Implementation of Pattern Generation Algorithm in Forming Gilmore and Gomory Model for Two Dimensional Cutting Stock Problem

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Abstract. Two dimensional cutting stock problem (CSP) is a problem in determining the cutting pattern from a set of stock with standard length and width to fulfill the demand of items. Cutting patterns were determined in order to minimize the usage of stock. This research implemented pattern generation algorithm to formulate Gilmore and Gomory model of two dimensional CSP. The constraints of Gilmore and Gomory model was performed to assure the strips which cut in the first stage will be used in the second stage. Branch and Cut method was used to obtain the optimal solution. Based on the results, it found many patterns combination, if the optimal cutting patterns which correspond to the first stage were combined with the second stage.

1. Introduction

The printing industry is trying to increase production on a large scale by managing the use of raw materials (stock). The planning aims to determine the minimum amount of raw materials used to fulfill the demand of items. One way that can be done to minimize the use of raw materials is to determine the proper cutting, or in the field of Optimization known as Cutting Stock Problem (CSP).

There are many types of CSP, one-dimensional CSP, two-dimensional CSP, and three-dimensional CSP. Dimension size is determined by the side of the remaining cuts (trim loss).The CSP faced by the paper industry in this study was a two dimensional CSP, which in the stock cutting process focused on the long and wide sides.

Previous research on CSP has been widely practiced. Column generation technique (CGT) can be used in the completion of two-dimensional CSP with guillotine-cutting type and fixed orientation, integer solutions obtained by rounding down on the optimal solution [10]. The completion of two-dimensional 2-stage CSP using the Branch and Price and Gomory Cutting Plane methods has also been done previously [2] by adding Gomory Cutting Plane to linear programming relaxation. Then Mrad [9] has used the Branch and Price method to solve two-dimensional CSP models of type guillotine.

The effective cutting methods can be done by cutting paper from one side to the other side of the line or commonly called guillotine cutting [4-6]. Then Mellouli and Damak [8] implemented an algorithm for determining the cutting pattern called pattern generation (PG) algorithm [12] on multiple two-dimensional CSP using CGT. The application of PG algorithm in arc-flow model can facilitate the

search of cutting patterns [1] in one dimensional CSP. For two-dimensional CSP, Octarina et al. [11] created an application for finding the cutting pattern based on the Modified Branch and Bound Algorithm.

Based on this background, this research used PG algorithm in forming Gilmore and Gomory model on two dimensional CSP. The model is extended from the CGT approach proposed by Gilmore and Gomory on a one dimensional CSP. The Gilmore and Gomory model is used to complete two-dimensional CSP on type guillotine and the optimal solution obtained by Branch and Cut method.

2. Research method
Steps conducted in this research are as follows.
1. Determine the data include products name, length and width, and also the demand.
2. Group the data that will be implemented on the PG algorithm in decreasing order.
3. Implement the PG algorithm on two-dimensional CSP to obtain the first cutting pattern and the second cutting pattern.
4. Create a searching tree by using the PG algorithm.
5. Form the cutting pattern table along with cut loss which is depicted from the search tree sequentially by branch.
6. Implement the first and second stage cutting patterns into Gilmore and Gomory models by:
   a. Defines the variables used.
   b. Determine the objective function that produces the minimum stock quantity to meet the demand of each item.
   c. Determine the constraints that ensure strip produced in the first stage is the strips used in the second stage, and will be cut into the requested items. Next define the constraints that ensure all requests item are met.
7. Find the Gilmore and Gomory model solution using the Branch and Cut method and use the LINDO 6.1 program in completing the relaxation LP model.
8. Conclude and obtain the optimal solution.

3. Pattern Generation Algorithm
Let the stock with standard width w'_k and length l'_k, $(k = 1, 2, …, h)$ will be cut to n items with the width w_i and length l_i $(i = 1, 2, …, n)$. The model of Cutting Stock Problem can be seen in (1).

$$\text{Minimize } z = \sum_{k=1}^{h}\sum_{j=1}^{m_k} c_{jk}x_{jk} + \sum_{i=1}^{n} w_i s_i \tag{1}$$

Subject to:

$$\sum_{k=1}^{h}\sum_{j=1}^{m_k} a_{ijk}x_{jk} - s_i = l_i \quad \text{for all } i, \tag{2}$$

$$x_{jk}, c_{jk}, s_i, a_{ijk} \geq 0 \quad \text{for all } i, j, k, \tag{3}$$

where s

a_{ijk} is the number of item with width w_i which cut according to j^{th} cutting pattern from k^{th} stock $(i = 1, 2, …, n; j = 1, 2, …, m; k = 1, 2, …, h)$.

x_{jk} is the length of kth stock which cut according to j^{th} cutting pattern.

c_{jk} the trim loss from k^{th} stock which cut according to j^{th} cutting pattern.

s_i is the remaining length which will produce item with width w_i.

A search tree is used to generate the feasible cutting patterns. The level of the tree represents the required width, which is arranged in decreasing order, the largest size is in the first level while the smallest size is placed at the last level of the tree. The initial vertex from first level is the standard width w'_k which use to generate the cutting pattern.

The branch from i^{th} level of search tree is the multiplication of the number of item and width w_i which represent the number of width item from k^{th} stock to meet w_i width. Vertex from 2nd level to n^{th} is the remaining width after fulfil the certain cutting pattern from the last $i-1$ branch. The last vertex from search tree is the trim loss from different cutting pattern. Search tree was built from up to down and left to right.

The steps for Pattern Generation (PG) algorithms are as follows:

1. Set the width size $w_i(i = 1,2,...,n)$ in decreasing order.

2. Use $a_{i1k} = \left| \dfrac{w'_k - \sum\limits_{z=1}^{i-1} a_{z1k} w_z}{w_i} \right|$ to fill the first column $(j = 1)$ in matrix.

3. Use $c_{jk} = w'_k - \sum_{i=1}^{n} a_{ijk} w_i$ to find the trim loss from each cutting pattern.

4. Set the level index (row index) i to $n-1$.

5. Check the vertex in i^{th} level, e.q. vertex (i,j). If the vertex has the value equals to zero $(a_{ijk} = 0)$, go to Step 7. If not, generate the new column $j = j + 1$ with the following elements:

   a. $a_{zjk} = a_{z(j-1)k}$ $(z = 1, 2, ..., i - 1)$ are elements to fill the vertex which precedes vertex (i, j).

   b. $a_{ijk} = a_{i(j-1)k} - 1$ are elements to fill vertex (i, j).

   c. Fill the remaining vertex from j column. For example, $a_{(i+1)jk}, a_{(i+2)jk}, ..., a_{njk}$ using

   $$a_{ijk} = \left| \dfrac{w'_k - \sum\limits_{z=1}^{i-1} a_{zjk} w_z}{w_i} \right|$$

6. Use $c_{jk} = w'_k - \sum_{i=1}^{n} a_{ijk} w_i$ to find cut loss from the jth cutting pattern. Go back to Step 4.

7. Set $i = i - 1$. If $i > 0$, repeat Step 5. Otherwise stop the iteration.

4. Result and Analysis

This section discusses the two dimensional CSP by using the same data from [11]. The name, size and number of each product can be seen in Table 1. The stock has the length and width with the size is 1,090 mm and 970 mm respectively.

Table 1. Products size and demand

No	Product Name (Item)	Length (mm)	Width (mm)	Number of order (pieces)
1	Invitation	325	225	300
2	Business Card	90	60	1,000
3	Pamphlets	210	150	3,000
4	Yasin/Book	230	160	500

PG algorithm was applied to find the cutting patterns. The cutting patterns corresponding to the width and length can be seen in Table 2 and 3 respectively.

Then all the cutting patterns are formed to Gilmore and Gomory model that can be seen in (4).

Minimize $z = \lambda_1^0 + \lambda_2^0 + \lambda_3^0 + \lambda_4^0 + \lambda_5^0 + \lambda_6^0 + \lambda_7^0$

with constraints:　　　　　　　　　　　　　　　　　　　　　　　　　　　　(4)

$\lambda_2^0 + 2\lambda_3^0 + 7\lambda_5^0 + 5\lambda_6^0 + 12\lambda_7^0 - \lambda_1^1 = 0$

$\lambda_1^0 + \lambda_2^0 + \lambda_3^0 + 3\lambda_4^0 + 3\lambda_6^0 - \lambda_1^2 - \lambda_2^2 - \lambda_3^2 = 0$

$\lambda_1^0 + 2\lambda_2^0 + 3\lambda_3^0 + 2\lambda_4^0 + 2\lambda_5^0 - \lambda_1^3 - \lambda_2^3 - \lambda_3^3 - \lambda_4^3 - \lambda_5^3 - \lambda_6^3 - \lambda_7^3 = 0$

$2\lambda_1^0 + \lambda_2^0 - \lambda_1^4 - \lambda_2^4 - \lambda_3^4 - \lambda_4^4 - \lambda_5^4 - \lambda_6^4 - \lambda_7^4 = 0$

$16\lambda_1^1 + 11\lambda_1^2 + 6\lambda_2^2 + \lambda_3^2 + 11\lambda_1^3 + 6\lambda_2^3 + \lambda_3^3 + 8\lambda_4^3 + 3\lambda_5^3 + 3\lambda_6^3 + 7\lambda_1^4 + 2\lambda_2^4 + 6\lambda_3^4 + \lambda_4^4 + 6\lambda_5^4 + \lambda_6^4 + \lambda_7^4 \geq 1.000$

$2\lambda_1^2 + 4\lambda_2^2 + 6\lambda_3^2 + \lambda_1^3 + 3\lambda_2^3 + 5\lambda_3^3 + 2\lambda_5^3 + \lambda_6^3 + 2\lambda_2^4 + \lambda_3^4 + 3\lambda_4^4 + 2\lambda_6^4 \geq 3.000$

$\lambda_1^3 + \lambda_2^3 + \lambda_3^3 + 3\lambda_4^3 + 3\lambda_5^3 + 4\lambda_6^3 + 6\lambda_7^3 + 2\lambda_1^4 + 2\lambda_3^4 + \lambda_5^4 + \lambda_6^4 \geq 500$

$\lambda_1^4 + \lambda_2^4 + 2\lambda_3^4 + 2\lambda_4^4 + 2\lambda_5^4 + 2\lambda_6^4 + 4\lambda_7^4 \geq 300$

$\bar{\lambda} \geq 0$ and $integer$

with $\bar{\lambda} = (\lambda_1^0, \ldots, \lambda_j^0, \lambda_1^1, \ldots, \lambda_j^1, \lambda_1^2, \ldots, \lambda_j^2, \ldots, \lambda_1^{m'}, \ldots, \lambda_j^{m'})^T$

Table 2. Cutting patterns corresponding to the width

The jth pattern	Number of items				Cut Loss (mm)
	225 mm	160 mm	150 mm	60 mm	
1	4	0	0	1	10
2	2	1	2	1	0
3	2	1	0	6	0
4	2	0	3	1	10
5	2	0	1	6	10
6	1	2	2	2	5
7	1	2	0	7	5
8	0	6	0	0	10
9	0	4	1	3	0
10	0	3	2	3	10
11	0	3	0	8	10
12	0	1	5	1	0
13	0	1	3	6	0
14	0	1	1	11	0
15	0	0	6	1	10
16	0	0	4	6	10
17	0	0	2	11	10
18	0	0	0	16	10

Table 3. Cutting patterns corresponding to the length

The jth pattern	Number of items				Cut Loss (mm)
	325 mm	230 mm	210 mm	90 mm	
1	2	1	1	0	0
2	1	2	1	1	5
3	0	3	1	2	10
4	0	2	3	0	0
5	0	2	0	7	0
6	0	0	3	5	10
7	0	0	0	12	10

Next the model in (4) can be rewritten in Table 4. Optimal solution of Gilmore and Gomory model showed the value of $z = 142$ with $\lambda_1^0 = 38$, $\lambda_4^0 = 99$, $\lambda_6^0 = 5$, $\lambda_1^1 = 25$, $\lambda_1^2 = 3$, $\lambda_3^2 = 347$, $\lambda_3^3 = 182$, $\lambda_4^3 = 1$ $\lambda_7^3 = 53$, $\lambda_3^4 = 2$, and $\lambda_7^4 = 74$. It means the 1st, 4th, and 6th cutting patterns can be used in the first stage, whereas the cutting patterns that can be used in the second stage are as follows.

1. The 1st cutting pattern will be used in the strip with length of 90 mm.
2. The 1st and 3rd cutting patterns will be used in the strip with length of 210 mm.
3. The 3rd , 4th, and 7th cutting patterns will be used in the strip with length of 230 mm, and
4. The 3rd and 7th cutting patterns will be used in the strip with length of 325 mm.

Table 4. Gilmore and Gomory Model

(l_i, w_i)	λ_1^0	λ_2^0	λ_3^0	λ_4^0	λ_5^0	λ_6^0	λ_7^0	λ_1^1	λ_1^2	λ_2^2	λ_3^2	λ_1^3	λ_2^3	λ_3^3	λ_4^3	λ_5^3	λ_6^3	λ_7^3	λ_1^4	λ_2^4	λ_3^4	λ_4^4	λ_5^4	λ_6^4	λ_7^4	
(90, 60)	0	1	2	0	7	5	12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	= 0
(210, 150)	1	1	1	3	0	3	0																			= 0
(230, 160)	1	2	1	2	2	0	0																			= 0
(325, 225)	2	1	0	0	0	0	0																			= 0
(90, 60)								16	11	6	1	11	6	1	8	3	3	0	7	2	6	1	6	1	1	≥ 1.000
(210, 150)								0	2	4	6	1	3	5	0	2	1	0	0	2	1	3	0	2	0	≥ 3.000
(230, 160)								0	0	0	0	1	1	1	3	3	4	6	2	2	0	0	1	1	0	≥ 500
(325, 225)								0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	2	2	4	≥ 300
Objective function	1	1	1	1	1	1	1																			

Figure 1. First Combination

Figure 2. Second Combination

Figure 3. Third Combination

Figure 4. Fourth Combination

Based on the implementation of pattern generation algorithm on cutting pattern determination, it found 57 cutting patterns corresponding to the width and 34 cutting patterns corresponding to the length. The cutting pattern chosen is the pattern that has the minimum cut loss and at the same time can meet the availability of items ordered, so the selected pattern is a pattern with a cut loss of no more than 10 millimeters as many as 18 cutting patterns corresponding to the width and 7 cutting patterns according to the length.
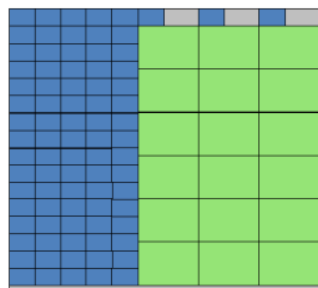
The selected cutting patterns, formed into the Gilmore and Gomory model and completed using Branch and Cut method and LINDO 6.1 program. The addition of Gomory restrictions in this study did not adequately affect the solution obtained due to the large number of variables. The number of items resulting from stock cuts are 142 sheets based on the optimal cutting pattern. The optimal solution obtained is substituted into Gilmore and Gomory model constraints, resulting in 1,056 sheets of business cards, 3,000 pamphlets, 503 pieces of Yasin / books, and 300 pieces of invitations. Each item produced is not less than the request, the resulting excess amount can be used for the next order.

The optimal solution from the first and second stage can be combined to the new cutting patterns, as shown in Figures 1-7.
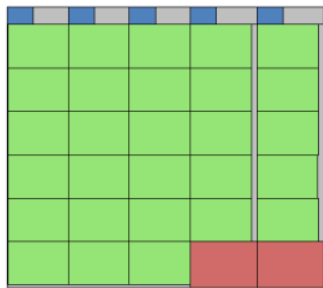
Figure 5. Fifth Combination   **Figure 6**. Sixth Combination   **Figure 7**. Seventh Combination
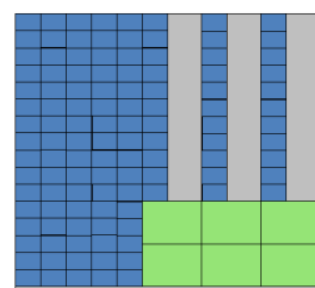
Note: ▢ means trim loss   ▢ means pamphlets   ▢ means invitations

▢ means business card   ▢ means Yasin/books

Figures 1-7, shows that there are 7 combinations of cutting patterns that can be used. Trim loss from each cutting patterns can be used again to produce the other items that fixed the size.

5. Conclusion

From the result, it can be concluded that implementation of PG algorithm helps the searching of cutting patterns become easier. Then, the patterns can be modelled as Gilmore and Gomory model whereas the objective function is to minimize the number of cutting stock and the constraints assure that the strips from first stage can be reused in the second stage in order to meet all the demands.

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