much time for each $\frac{1}{5}$parts of the wall if it is known that for the $\frac{2}{5}$ of wall, the time required is $\frac{3}{4}$ hour. They were reasoning that $\frac{1}{5}$ is half of $\frac{2}{5}$. In other word $\frac{1}{5}$ is obtained from $\frac{2}{5} \div 2$. Therefore to find the time in each $\frac{1}{5}$ then the time for $\frac{2}{5}$wall divided into two as well, i.e. $\frac{3}{4} \div 2 = \frac{3}{8}$ hours. To answer the given problem, students saw the number of units in all partition walls intact i.e. there are five sections. Therefore, students multiplied the time for each $\frac{1}{5}$ to five times. Students earn $\frac{3}{8} \times 5 = \frac{15}{8}$ hours.

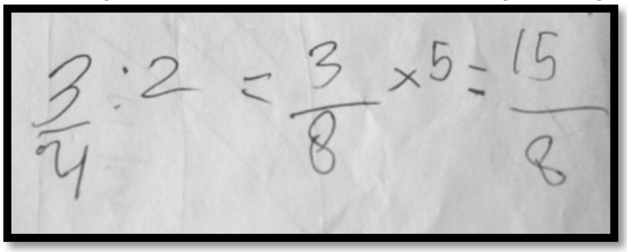
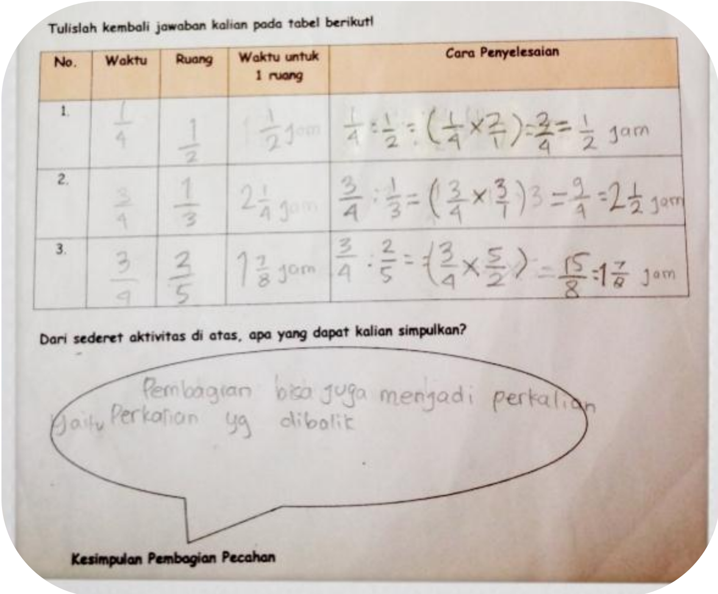


Figure 4. Students' work by Finding for Every Unit Partition Strategy

From the overall strategies, students were geared towards fill conclusion table in order to view the threepatterns of solving the three given problems. After all the groups filling the table, then a representative from each group whose different answer was asked to fill the table on board. Next,after a class discussion, students were able to conclude that the division of fractions can be solved by multiplication the inverse of the second fraction to first fraction, then it is called "invert and multiply" algorithm.


Tulislah kembali jawaban kalian pada tabel berikut!

No.	Waktu	Ruang	Waktu untuk 1 ruang	Cara Penyelesaian
1.	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ jam	$\frac{1}{4} : \frac{1}{2} = (\frac{1}{4} \times \frac{2}{1}) = \frac{2}{4} = \frac{1}{2}$ jam
2.	$\frac{3}{4}$	$\frac{1}{3}$	$2\frac{1}{4}$ jam	$\frac{3}{4} : \frac{1}{3} = (\frac{3}{4} \times \frac{3}{1}) = \frac{9}{4} = 2\frac{1}{2}$ jam
3.	$\frac{3}{4}$	$\frac{2}{5}$	$1\frac{7}{8}$ jam	$\frac{3}{4} : \frac{2}{5} = (\frac{3}{4} \times \frac{5}{2}) = \frac{15}{8} = 1\frac{7}{8}$ jam

Dari sederet aktivitas di atas, apa yang dapat kalian simpulkan?

Pembagian bisa juga menjadi perkalian
yaitu Perkalian yg dibalik

Kesimpulan Pembagian Pecahan

Figure 5. Conclusion Table

Retrospective Analysis

The activity aims to bridge studentson formal conclusion to the algorithmic "invert and multiply" on the division of fractions. The findings in this study showed that students'strategies which used in solving the given three partitive problem. Those strategies were in accordance with the researchers conjectures.

First, students added time for the known wall part then find time to put wallpaper on the entire wall. For instance in the first problem, students counted $\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$hour.Furthermore, the concept that repeated addition can be written in the multiplication form. It was used to write down the solution back into $\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2}$ hour. Other strategy wasstudents converted hour to minute, so the fraction was changed to integers. As in first problem 1/4 hour was changed to 15 minutes.With the modeling wall partition into two parts$\frac{1}{2}$, students knew that there are two parts $\frac{1}{2}$ in one wall. Such that, students multiplied the time twice, then students get $15 \times 2 = 30$ minutes.

Division of fractions problems on firsttwo problems were still involved unit fractions, then students easily find the time to overall wall. It based on the partition wall in the students' models pictures. However, students begin encountering obstacles when solved the third problem($\frac{3}{4} \div \frac{2}{5}$) as to the context involving non-unit fraction. Students tried to find how many times the part of wall in a whole wall. They found that there are two and half of partition $\frac{2}{5}$ in 1 wall. Next, they changed two-half ($2\frac{1}{2}$) to $\frac{5}{2}$. Therefore, they used the previous strategy that multiply the time $\frac{3}{4}$ hour of known part to $\frac{5}{2}$ or can be written $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$. Such that, students obtained time for a whole wall that is $\frac{15}{8}$ hours.

Another strategy was finding time for each partition unit. It is important to students to know the unit and non-unit fraction. Because students knew the time to paste wallpaper on $\frac{2}{5}$ part of wall is $\frac{3}{4}$hour. Thus, they eager to find time for $\frac{1}{5}$ part of wall. They could find by dividing time for $\frac{2}{5}$ i.e. $\frac{3}{4}$hour by two and got $\frac{3}{8}$ hour for $\frac{1}{5}$ part of wall. In advance, to obtain the time for whole wall that isfive times$\frac{1}{5}$ then multiplied $\frac{3}{8}$ by 5. In other word, students divided numerator of second fraction first, then it multiplied by its denominator.

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \div 2 \times 5 = \frac{3}{8} \times 5 = \frac{15}{8}\text{hours}$$

CONCLUSION

The contribution of students in finding a way or strategy is very important in PMRI approach because this is a process for students to construct their own knowledge to reinvent the mathematical concept by teacher guidance. The overall strategies and resolution on the conclusion table in students' worksheet were delivering to formal conclusion, which is division of fractionscan be completedby way of multiplying the first fraction to the inverse of second fraction.

Thus, it can be concluded that those strategies bridging students in constructing aformal knowledgeof "invert and multiply" algorithm on the division offractions.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Students were also able to find out the meaning behind the procedure. Division of fractions can be solved by reversing or inverting the second fraction and then multiply it to the first fraction. Because the second fraction has to be changed, become 1, such that it multiplied by its opposite or its inverse. It is also based on the concept of partitive division which focus on how much in one, therefore in division of fraction we also make the second fraction become one.

REFERENCES

Andrew, P. (2004). International comparisons of mathematics teaching: Searching for consensus in describing opportunities for learning. In *Paper presented to discussion group 11: International Comparisons in Mathematics Education, the Tenth International Congress on Mathematics Education (ICME-10).* Copenhagen: Danish Technical University.

Bulgar, S. (2009). A Longitudinal Study of Students' Representations for Division of Fractions. *TMME ,6*, 165-200.

Charalambous, Y & Pitta, D. (2007). Children's strategies for division by fractions. *Springer:Educational Study in Mathematics, 64*, 293–316. DOI: 10.1007/s10649-006-9036-2

Coslett, D. (2004). *Closer Look at Fraction Division: How Does Comprehension and Confidence of the Subject Change Preservice Teachers' Perception of Instructional Practice?* http://www.math.vt.edu/people/plinnell/Ugresearch/coslett1.pdf accessed on 10 October 2013

Empson, S. (2010). Equal Sharing and Shared Meaning: The Development of Fraction Concepts in a First-Grade Classroom. *JSTOR, 17,* 283 – 342.Stable URL;

http://links.jstor.org/sici?sici=0737-0008%282010%2917%3A3%3C283%AESASMT%3E2.0.CO%3B2-D

Petit, Laird, & Marsden. (2010). A Focus on Fractions: Bringing Research to the Classroom. New York: Routledge Taylor & Francis.

Fosnot, C.T &Dolk, M. (2002). *Young Mathematicians at Work*: *Constructing Fractions, Decimals, and Percents*. Portsmouth: Heineman

Freiman, V. &Volkov, A. (2004). *Fractions and fractions again? A comparative analysis of the presentation of common fractions in the textbooks belonging to different didactical tradition*. Paper presented at ICME-10, Discussion Group 14, Copenhagfen, Denmark.

Gravemeijer,K. (1994). Developing Realistic Mathematics Education. Utrecht: Technipress, Culemborg.

Gregg, J. & Gregg, D. U. (2007). Measurement and fair-sharing models for dividing fractions. *Mathematics Teaching in the Middle School, 12 (9),* 490–496.

Ma, L. (2010). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States.* New York & London: Routledge Taylor & Francis.

Muchsin, et al. (2014). Designing the Teaching and Learning of Division of Fractions Using Partitive Model in Fifth Grade. Thesis of Sriwijaya University.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *Journal of Mathematical Behavior, 22*, 405–435. http://dx.doi.org/10.1016/j.jmathb.2003.09.002

Sharp, J & Adams, B. (2002). Children's Constructions of Knowledge for Fraction Division After Solving Realistic Problems. The Journal of Educational Research. 95 (6), 333-347 DOI: 10.1080/00220670209596608

Streefland, L. (1991). *Fractions in Realistic Mathematics Education: A Paradigm of Development Research*. Dordrecht: Kluwer Academic Publishers.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: the case of division of fractions.*Journal of Research in Mathematics Education, 31 (1)*, 5 – 25.

Van Nes, F., & Van Eerde, D. (2010). Spatial structuring and the development of number sense: A case study of young children working with blocks. The Journal of Mathematical Behavior, 29(3), 145-159. http://dx.doi.org/10.1016/j.jmathb.2010.08.001

Van de Walle, John A. (2010). *Elementary and Middle School Mathematics: Teaching Developmentally*. Boston: Allyn & Bacon.

Zaleta, C. (2006). *Invented strategies for division of fractions*. Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Mexico: Universidad Pedagogica National.

Zulkardi. (2002). *Developing a Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers*. Thesis University of Twente. The Netherlands: PrinPartnersIpskamp-Enschede

Zulkardi&Putri, R. (2006). MendesainSendiriSoalKontekstualMatematika. *Prosiding KNM 13*. Semarang: UniversitasNegeri Semarang.