SUPPORTING THE DEVELOPMENT OF STUDENTS' REFERENCE POINTS FOR LENGTH ESTIMATION

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Abstract

This study reports a part of a full study aimed to improve local instructions theory on mathematics education for supporting to develop students' reference points for length estimation. We design 5 classroom activities using Realistic Mathematics Education (RME) approach with Indonesian contexts through approximating and estimating tasks. Measuring relatively long object in lompat katak and lompat pocong game and approximating from Indonesian traditional games photographs build understanding of physical unit iteration and naturally shift from the use of rulers to use reference points.. Meanwhile, estimating the height of a classroom, and the height of a flag pole push the use of mental reference points and connecting body parts to external object as new reference points. The participants of this study are four 4th graders of MIN 2 Palembang. We collect data from students' work, classroom observation and interviews with the participants then we analyze the data mostly in qualitative ways. The results suggest that the students use guessing, employ reference points in form of body parts (arm spans, body height, etc) or external objects (height of a bookshelf, height of a room, etc.) and also mentally iterate IFR especially one meter unit by imagine the length of an arm spans to the to-be-estimated objects.

Keywords: *Reference points*, *Estimation*, *Approximation*, *PMRI*, *RME*, *local instructions theory*, *design-based research*, *hypothetical learning trajectory*

INTRODUCTION

Measurement estimation is a very important skill in life. Jones and Taylor (2009) stated that individual in a variety of professions argue that estimating skills are essential for their careers. For instance, an architect estimates the space needed for a body to pass a doorway, a butcher estimates the dimension of meat being cut off to meet certain weight, and a park ranger estimates the distances between landmarks. The action in the activities come naturally without the use of standard measurement tools.

However, many studies reported low performances on length estimation tasks (Hildreth, 1983; Joram, Subrahmanyam, & Gelman, 1998). For instance in Hildreth (1983), he interviewed 24 students to solve 24 estimation tasks (length and area estimation), 40% of the students were categorized as using inappropriate strategies including wild guessing. Moreover, only a few studies that focus on developing students' skill in length estimation tasks (G. Jones, Taylor, & Broadwell, 2009). Yet little is known how it can be embedded into instructional activities (Joram et al., 1998). Furthermore, Markovitz and Hershkowitz (1997) followed by Hogan and Brezinski (2003) stated that a different approach is needed to teach and support students developing their skill in length

estimation such as enriching visual or spatial experiences and developing personal point of references. Hence, teaching and learning about length estimation need more attention.

Including in Indonesia, length estimation is not explicitly taught in the Indonesian curriculum (BSNP, 2006). The curriculum tends to focus on how to convert and use standard units of length (metric units) and the standard measurement tools. Less attention is given to an activity of making sense of units for estimating. It seems that there is a need to improve this educational gap by designing a series of mathematics lessons/activities that can support students in developing their skills in estimating.

The goals of the present study are to investigate what strategy used by students use in estimating length and students' sense of length (point of references). In general, we want to contribute to local instructions theory in mathematics education about how to support students' skills in length estimating especially in the measurement strand. Therefore our intentions can be summarized as a general research question: *How can we support the development of students' reference points for length estimation?*

THEORETICAL FRAMEWORK

Measurement Estimation

There are three type of quantitative estimations: computational estimation, numerosity estimation, and measurement estimation. The first, computational estimation is related how ones can compute flexible and creatively (number sense) to find certain calculation of number. The second, numerosity estimation is how ones can estimate number of discrete objects. The last, measurement estimation (length) is related how ones can make an educated guessing about how long a continuous object without the use of measurement tools (Bright, 1979; Smart, 1982).

Measurement estimation does not relate to the development of general mathematical ability as the computational estimation. Instead, it should be addressed as part of spatial ability (Hogan & Brezinski, 2003). The learning instructions should enrich students' visual experience through interaction with physical objects (Markovits & Hershkowitz, 1997; Smart, 1982). In other words, students should first experience a process of rough physical measuring (approximating) before mentally measuring the to-be-estimated objects (estimating). It is expected that the use of concrete objects can enhance students' feel of length internalized into known objects called reference points.

Point of References

When students are given length estimation tasks, various strategies will be employed depending on their proficiency in estimating (Hildreth, 1983). A less skilled estimator tends to wild guess, mention inappropriate units or numbers without a reasonable explanation. Meanwhile, a skilled estimator employs educated strategies such as the use of prior knowledge (recall identic objects), mental meter, and or reference points.

Following is a summary of the development of students' strategies in approximating and estimating.

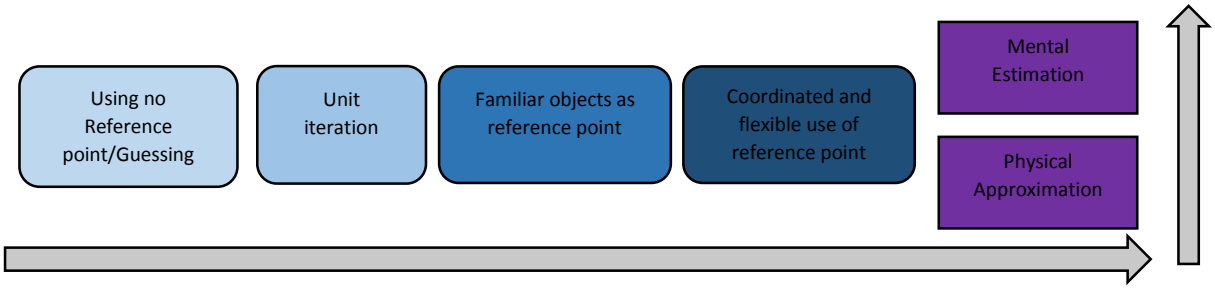


Figure 7: Two dimensional development of reference points and estimation skills

Personal point of references are object whose lengths are known physically or mentally (in mind) to be used for approximating or estimating. Reference points help estimator to approximate or estimate by imagining a specific object and compare it to the to-be-estimated (Joram, et al., 2005). This imaginable objects develop through everyday experience but the development can be enhanced through appropriate approximating and estimating tasks. For instance, one might use their body height to estimate the height of a classroom or even imagine a 20-feet-long crocodile to estimate the length of a classroom as found in Hildreth's (1983) study.

Furthermore, developing students' point of reference should be the primary goal of length estimation instructions, it serve as a critical point to develop students' sense and understanding of measurement (Bright, 1976; Clements, 1999; Lang, 2001). However, students often do not spontaneously employ reference points to approximate or estimate length caused by complex interaction among students' preferences, context of the tasks and the nature of estimation activity (Gooya, Khosroshashi, & Teppo, 2011; Hildreth, 1983; Joram, Gabriele, Bertheau, Gelman, & Subrahmanyam, 2005). Therefore a learning instruction in length estimation should carefully be designed by considering the aforementioned aspects. In this study, we employ Realistic Mathematics Education approach to design the activities.

Realistic Mathematics Education (RME)

RME promotes mathematizing, a process in which students be able to mathematize everyday problem situation in mathematical terms and employ it within mathematics itself (Freudenthal, 1991; K. Gravemeijer, 1994; K. P. E. Gravemeijer, 1994). RME provides a framework for designing an instructional task for the progressive mathematization (Treffers, 1987) determined by its five tenets. In Indonesia RME is adapted into PMRI (*Pendidikan Matematika Realistik Indonesia)* (Sembiring, Hadi, & Dolk, 2008; Zulkardi, 2002) which focus on supporting students with Indonesian local contexts for learning mathematics (Zulkardi & Ilma, 2006). We briefly describe how the five tenets employed to the topic of our study.

Approximating/estimating are *context*-bounded tasks, the instructions should be started and ended in a meaningful real world situations. Promoting *the use of models* such as drawing, sketch, as proposed in RME, may enhance students understanding how unit is iterated and used in efficient ways. Moreover, process of developing personal reference points is a *students' own construction* process, it cannot be forced to the students because it relates to a mental perception depending what they perceive and experience. Hence, an *interactive instruction* should be formulated in order to trigger discussion about students' personal reference points. It is important because from listening and negotiating other perceptions and perceptive of length, one may build a good personal reference points.

In addition, it also important to note that in order to conduct a powerful instruction as proposed in RME tenets, change in class socio norms and socio mathematical norms should be promoted (Yackel & Cobb, 1996). The students should be realized that they cannot always go to the teacher for clarification of their answers. Moreover, the teacher should be able to establish agreement what kind of answers/strategies are considered as good guesses or efficient and how to judge which estimation is better in vague and inexactness of estimating tasks (Forrester & Pike, 1998). Therefore, the role of the teacher is changed from the information transmitter and the one who always give judgement to a facilitator and guide of students' own discovery.

METHOD

Research Approach

Considering our research question, it seems to be logic that we employ an approach that directly speaks how to design a learning trajectory such as classroom activities and the teaching and learning materials. Hence, we employ design-based research as research approach of this study.

DBR is characterized by its cyclical or iterative process of designing-revising the educational materials specifically the learning trajectory (Bakker & van Eerde, 2013; Barab & Squire, 2004; Edelson, 2002). The learning trajectory is designed and tested in 3 phases (preparation and design, teaching experiment, and retrospective analysis) and be revised in several cycle (one cycle consists of the three phases) (Bakker & van Eerde, 2013; Gravemeijer, 2004). Further, DBR does not only also speaks how to design, but also how to describe the students' learning development which is used to advice for better teaching and learning action. Therefore, we will design a learning trajectory and its hypothetical learning trajectory (HLT) describing a learning instruction and the possibilities of students' thinking to support students' development of reference points for estimating. In addition, we will also explain how the students' learnings take place and give practical advice about it.

Subject, Data Collection and Data Analysis

This study is the first cycle of two cycle DBR that we conducted. The subject of this pilot experiment is 4 fourth graders (two boys and two girls) of an elementary school in Palembang, Indonesia.

Data were collected from the preparation phases and the teaching experiment phases of design-based research and a posttest afterward. The data were collected by semi-structured interview (teacher and students), classroom observation, and students' written tests.

Data gained were analyzed using triangulation system, we analyzed students' written work, registered video of the learning and field note of the observer. We compare interesting fragments of students' written works or registered video in which the learning takes places or not compare to the HLT. The analysis was mostly done in a qualitative ways and in modest quantitative ways.

RESULT AND DISCUSSION

The learning activities that we design consist of 5 lessons. Before implementing the design, we conduct a pretest and interview to the students then a posttest afterward. It is important to note that both pretest and postest are not aimed to measure how far the design work. Instead, we analyse each lesson to know how the design and how the students' learning occurs. Following is the overview of the 5 activities.

Table 1
Overview of Activities and Main goals on supporting the development of students' reference points for length estimation

Activity	Main Goal
Frog Jumping and *Pocong* Jumping	Shift students' from using rulers to use other reference points such as body part for approximating.
Measure and Use Your Body Parts	Knowing accurate lengths of body parts to be used as reference points.
Length, Width and Height of the Building	Develop external reference points and ordering their efficiency. Visualizing iteration of reference points.
17th of August Decoration: Balloons and the Flag Pole	Internalization of standard units of measurement. Shift students to do mental iteration (estimation) using reference points.
17th of August Decoration: Plastic Flag and the Rope	To spot, use and reason using reference points for solving length estimation problems involving social arithmetic.

We may see from table 1, the lessons are sequenced to support students' development of using references point in a physical ways (approximation) to the mental one (estimation). We will analyse and discuss each of the lessons on the next paragraphs.

First of all, the prestest, it is aimed to get information about students' prior knowledge about the topic of references points for approximation/estimation. The result of the prestest and the interview afterward indicated that students have difficulties to use a proper unit of length. In addition, they also improperly represented a magnitude of objects. For instance, if they think the object is very long they will write a relatively very big number. One student said that the length of a A4-sized paper is 450 cm, and another student said it is 5 or 7 cm. The students had not yet developed good sense of units marked by either improper use of units and numbers. In addition, most of the students use guessing methods when they were asked how they know their reference points. Hence, in most cases, students did employed reference points but they have no idea about the length of the reference points.

Lesson 1, frog jumping and *pocong* jumping is aimed to accommodate learning through a game, raise awareness to compare lengths, stimulate conflict about used reference points and to shift students from using rulers to more efficient ways. Students in pair, were asked to measure/approximate a relatively long distance through a game called "Frog Jumping and *Pocong* Jumping". The game was played on the school yard, each students in their group alternately jump like a frog 4 times, jump like a *pocong* (Indonesian ghost) 4 times, mark their distance and then approximate whether the distance is 30 meter or not.

We found a confirmation to our HLT that the students did not want to use a ruler since it would be tedious to use. Instead, the students used arm spans as one meter unit. Indeed, there was also a conflicting situation in which the students argue whether their arm span is one meter long or not. Following fragment shows their reasoning.

Fragment 1: Conflict of arm span length

1. Teacher : Do you agree that they got 54 (meter)?"
2. Sugi and Aziz : No!
3. Teacher : Why? Why?
4. Sugi : (pointing to Fitria) her arms are short.

What we can say from fragment 1 is that Sugi refused to believe that the girl group got 54 meter because the Fitria's arms are too short to represent one meter.

Overall, lesson one provides a sufficient situation to raise students awareness to employ references points. What is need to be improve is that the game should facilitate the use of several reference points (not only arm spans) to emerge a rich discussion.

Lesson 2, measure and use your body parts, aimed to provide students to physically approximate object in photographs using reference points on the photographs. The students were given a set of familiar photographs about Indonesian games. The photograph were selected in such a way we embeded clues of body parts that possibly could be used for the students to approximate. They were asked to find a real length or distance of an object in the photograph. For instance, they are asked to approximate the length of a *bakiak* (long-wooden sandals) given clues such as feet, tiles, etc.

Figure 2: *Bakiak* Problem

We found that most of the students could figure out the lengths of the objects employing reference points such as hand spans, body height and arm spans as predicted on the HLT. For instance, the students first measure their hand spans using a ruler as a reference points to be used on the photograph. It is however, the students did not spontaneously perceive feet as a reference points for approximating especially for the *bakiak* problem. The students tend to guess the length of the *bakiak* by imagine their own version of one meter (mental) in that photograph. It could be probably because the students are not familiar to use feet for measuring something. Overall, lesson 2 could trigger students' sensitivity to make body parts as reference points. A point to improve is that the photograph of the *bakiak* should be revised in order to stimulate students' awareness of the use of feet as reference points.

In lesson 3, the students were asked to approximate two photographs (a greasy pole and a building). Still, we found that the students used guessing to approximate at the very beginning. However it turned to be out that for the first problem, the students employed their thumb as a reference for one meter on that photograph. It is probably that the students first assume the one meter, then assign it to the thumb. Moreover, one student named Aziz use a person on the photograph as clue for imagining a person's arm span. Hence, it seems students tend to imagine or iterate standard unit of one meter to approximate the problem rather than using a ready-made point of reference such as body height or the height of the house.

For the second problem of lesson 3, we found that the students got difficulties to cope with the context of the problem. The students were not familiar with the building and got confused about 2 dimensional representation of a 3D object. It was not surprising also that when they were asked to find the most efficient point of references, they only focus on finding the longest objects as point of references such as a road lamp and a bus even though they are not familiar with the length rather than using objects which they know the lengths. Based on this finding, we intend to revise the problem of lesson 3 as such we will involve more students' investigation on connecting body parts to external objects to develop students' reference point for approximating or estimating.

In lesson 4, students were engaged in two activities. The first, students were asked to make a one-meter-long rope in the context of eating *kroepoek* contest. The aim is to

promote internalization of one meter unit as reference point which is important for mental estimation. The second activity is the students were asked to estimate the height of their classroom and the height of the school flag pole. It is aimed so that the students shift to begin imagine mentally their reference points instead of iterate them physically.

We found that in the first activity, the students again tended to use arm span to make the one meter long rope. It is however, make them tediously shift to employ other reference points. For instance, in activity two, as predicted in HLT, they try to approximate the height of the school flag pole and the height of the classroom using arm span as shown in the following photograph.

Figure 3: Aziz approximates the height of the wall using arm spans

From figure 3, we may see that, Aziz used his one and a half arm spa to approximate the height of the classroom. Realizing that it was difficult, the students discussed and found out to use other references as shown in the following fragment.

Fragment 2: Discussion on the height of a bookshelf

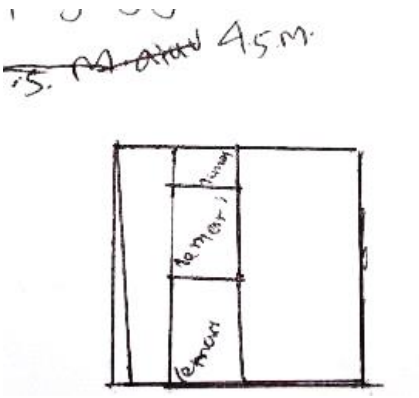
1.	Fitria	:	Ouu that is three meters high (pointing at a bookshelf).
2.	Teacher	:	Which one? The bookshelf? How high is the bookshelf?
3.	Sugi	:	Two, two, one meter and a half!
4.	Fitria	:	Two meters.
5.	Teacher	:	How do you know this is 2 meter high?
6.	Fitria	:	Because…..
7.	Aziz	:	It can't be 2 meter high! You know that my body height is 151, don't you?

From fragment 2, we may see that Aziz connected his body height to make sense the height of the bookshelf to be used to estimate the height of the classroom. The next fragment shows how they use the bookshelf for estimating.

Fragment 3: Discussion on the height of the classroom

1.	Teacher	:	Could you explain how to find out the height of this room?
2.	Sugi	:	Errrr, the bookshelf is two meter high. This one should be added by two meter and a half. Therefore, it becomes three meters and a half.

We may see from the fragment 3 above that Sugi imagined the left over space between the top of the bookshelf and the ceiling is one and a half meter. It turned that FItria disagreed with Sugi's opinion. Fitria and Dytha as a group purposed the following answer:



Lemari: Bookshelf

Figure 4. Dytha and Fitria's Strategy

From figure 4, we see that Fitria and Dytha employed a mental estimation using the bookshelf as reference point. They imagined that there will be one and a part of bookshelf again that could be stacked to the ceiling. By this strategy they found out that the height of the classroom is 2 meter + 2 meter + 0.5 meter which is 4.5 meter. It shows that the students shift to use external object as reference points and shift from physical approximation to mental estimation. Nevertheless, there are several remarks for improvement for lesson 4. Activity 1 of lesson 4 would be deleted since it makes students too rely on arm spans and make it difficult for students to use other reference points.

In lesson 5, students were engaged in a real situation in which they observed their school to determine how many plastic flags and how long the rope should be hanged on outer ceiling for 17 august decoration. The aim of the activity is to give students chance to spot and employ reference points at the school. We found that the students again use arm spans as reference point for one meter. They iterated their arm spans tediously, sometimes they got lost to cover the desired part of the school. As we predicted in the HLT, one of the group (the boys group) creatively used distance between two pillars which they approximate using arm spans first and then count the number of the pillars to figure out the number of the plastic flag and the length of the rope needed. In other words, the students employed a new reference for the approximation but they still rely on arm spans. For refinement purpose, we see the need to emerge various strategies or reference points to be able to stimulate a rich discussion, not only the use of arm span.

CONCLUSION

The research question of this study is how can we support the development of students' reference points for length estimation? In this part we will show how our design support the development of the students' from guessing to use reference points and from approximating to estimating. After summary of the result we discuss the limitation of this study and suggestion for further study.

In the teaching experiment, lesson 1 we see that students naturally use their very basic-familiar reference point which is arm span instead of using a ruler. This implies that the condition creates by the frog jumping and *pocong* jumping game could shift student into approximating mode from what they usually do in class, measuring. Meanwhile in lesson 2, the students observed, listed and reasoned using body parts on the photograph. Giving body parts as clues on the photograph could support the students to gain new reference points and for approximating. However in lesson 3, we found that students got a little problem to cope with unfamiliar context of building and 2 dimensional of the 3d objects. We also get insight that the students tend to claim several objects as references point even they do not familiar with the lengths. In this case, we think we need to simplify the problems and focus more on connecting body parts to external objects as new reference points. We see in the lesson 4 that the students try to estimate by making sense of objects with their body height. The students imagine a stack of bookshelf to estimate the height of the classroom. We see students grasp with the idea of mental estimation and shift from physical approximation. In the last lesson, the students use the distance between two pillars of the school as a new reference points which they obtain by using their arm spans. In general, our design activities could answer the research question, students which are used to use ruler shift to approximate using arm spans and then make sense object by their body parts for estimating.

Nevertheless, we realize that our study is only a preliminary study of two cycles of design-based research. We only use 4 students as the subject of this study. Hence this learning situation is far from the real situation of classroom in Indonesia. It is certainly needed to revise and reformulate some points of the design for the next implementation. We point out that for bigger classroom, the designer should facilitate the teacher to sufficiently understand to establish well socio norms and socio-mathematical norms in inexactness of approximation and estimation. Moreover, the designer should also be able to predict and give suggestion for the teacher to manage the dynamic classroom (e.g. field activity).

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