SPATIAL STRUCTURING ABILITY SUPPORTING FIRST GRADE STUDENTS' STRATEGIES IN LEARNING ADDITION OF NUMBER 1 TO 20

Ummy Salmah.[1], Ratu Ilma Indra Putri[2], Somakim[2]
Student of International Master Program on Mathematics Education Sriwijaya University[1]
Faculty of Teacher Training and Education, Sriwijaya University[2]
1)ummysalmah@ymail.com, 2)ratu.ilma@yahoo.com, 3)somakim_math@yahoo.com

Abstract

The aim of this study is to design learning activities that can support students to develop strategies for the addition of number 1 to 20 in first grade by involving students' spatial structuring ability. There are three activities that have been conducted in this study, namely dice games, rubber ball box activity, ten-box activity. In this paper, one of three activities is discussed namely rubber ball box activity. The aim of this activity is to introduce and develop double-structure to be a students' strategy in addition of number 1 to 20. PMRI underlie the context and activity. The method used in this study is design research through three steps namely preliminary design, teaching experiment and retrospective analysis. This study was conducted in SD Negeri 179 Palembang as a partner school of PMRI by involving 27 students of Grade 1. The result of the study indicates that rubber ball box activity can help students to develop double-structure as a strategy in addition of number 1 to 20. Based on the learning activities, students can develop and use spatial structures to develop their strategies, either using double structure as strategy to determine the number of structured balls or unstructured balls. Students can explore their strategies in solving problem of addition of number 1 to 20. As a recommendation, PMRI can be implemented as an approach of teaching and learning addition 1 to 20.

Keywords: Addition, double-structure, spatial structure, PMRI.

INTRODUCTION

Numbers is one of important topics because it will be the basis for the mastery of other mathematical concepts (Freudenthal, 1973; NCTM 2000). In the early stages of learning about addition as shown in elementary school, there are many students who have not been able at doing the sums of numbers. This is because in the early stages of studying the numbers, more students use strategies that involve the calculation of one by one (Van Nes, 2007; Collins, 2009; Son, 2011) and rely on real objects to be counted (Collins, 2009). Counting one by one is a basic strategy of students in solving addition problems (Nursyahidah, 2013). In the early counting phase, students will count the number of things one by one.

In Indonesia, the teachers still give the standard algorithm or procedure without any emphasis on teaching the concept (Armanto, 2002). Therefore, it is necessary to use a strategy that can help students not only to understand but also to be proficient in performing addition in particular addition 1 to 20. In addition of numbers 1 to 20, strategy which student uses can be different to the other students. There are some strategies that usually students use in addition number 1 to 20 (Thompson, 1999). Some of them are (1) counting on from first number, (2) counting on from larger, (3) double (near-double system, (4) using fives, and (5) bridging through ten.

To support the understanding of students about addition of number 1 to 20, researchers are encouraged to develop and to use spatial structuring ability to help students do the addition. The spatial structuring ability is the mental operation of constructing an organization or form for an object or set of objects (Battista and Clements, 1998). The spatial structure that subsequently arises can help the student recognize (part of) the quantity and consequently abbreviate the counting procedure (Van Nes, 2009).

A lot of research that has studied on how students' spatial and beginning numerical ability can influence the development of students' mathematical thinking (Clements, 2004, Mulligan, 2006). Spatial structure is considered to contribute to insight into important mathematical procedures and concepts such as patterning, algebra, and the recognition of geometric shapes and figures (Carraher, 2006; Clements & Sarama, 2007; Mulligan, Mitchelmore, & Prescott, 2006; Papic & Mulligan, 2007). It takes an important role to help students for developing and simplifying their strategies to define, compare, and perform addition, subtraction, and multiplication (Battista, 1998; Papic, 2005; Van Nes, 2009). Meanwhile, a study conducted by Mulligan et.al., (2006) found that students with sophisticated awareness of patterns and structures excelled in mathematical thinking and reasoning compared to peers and vice versa.

Learning by mathematical structures and pattern could stimulate students' learning and understanding of mathematical procedures and concepts. This coincides with Battista views about how students must learn to construct a meaningful structure and that students could improve their own use of structure if they recognize errors in their counting as the result of inadequate spatial structuring. Spatial structuring ability should be foster as a key factor in the development of number sense, particularly regarding insight into numerical relations (Van Nes, 2007).

Based on that background, the researchers designed the context and learning activities that can help students to develop their strategies in solving addition of numbers 1 to 20. Pendidikan Matematika Realistik Indonesia (PMRI) underlying the design context and activities undertaken.

Pendidikan Matematika Realistik Indonesia (PMRI) is an adaptation of the Realistic Mathematics Education (RME) approach that has been developed in the Netherlands since the early 1970's. RME is determined by the view of Hans Freudenthal about mathematics. Two his important views are "Mathematics must be connected to reality and mathematics as human activity" (Zulkardi, 2002). Firstly, mathematics should be close to the students and relevant to their reality. Secondly, Freudenthal emphasizes that mathematics is as human activity, so that students must be given opportunity to do activities in every topic in mathematics.

The research question is *how do spatial structuring ability help first grade students in developing of addition 1 to 20?*

THEORITICAL FRAMEWORKS

1. *Pendidikan Matematika Realistik Indonesia (PMRI)*

Pendidikan Matematika Realistik Indonesia (PMRI) is an adaptation of the Realistic Mathematics Education (RME) approach that has been developed in the Netherlands since the early 1970's (Putri, 2011). Realistic Mathematics Education (RME) is determined by the view of Freudenthal about mathematics. Two his important views are "*Mathematics must be connected to reality and mathematics as human activity*". Firstly, mathematics should be close to the students and relevant to their reality. Secondly, Freudenthal emphasizes that mathematics is as human activity, so that students must be given opportunity to do activities in every topic in mathematics. As the basis of this research, PMRI approach is defined elaborately through five tenets.

1) Phenomenological Exploration or the Use of Contexts
   Learning math is a constructive activity. Students are introduced to concepts and abstractions through concrete things and it is started from the experiences of students as well as derived from their environment. Contextual issue not only serves as a source of mathematizing, but also as a source for applied mathematics again. Contextual problem as a topic of preliminary learning should be simple problems from students' daily activities. In the first activity, students were introduced with the five-structure and double-structure using the context of arranging balls in the box. In the second activity, the context of arrange the ball in the box of tens was used to discover and use the structure of tens as one strategy in addition.

2) The Use of the Models or Bridging by Vertical Instruments
   The terms of model relates to the situation model and mathematical model developed by students, themselves (*self developed models*). Its role is the bridge for students from real situation to the abstract situation or from informal mathematics to formal mathematics. It means that students make their own model in solving problem. First is the kind of situation close to the students' real world. Generalization and formalization of those models will turn to *model of* problem. Through mathematical reasoning, *model of* will be *model for* of the same problem. Eventually, it will be the model of formal mathematics.

3) The Use of the Students own Productions and Constructions
   The ideas of students need to get noticed and appreciated in order to exchange idea in the learning process. Students produce and construct their idea, so that learning process will be constructive and productive. Students' ideas are communicated to other students and teacher so that mathematics learning appears from either individual activity or group activity. Both on the first activity and the second activity, the students were involved actively to state the way which they choose to solve addition problems.

4) The Interactive Character of the Teaching Process or Interactivity
   In learning mathematics, good interaction has appeared among students concerning the result of student thinking. The teacher facilitates students in discussion interactively. So, the interaction among students, students and teacher, and students and learning tools is an important thing.

5) The Intertwinning of Various Learning Stands
   The addition of number 1 to 20 could be taught by associating with students' spatial structuring ability. Teaching addition by involving spatial structuring ability can help students to find the strategy in order to make addition 1 to 20 more easily. Spatial

structure can be use in learning addition such as finger patterns, five-structure, double-structure, and the ten-structure.

2. Spatial Structuring Ability

The spatial structuring ability is the mental operation of constructing an organization or form for an object or set of objects (Battista et al., 1998). The spatial structure that subsequently arises can help the student recognize (part of) the quantity and consequently abbreviate the counting procedure (Van Nes, 2009). Battista and Clements suggest that spatial structuring is the essential mental process underlying students' quantitative dealings with spatial situations (Battista et al., 1998). Spatial structure is considered to contribute to insight into important mathematical procedures and concepts such as patterning, algebra, and the recognition of geometric shapes and figures (Carraher, 2006; Clements & Sarama, 2007; Mulligan, Mitchelmore, & Prescott, 2006; Papic & Mulligan, 2007).

Research by Mulligan et al. (2006) found that students with sophisticated awareness of patterns and structures excelled in mathematics thinking and reasoning compared to their peers and vice versa. In this case, we can say that the students can understand more than to count the sum.

Learning by mathematical structures and pattern could stimulate students' learning and understanding of mathematical procedures and concepts. This coincides with Battista views about how students must learn to construct a meaningful structure and that students could improve their own use of structure if they recognize errors in their counting as the result of inadequate spatial structuring. Spatial structuring ability should be foster as a key factor in the development of number sense, particularly regarding insight into numerical relations (Van Nes, 2007).

3. Addition of Numbers 1 to 20

According to Freudenthal (1991) in his other book, he states that counting is a the first student's verbal mathematics. Counting plays a very important role to develop in student's number sense students earlier.

Counting one by one is a basic strategy of students in solving addition problems (Nursyahidah, 2013). In the early counting phase, students will count the number of things one by one. In addition numbers 1 to 20, strategy which student uses can be different to the other students. There are some strategies that usually students use in addition number 1 to 20 (Thompson, 1999). Some of them are (1) counting on from first number, (2) counting on from larger, (3) double (near-double system, (4) using fives, and (5) bridging through ten.

METHOD

The method of this study is design research. Design research encompasses three phases: developing a preliminary design, conducting the teaching experiment, and carrying out the retrospective analysis (Gravemeijer, 2004). These phases will explain below:

1. Preliminary Design
   In this step, we study some literatures about addition 1 - 20, spatial structuring ability, Realistic Mathematics Education (RME) and analysis addition 1 to 20 in Curriculum 2013. Learning trajectory and Hypothetical Learning Trajectory (HLT)

were designed. HLT describes learning goals, learning activity, and media to support learning process. The sequence of learning activities is conjecture as guidance to anticipate students' strategy and thinking appeared and developed in the learning activities from informal level to formal level. The conjecture of student thinking is dynamic with the result that can be adjusted to the act of learning and revised during teaching experiment.

2. Teaching Experiment
   In this step, the learning design from preliminary design is tested to explore and to know students' strategies in learning addition of numbers 1 to 20. There are two stages, pilot experiment and teaching experiment. Pilot experiment is as the first cycle. In this first cycle, HLT was tested to 6 students who did not come from teaching experiment class. It conducted to improve the quality of HLT which can be used in actual teaching process as the second cycle. Those six selected students were chosen by teacher who knew their abilities. They consisted of low, medium and high ability students. In the second cycle, revised HLT from the first cycle was implemented in order to explored student's strategy and thinking as data that was used to answer research question. This stage is a cyclic process from thought experiment to instruction experiment. Cyclic process was conducted until we get a learning trajectory, a revision from tested learning materials.

3. Retrospective Analysis
   The aim of retrospective analysis is to develop Local Instructional Theory (Akker et al, 2006). After teaching experiment, all data collected during learning process were analyzed. HLT was used as the main guidance and reference for answering research question. HLT was compared to learning activities undertaken by students. In this case, those are strategies and thinking of students during the learning process. The result of retrospective analysis was also used to make conclusion and provide recommendation on how the HLT was developed for further research.
   Data was collected by interviewing the teacher and some students, observing the students' activities, taking some pictures and videos, and using field notes. In the first cycle, researcher was interviewed the teacher to find some information about the students' condition, materials, and teacher's statements about our HLT. Moreover, the students were interviewed to know the strategies that they always use to solve addition problem. In the second cycle, did the same interview to the teacher and six selected students who have different level of knowledge. Observation was conducted to the research subject in teaching experiment phase. Data from observation was collected by using observation sheets and field notes. In the beginning of the first and the second cycle, researcher gave pre test for students before doing the interview.

The research subjects in the teaching experiment are 21 students for Grade 1 SD Negeri 179 Palembang. This school has been involved in PMRI Project since 2008. The students were about 6-7 years old and they had known some spatial structure in kindergarten and in the early of grade 1 semester 1. There are three instructional activities which were implemented in the second cycle to develop students' strategies in addition of number 1 to 20. Those three activities are dice games, rubber-ball box activity, and ten-box activity. In this paper, we are focus on the second activity namely rubber-ball box activity. Rubber-ball box activity can help students in developing their double-structure as a strategy in solving addition of number 1 to 20.

RESULTS AND DISCUSSION

The findings of this study are (1) Arranging rubber ball box activity can help students to develop their spatial structures; (2) spatial structure such as five-structure and double-structure can be used by students as a strategy to solve addition problem; (3) spatial structure plays an essential role in bridging the students to arrange structures and patterns that is useful for students in solving numerical problem especially addition of number 1 to 20.

In the rubber-ball box activity, doubled-structure and five-structure were introduced to the students as the strategies that can be used to determine the number of objects without counting one by one. There were three problems that have to be solved by students. In the first problem, students have to rearrange 6, 8, and 10 balls in the box. It is important to guide students to doubled-structure and five-structure.

It is as shown in Figure 1 where students used double-structure to determine 8 rubber balls. By using rubber balls and box, students arranged balls into 4 plus 4. In this arrangement, students were not influenced by the box arrangement to put 4 balls in the first line and the other 4 balls in the second line.

Iqbal's group arranged balls into four-four and was not influenced to put 4 balls in the first line and the other 4 balls in the second line. Iqbal put each four balls on the right and left side of the box, as shown in Figure 1. Another arrangement was shown by Vania's group in the Figure 2.. They arranged balls by putting 4 balls in the first line and the other four balls in the second line of the box.

Figure 1. Iqbal's group arrangement

Figure 1. Vania's group arrangement

Through these problems, it appears that spatial structure can help students to develop their counting ability and addition strategies. Although students were assisted with arranging ball in the box to build those structures, it appears that students can classify objects without being influenced by the box arrangement. In arranging the balls, students understood that they can use the five-structure and double-structure to determine the number of objects counted.

In the second problem, students were asked to determine the number of structured balls and write their strategies to determine the number of balls. When students determine the number of 13 balls, every group can use some different strategies. One group counted the balls by using addition 2 + 2 + 2 + 2 + 2 + 2 + 1. In this answer, we can see that students use the doubled-structure in finding the number of balls. Although this way was still have more time to do. The other groups arranged balls by divided balls into two groups which consist of 6 balls and add the rest of balls. So they found 6 + 6 + 1. The Figure 3 shows the students strategies in determining the number of 13 balls.

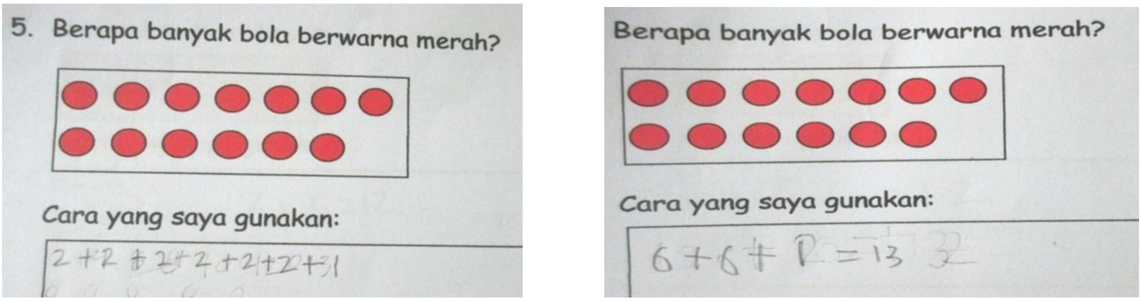

Figure 3. Strategies that students used

In the last problem of this second activity, students were asked to determine the number of unstructured balls. This problem aims to use the doubled-structure and five-structure that students learned in the previous problems in this activity. In this part, there were two groups of unstructured balls that have to be counted. In the first part, there were 14 balls in unstructured arrangement. In order to determine the number of these balls, students' strategies were appeared and they did not use one-by-one counting strategies. One group used a strategy by grouping balls so they found 5 + 5 + 4. We can see in Figure 4 below.

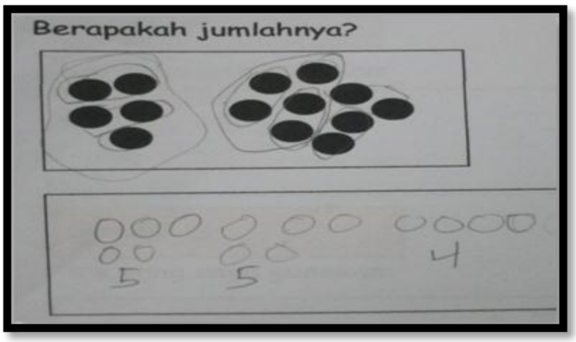

Figure 4. One group's strategy in determining the number of balls

In Figure 4, students used doubled-structure to find the number of balls. In the first way, students counted the balls by making a group of two balls to arrange the group of five balls. They encircled five balls into one group. In this way they found two groups of five balls and the rest which consists of four balls. At the end of their counting, they found that the number of unstructured balls was 14. Another strategy that students used in this problem is by using a doubled-structure. The students' strategy can be shown in Figure 5 below.

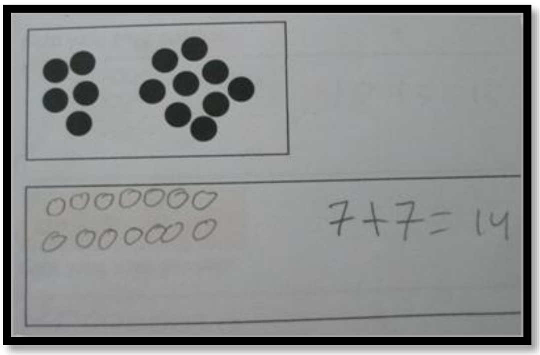

Figure 5. Students used a doubled-structure

According to the Figure 5, students used a doubled-structure by making a group of seven balls. In this way, students did not encircle the balls into one group as what the students did in the Figure 4. They directly grouped 7 balls in every group as their strategy to determine the number of 14 balls. They used a different strategy with a doubled-structure that consists of 7 balls.

At the end, students stated that addition by using five-structure and doubled-structure was the easiest way that they can use the strategies in solving addition of number 1 to 20. Through this activity, students can understand and apply the simplest spatial structure in order to abbreviate their counting.

CONCLUSIONS

The pretest and the interview provided information that most of students used one-by-one counting strategy to solve addition problems. Most of students used the strategy which teacher gave to them in the class. They counted by using finger pattern. The first number will be remembered then the second number will be added by the help of finger pattern. After doing Rubber Ball Box Activity, students are able to use and to choose the best structure to be their strategy in solving addition problems.

Based on the activities undertaken by students, it can be concluded that spatial structuring ability can help students to organize objects so that they can find easily the better strategy to count the objects. Students no longer use one-by-one counting, but organize objects by arranging structures that is easier to count. The use of arranging ball context can help students to discover the five-structure and double-structure as strategies in addition of number 1 to 20.

As a recommendation, RME approach can be a basis and an approach in teaching addition 1 to 20. However, learning addition in lower grade is still considered as the most difficult topic for some students. By designing an interactive learning, students will understand the concept and develop strategies in addition number 1 to 20 better. Those ways are applying RME approach and involving spatial structuring ability of students. By linking the addition of number 1 to 20 with spatial structures, students are expected to develop their abilities in counting number 1 to 20 and not to be influenced to use one-by-one counting.

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