SUPPORTING 7th GRADE INDONESIAN STUDENTS' UNDERSTANDING OF AREA MEASUREMENT OF QUADRILATERALS

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Abstract

This study reports a part of a full study on how to support students understanding of area measurement. Six grade 7 students participated a preliminary teaching in the first cycle. We designed instructional sequence activities consisting of reallotment and tiling activities in six meetings. Reallotment activities embed the concept of conservation of area. Meanwhile, the tiling activities embed the square unit of measurement. Comparing areas of leaves and rice fields, reshaping are activities using reallotment activities. Comparing areas of tiled floors are activities using square unit of measurement. The result indicated that in comparing areas, students could develop their understanding of conservation of area by overlapping, cutting, and pasting. Students could reshape quadrilaterals into a rectangle. In reshaping, students understood that an area of a figure will not change when it is reshaped. Therefore, students applied area formula of rectangle and derived the area formulas of quadrilaterals and triangles.

Keywords: Area, conservation of area, unit of measurement, area formulas, PMRI, RME, local instructional theory, hypothetical learning trajectories.

INTRODUCTION

Some studies on area measurement revealed that students in all levels experience difficulties in dealing with area concepts (Cavanagh, 2007). Cavanagh (2007) shows that many students confused between area and perimeter and about the height of a figure. In Indonesia, the emphasis in teaching and learning mathematics is on algorithm and the use of formulas (Fauzan, Slettenhaar&Plomp, 2002). In addition to this, learning by memorizing formulas and applying them will not support students' understanding of the concept of area. In line with this, Zacharos&Chassapis (2012) argue that the lack of understanding of the mathematical concepts is due to the use of traditional teaching methods overstressing formulas and algorithms without giving attention to students' comprehension of the concepts. Related to area measurement, Martin &Strutchens (2000) (cited in Kamii and Kysh, 2006) state that the concept of area is often difficult for students to understand, and that this is perhaps due to their initial experiences in which it is tied to a formula (such as area as length × width) rather than more conceptual activities. Zacharos (2006) states that research in the field of mathematical education often reveal poor understanding of the processes used for area measurement of plane figures. Özerem (2012) reports that seventh year secondary school students have a number of misconceptions and a lack of knowledge related to geometry subjects, such as using the wrong formula due to the lack of understanding of the concept of area and the memorization of formula.

An experimental study by Zacharos&Chassapis (2012) in grade 6 of a Greek elementary school shows students who learned Eucledian method area of comparison and the principles of overlapping have more successful strategies dealing with area measurement problems.

However, we still do not know how and why the treatment works. In addition, it is also important to implicitly let students understand the concept of conservation of area, in which reshaping the area of a shape does not change its area. In Indonesia, no studies have been conducted in secondary level in which students have learned area formulas to measure areas in elementary schools. Since most ofIndonesian teachers taught area formulas, it is believed students only perceived area measurement as applying formulas.Hence, secondary school students still do not understand why the formulas work because they do not know the concepts. Zacharos&Chassapis (2012) suggest finding teaching interventions that focus on the comprehension of concept of area in order to lead the transition from overlapping practices to formulas.In Indonesia, some studies related to area measurement have been conductedin elementary schools (see Fauzan, 2002; Yuberta, 2011;Febrian, 2013;Fiangga, 2013; Funny, 2013;Putrawangsa,2013). Fauzan (2002) recommends that the development of the local instructional theory should begin with teaching and learning of mathematical topic from the lower grades and gradually to the higher grades. Funny (2013) suggests to further study on the effect of learning of conservation of area when students are learning or after they learn area measurement. Yuberta (2011) suggests to further study on how students achieve the area formulas.

Based on the aforementioned discussion, there is still a need to investigate secondary school students' understanding of area measurement. Therefore, this study will investigate the students' understanding of area measurement by focusing on the concept of conservation of area integrated with unit measurement to support students to understand area formulas to measure area of quadrilaterals in secondary school. This study will use "reallotment" which is the act of reallocation or redistribution of something by cutting and pasting. Using the squares unit of measurement in estimating area also involve the concept of conservation of area. The researchers tried to understand how and why the designed activities support students understanding of area measurement. This study is aimed at contributing to a local instructional theory of area measurement. Therefore, the researchers posetwo research questions as the following:

1. *How can reallotment activities support students' understanding of the concept of area measurement?*
2. *How can reallotment activities support students to measure areas of quadrilaterals and triangles?*

THEORETICAL FRAMEWORK

In Euclidean geometry, in dealing with area measurement, toshow an equality area of two figures, ones can divide one of the figures into parts and then fit those parts in certain ways to produce the second figure (Bunt, Jones &Bedient, 1988). In the criteria for equality of triangles this strategy is called overlapping‟ or epithesis‟ and this strategy can be used extensively to determine the equality of areas as well (Zacharos, 2006). In addition, dividing or parting and rearranging to produce a new figure involve knowledge of the concept of conservation of area. It means that breaking up the first figure into parts and rearranging from those parts to produce the second figure will not change the area of the original figure.More implicitly, in this study, reallotment term is used to name this activity.Kordaki (2003) states that students can master the concept of conservation of area through the *cut, move* and *paste* activities.

The procedure to measure an area involves the surface to be measured or compared. Therefore, area is closely related to surface. Baruto&Nason (1996) define area as an amount of region (surface) enclosed within a boundary and this amount of region can be quantified. In everyday words, the area of a figure or object is the amount of stuff needed to cover the figure (Konya and Tarcsi, 2010). Moreover, there is a need to find the stuffin order to make it easier to determine the area of a shape. Cavanagh (2007, 2008) states that area measurement is based on partitioning a region into equally size units that cover it without any gaps or overlaps. Here, the stuff needed to measure an area is a unit of measurement.

Reynolds & Wheatley (1996) state that to determine an area of a region can be done by comparing that region to another region like a square unit. They argue that in comparing regions that assigns numbers, there are four assumptions. The four assumptions are (1) a suitable two-dimensional region is chosen as unit, (2) congruent regions have equal areas, (3) regions do not overlap, and (4) the area of the union of two regions is the sum of their areas. Therefore, learning and teaching of area measurement can be taught though tiling activity (Reynold& Wheatley, 1996). In line with this statement, tiling activity can be used to teach students that area is a measure of covering (Konya &Tarcsi, 2010). Tiling activities use the idea of covering a region without any gaps or overlaps within certain tiles as units of measurement. Stephan & Clements (2003) argue that there are at least four foundational concepts that are involved in learning of area measurement: (1) partitioning, (2) unit iteration, (3) conservation, (4) structuring array. In this study, the researchers integrate all concepts all four foundational concepts in the designed-instructional activities

METHOD

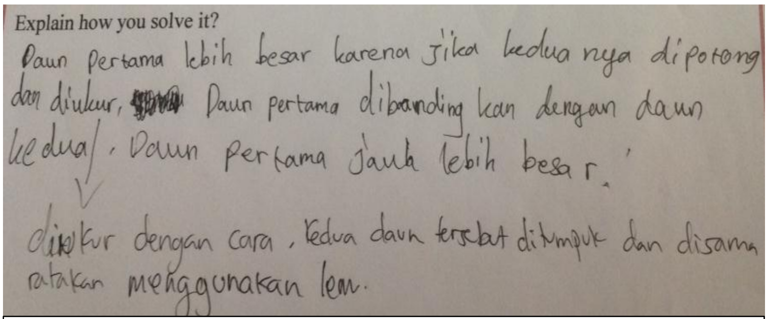
PendidikanMatematikaRealistik Indonesia (PMRI) approach as an adaptation of Realistic Mathematics Education (RME) is used in this study. By using design research approach, the researcher developed a Hypothetical Learning Trajectory (HLT) and mathematical activities to get better understandings of how secondary students' (aged 12-13) understanding of area measurement may be fostered. The researcher designed six meeting involving comparing areas. Reallotment activities and tiling activities are embedded in the comparing areas. This study has two cycles but this paper only describes the first cycle. The first cycle, a teaching experiment was conducted in a small group of students. The researcher acted as a teacher in the first cycle. Meanwhile the second cycle, the teaching experiment is conducted in a real classroom setting with the home class teacher. Data were collected through video recording, students' work, pre-test, post-test, and interviews. The data were analyzed by comparing the HLT with actual learning in order to know what students learned and did not learn from the designed activities.

RESULT AND DISCUSSION

The researchers zoomed out only the three activities in this paper. The first activity is comparing leaves (leaf A and B) in meeting 1. Students had to determine a leaf getting more sunlight for photosynthesis. Provided with cutting tools, students cut out the leaves from the worksheet and overlapped them (see figure q). As conjectured in HLT, students would overlap the leaves, cut and paste non-overlapping parts of the first leaf and paste them on the non-overlapping part of the second leaf.

Figure.1 Student overlapped the leaves

Students solved the problem by overlapping, cutting and pasting to fit one leaf to another one. The following figure shows how students explain their strategy (figure 2).



Explain how you solve it?

Daun pertama lebih besar karena jika keduanya dipotong dan diukur, Daun pertama dibanding kan dengan daun kedua. Daun pertama jauh lebih besar.

diukur dengan cara, kedua daun tersebut ditumpuk dan disama ratakan menggunakan lem.

Explain how you solve it?

The first leaf is larger because if both leaves are cut and measured by overlapping both leaves, cutting and pasting with glue.

Figure.2 Students overlapped, cut and pasted the leaves

One student also grasped the concept of conservation of area. The researcher posed questions when student presented their work.

Researcher : This parts belong to this leaf (pointing leaf B), is the area of this cut-leaf equal to the uncut-leaf? Does the area change?

Gio : **No**

Researcher : It does not change, Why?

Gio : Because, these cut-parts are not... (having difficulty to say)

Researcher : Did you throw away some parts?

Gio : **No, we glued them all**

Researcher : You glued them all, and united them right? But the shape changed, didn't it?

From this segment, we can see that student grasped the concept of conservation of area when he cut and pasted non-overlapping part of leaf B. Gio understood the area of leaf B remains the same after its parts were cut and pasted again. He knew that there were no parts thrown away.

Reshaping activity was conducted in the third meeting. Students had to reshape quadrilaterals into a rectangle. This activity is aimed at deepening their understanding of the concept of conservation of area. As conjectured, students reshaped the quadrilaterals into a rectangle by cutting and pasting. However, students had different result of rectangle. Some students still had difficulties reshaping the rhombus and the kite. To reshape the parallelogram and the isosceles trapezoid is much easier for students.

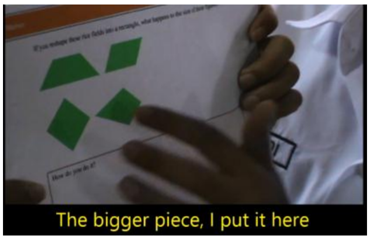


Figure. 3 Students explained how to reshape the kite

The following segment describes how Gio reshaped the quadrilaterals into a rectangle.

Gio	: I did it by dividing this into two
Researcher	: Please, pay attention
Gio	: Then, the lower part, I placed it here
Researcher	: Heem (letting to continue)
Gio	: Then the upper part is divided into two, I cut it from this to this
Researcher	: Then?
Gio	: The smaller piece, I put it here The bigger piece, I put it here
Researcher	: Did it work?
Gio	: Yes
Researcher	: Did it become a rectangle?
Gio	: Heem (agreeing)
Researcher	: Ok, one question, does the area change?
Gio	: No
Researcher	: **Why doesn't the area change?**
Gio	: **Because, i did not throw away anything**
Researcher	: Nothing was thrown away; you pasted it again, didn't you?
Gio	: Yes, I pasted it again
Researcher	: So you pasted again, and nothing was thrown away
Gio	: No, I did not (throw way anything)

Researcher : **So what happens to the area?**

Gio : **The area does not change, the shape changes**

From this conversation, Gio once again claimed that the area does not change because he did not throw away anything. He pasted again the part he cut to the quadrilaterals to make a rectangle. He has understood that reshaping the quadrilateral into a rectangle will not change their areas. In another word, he has sensed that the area of a figure is conserved when it is reshaped.

In the last meeting, students had to determine areas of quadrilaterals and triangles. In this occasion, the researchers only explain how students dealt with the rhombus and the kite. On the worksheet, students might see the hint to reshape the figures. Firstly, students reshaped the rhombus and the kite into a rectangle then determined their areas using area formula of rectangle. Students had different way of reshaping the rhombus. The figure 4 shows Dean and Gio solved the problem.

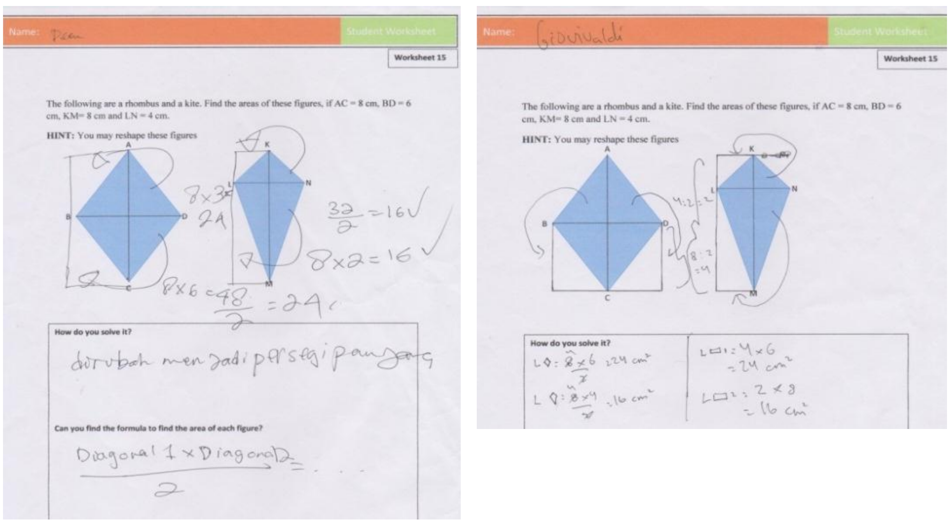


Figure. 4 Students (Dean and Gio) reshaped the rhombus and the kite

From the students' work, it is clear that students reshaped the rhombus and the kite into a rectangle. The determined the area of those quadrilaterals by applying area formula of rectangle. To support this evidence, one student presented his work. The following fragment tells about the relation of the base and the height of the rectangle to the diagonals of the kite.

Gio : The diagonal 1 is from K to N

Researche
r : heem, from L to N is the diagonal 1, do you agree?

Students : (Nodding)

Gio : From K to M is diagonal 2. if the kite is reshaped into a rectangle L to N is divided by two becoming the base of the rectangle

Researcher : And then?

Gio : The height of the rectangle does not change (diagonal 2), and KM is the height of the rectangle

Researcher : What is it at first?

Gio : The diagonal 1

Researcher : Then, what is the area?

Gio : Which area?

Researcher : The area of the rectangle

Gio : The base is taken from LN divided by two

Researcher : Then what is the height?

Gio : KM

Researcher : If you used the diagonals, is the area formula same?

Gio : Yes, It is

Researcher : What?

Gio : D1xD2/2 (diagonal1 times diagoanal2 divided by two)

Researcher : If you used the rectangle, what is the area formula?

Gio : base x height

Researcher **: The base is…?**

Gio **: Half the diagonal1**

Researcher **: The height is…?**

Gio **: The height is one, eh… the diagonal 2**

From the fragment, it is obvious that Gio could see the relationship between the base and the height of the rectangle and the diagonals of the kite. In the end of meeting, the teacher asked students to know why the area formula of the rhombus is the product of its diagonals.The teacher and students made conclusions of the area formula of rhombus and kite.

Researcher	: Then, why the area of rhombus is diagonal 1 x diagonal 2?
Researcher	: Because if we reshape it into…
Students	: Rectangle
Researcher	: Into a rectangle, what is the diagonal 1?
Mia	: The length
Researcher	: The length (repeating), what is the diagonal 2?
Gio	: A half
Raudy	: A half
Researcher	: Yes, a half diagonal 2 is the width
Researcher	: So, that is why we get 1/2 d1xd2 and for kite, it applies the same by…?
Students	: **Reshaping**
Researcher	: **If we do reshaping, what happens?**
All students	: **The shape changes, the area does not change**

From this segment, we can conclude that to some extent students could derive the area formula of quadrilateral through reshaping. To be noted, the worksheet provided a hint that students could reshape the figures. In the second cycle this hint was omitted. The researchers tried to know whether students could make use their knowledge of reshaping or not. I addition, in this first cycle, reshaping activity could help students to measure area of isosceles trapezoid but it seems to be difficult to derive its area formula. Therefore, in the second cycle, to derive area formula of trapezoid, students will divide the trapezoid into triangles.

CONCLUSION

Students understand that reshaping a figure will not change the area but just the shape. Based on our analysis and findings, we answer to the research questions as the following:

1. *How can reallotment activities support students' understanding of the concept of area measurement?*
   The reshapping activities can support students understanding of the concept of conservation of area because students did not throw away any parts when they reshaped a figure into another figure. As in the leaves problem, students cut and pasted one leaf to fit another leaf. It is more obvious when students reshaped the quadrilaterals into a rectangle.
2. *How can reallotment activities support students to measure areas of quadrilaterals and triangles?*
   Reallotment by reshaping quadrilaterals into a rectangle could support students to measure areas of the quadrilaterals. Students could apply area formula of rectangle after reshaping the quadrilaterals. Students could understand why the formula can be derived by seeing the relationship of the quadrilaterals and the rectangle produced.

Therefore, we can conclude that reallotmet activities through comparing areas, reshaping into a rectangle could support students understanding of the concept of conservation of area. This activity also could support students to measure area of quadrilaterals and derive their formulas.

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