Implementation of Branch and Cut Method on N-Sheet Model in Solving Two dimensional Cutting Stock Problem

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Implementation of branch and cut method on *n*-sheet model in solving two dimensional cutting stock problem

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Abstract. The problem of cutting raw materials to fulfill the size of demand using certain cutting pattern is called Cutting Stock Problem (CSP). CSP aims to find the optimal cutting pattern with the minimum trim loss. In this research, all possible cutting patterns were generated by pattern generation algorithm where the materials were cut based on the length on the first stage and the width on the rest stage. All patterns were modeled to *N*-Sheet model and solved using Branch and Cut method. Based on the results, it showed that the *N*-Sheet model ensured as minimum a possible trim loss to fulfill the demands.

1. Introduction

Competition in the field of production is increasingly complex. Producers only focus on quantity and quality. But now, manufacturers have begun to think of ways to optimize revenue without making the consumers loss or raising capital. One way to solve this problem is to optimize the use of raw materials. The problem of setting raw materials in Operations Research (OR) is known as Cutting-Stock Problem (CSP).

Research on CSP has been widely discussed. [1] examined the two-dimensional CSP using an Arc-Flow model with guillotine constraints. Furthermore, [2] completed the Pattern Generation algorithm with an Arc-Flow model for one-dimensional CSP. The resulting Arc-Flow model used only demand and non-negative constraints,otherswill not be used, because the selection of the cutting pattern is only the minimum cut-loss.

Previous research was conducted by Gilmore and Gomory in 1963, which examined one-dimensional CSP by linear programming formulation. Then Gilmore and Gomory in 1965 again conducted research for two-dimensional CSP in its application into the Column Generation Technique (CGT). Another study conducted by [3] regarding the two-stage CSP in the guillotine problem. Their research showed that the model can be modified to handle certain cases such as cutting the right two-stage guillotine without trimming.

[4] made a cutting pattern program for two-dimensional CSP using the modified Branch and Bound algorithm, but this program still produces many similar patterns. Furthermore, [5] continued their research using the Branch and Bound and Gomory Cutting Plane methods, and referred as the Branch and Cut method, so it concluded that the addition of Gomory constraints is less effective due to a large number of variables.

[6] proposed an exact algorithm for two-dimensional CSP with one raw material size on *N*-sheets and obtained a suitable model to minimize the remaining cuts. Based on [7], the Gomory Cutting Plane algorithm can be used quickly, but it often produces non-integer solutions. However, the Branch

and Bound algorithm produce an integer solution but the iteration process is too much. This caused the two algorithms to be combined into one, and are called Branch and Cut.Based on the background described earlier, this study implements the Branch and Cut method in the *N*-Sheet model to solve two-dimensional CSP.From the model, it can found the optimal cutting pattern with the minimal trim loss so can fulfilled the demand. The possible cutting patterns can be reduced using this model.

2. Literature Review

Literary studies that support and relate to this research, including Cutting Stock Problem (CSP), Pattern Generation (PG), *N*-Sheet Model, and Branch and Cut Method.

2.1. Cutting Stock Problem (CSP)

Cutting Stock Problem (CSP) is an optimization problem that has been studied extensively over the past five decades which involves determining the cutting pattern that optimizes the objective function so that the order is fulfilled. Cutting Stock Problem was first put forward by Leonid Kantorovich, a Russian scientist in 1939. The most well-known study was Gilmore and Gomory's research in 1961 and 1963.

2.2. Pattern Generation (PG)

The stocks with standard length (l'_k) and standard width $(w'_k)$where $k$indicates the number of raw materials used and$k = 1,2,3, \ldots, h$cut to the certain length $(l_i)$and width $(w_i)$with $n$sizes of demand and $i$is the number of demand sizesand$i = 1,2,3, \ldots, n$. The objective is to find the cutting pattern with minimum trim loss and fulfill the demand.

The steps in PG algorithm according to[8] are as follow:

1. Sort the length l_i $(i = 1,2, \ldots, n)$in decreasing order.
2. Fill the first column $(j = 1)$of the matrix
   $$a_{ijk} = \left\lfloor \frac{l'_k - \sum_{z=1}^{i-1} a_{zjk}\, l_z}{l_i} \right\rfloor \quad (1)$$
3. Count the cut loss from the cutting pattern
   $$c_{jk} = l'_k - \sum_{i=1}^{n} a_{ijk}\, l_i \quad (2)$$
4. Set index level (row index) $i$to $n - 1$.
5. Check the current vertices in i level, e.g. vertex (i, j). If the vertex has 0 value$\left(a_{ijk} = 0\right)$, go to Step 7. Otherwise, generate new column $j = j + 1$with the following elements:
   a. Elements to fill the preceeds vertex $(i, j)$is $a_{zjk} = a_{z(j-1)k} (z = 1,2, \ldots, i - 1)$. (3)
   b. Elements to fill vertex$(i, j)$is$a_{ijk} = a_{i(j-1)k} - 1$. (4)
   c. The remaining vertex from j, fill with Eq (1)
6. Count the cut lossfrom the-jth cutting pattern by using Eq(2). Go back to Step 4.
7. Reduction of index$i$, with $i_f = i_{f-1} - 1$, where$f = 1, 2, 3, \ldots, n - 1$. If$i_f > 0$redo Step 5. If $i_f = 0$,then STOP.

2.3. N-Sheet Model

The two-dimensional CSP model is considered to the important part of this research, where there are enough raw materials and many items of requests are given. If all possible cutting patterns are available and the requested item can be cut from one raw material, the optimal solution for two-dimensional CSP can be determined. The purpose of the *N*-Sheet model is to minimize the remaining cuts or trim loss. The *N*-Sheet model according to [6] can be modeled in the Model (5).

Minimize

$$\mathbf{Z(N, \beta)} = \sum_{j=1}^{n^\beta} w_j^\beta x_j \quad (5)$$

Subject to:

$$\sum_{j=1}^{n^\beta} I_{ij} x_j \le I_i \qquad i = 1, \ldots, m \quad (5.a)$$

$$\sum_{j=1}^{n^\beta} x_j = N \qquad j = 1, \dots, n^\beta \tag{5.b}$$

where
x_j is the number of j cutting pattern
$w_j^\beta \in W_\beta$ is the remain from j cutting pattern (trim loss)
I_{ij} is the number of i item which is cut by j cutting pattern
I_i is the number of i item
m is the number of item type
N is the number of stocks
n^β is the number of all cutting pattern with the trim loss in β
β is the number of trim loss that allows entering the process in PG

2.4. Branch and Cut Method

Branch and Cut method is one method of solving integer programming problem. Branch and Cut is a combination of the Cutting Plane method with the Branch and Bound method. This method completes the relaxation part of the linear program from the ILP problem. Branch and Cut is an optimization method for ILP which involves the Branch and Bound method and uses Cutting Plane to round off the results of its linear program.

3. Methodology

This study implemented the PG algorithm in the *N*-Sheet model and completed the model with the Branch and Cut method. This research used data of two-dimensional CSP problems. The steps taken in this study are as follows:

a. Describing data. The data includes the length and width of the raw material, the length and width of the product demand, and the number of raw materials used.
b. Sort the data in descending order to be used in the PG algorithm.
c. Implement the PG algorithm on data that has been processed in order to obtain the first cutting pattern and the second cutting pattern.
d. Create a search tree that is formed based on the PG algorithm in determining the first cutting pattern and the second cutting pattern.
e. Make a table of cutting patterns along with trim loss obtained from searching tree.
f. Implement the first stage of the cutting pattern and the second stage cutting pattern into the *N*-Sheet model.

4. Results

This research used the data with 3 platted material of the stocks. The size of each stock is 30 inches×24 inches. The products size and demand can be seen in Table 1.

Table 1.The Product Size and Demand

The-*i* th Demand	**Length(Inches)**	**Width (Inches)**	**Number of Demand**
1	9	5	9
2	10	5	5
3	11	7	8
4	3	2	10
5	7	5	5

Based on Table 1, there are 5 products with different sizes. The most number of demand is the product with 3 inches × 2 inches. After implemented the PG algorithm, there were 33 cutting patterns which cut according to the length and can be drawn in searching tree on Figure 1, 2, and 3.

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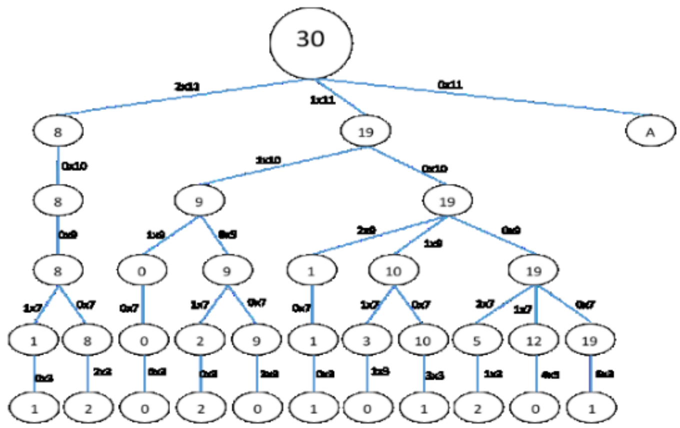

Figure 1.The Searching Tree of TheCutting Pattern According to Length (Part I)

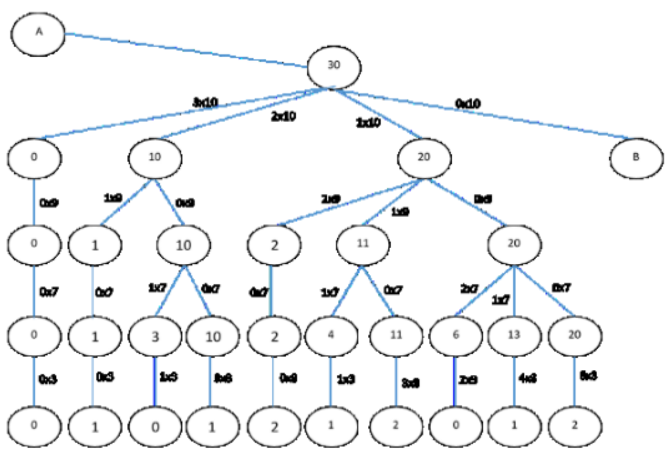

Figure 2.The Searching Tree of TheCutting Pattern According to Length (Part II)

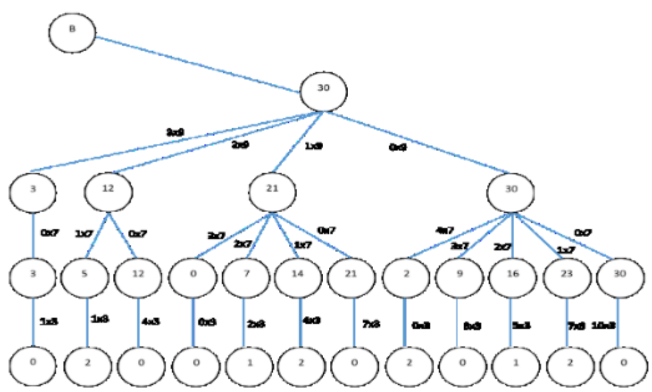

Figure 3. The Searching Tree of TheCutting Pattern According to Length (Part III)

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Figure 1 - 3 shows the searching tress of the cutting pattern. Those three pictures are one set of picture but separated to three parts. The stock with length of 30 inches, was cutted to 5 sizes of items. There are 11 inches, 10 inches, 9 inches, 7 inches and 3 inches. The size must be sorted in decreasing order. The cutting patterns from Figure 1-3 then rewrite in Table 2.

Table 2.Cutting patterns according to the length

Pattern-*i*	Number of Cutting					*Cut Loss* (Inches)
	11 Inches	10 Inches	9 Inches	7 Inches	3 Inches	
1	2	0	0	1	0	1
2	2	0	0	0	2	2
3	1	1	1	0	0	0
4	1	1	0	1	0	2
5	1	1	0	0	3	0
6	1	0	2	0	0	1
7	1	0	1	1	1	0
8	1	0	1	0	3	1
9	1	0	0	2	1	2
10	1	0	0	1	4	0
11	1	0	0	0	6	1
12	0	3	0	0	0	0
13	0	2	1	0	0	1
14	0	2	0	1	1	0
15	0	2	0	0	3	1
16	0	1	2	0	0	2
17	0	1	1	1	1	1
18	0	1	1	0	3	2
19	0	1	0	2	2	0
20	0	1	0	1	4	1
21	0	1	0	0	6	2
22	0	0	3	0	1	0
23	0	0	2	1	1	2
24	0	0	2	0	4	0
25	0	0	1	3	0	0
26	0	0	1	2	2	1
27	0	0	1	1	4	2
28	0	0	1	0	7	0
29	0	0	0	4	0	2
30	0	0	0	3	3	0
31	0	0	0	2	5	1
32	0	0	0	1	7	2
33	0	0	0	0	10	0

From Table 2, there are 33 cutting patterns with each cut loss. By using the first cutting pattern, it can got 2 pieces of 11 inches and 1 piece of 7 inches with 1 inch of cut loss. All the cutting patterns that have shown in Table 2, then form to *N*-SheetModel (6) and solved by Branch and Cut Method and also Program LINDO 6.1.The solution showsthe value of $z = 6$with $x_2 = 1$, $x_5 = 1$, $x_{16} = 1$, $y_{23} = 1$, and$y_{24} = 2$.

Minimize $Z = 2x_1 + x_2 + 2x_3 + 2x_4 + x_5 + 2x_6 + 2x_7 + x_8 + 2x_9 + x_{10} + 2x_{11} + x_{12} + x_{13} + x_{14} + 2x_{15} + x_{16} + x_{17} + 2x_{18} + 2x_{19} + x_{20} + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11} + y_{12} + y_{13} + y_{14} + y_{15} + y_{16} + y_{17} + y_{18} + y_{19} + y_{20} + y_{21} + y_{22} + y_{23} + y_{24}$ (6)

Subject to

$7x_1 + 5x_2 + 4x_4 + 2x_5 + x_6 + 6x_7 + 4x_8 + 3x_9 + x_{10} + 3x_{12} + 6x_{14} + x_{15} + 3x_{16} + 2x_{19} \leq 10$

$9y_1 + 4y_2 + 9y_3 + 4y_4 + 4y_5 + 4y_6 + 9y_7 + 4y_8 + 4y_9 + 4y_{10} + 4y_{11} + 4y_{12} + 4y_{13} + 8y_{14} + 3y_{15} + 3y_{16} + 3y_{17} + 3y_{18} + 3y_{19} + 3y_{20} + 2y_{21} + 2y_{22} + 2y_{23} + y_{24} \leq 10$

$x_1 + 2x_2 + 4x_3 + x_4 + 2x_5 + x_6 + x_8 + x_{10} + 2x_{15} + x_{18} + x_{20} \leq 5$

$y_1 + 3y_2 + 2y_4 + y_5 + 2y_8 + y_9 + y_{11} + 2y_{15} + y_{16} + y_{18} + y_{21} \leq 5$

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$x_4 + x_5 + 2x_6 + x_9 + x_{10} + 2x_{11} + x_{13} + x_{16} + 2x_{17} \leq 9$
$y_3 + y_4 + 2y_5 + 3y_6 + y_9 + 2y_{10} + y_{12} + y_{16} + 2y_{17} + y_{19} + y_{22} \leq 9$
$x_7 + x_8 + x_9 + x_{10} + x_{11} + 2x_{12} + 2x_{13} + x_{18} \leq 5$
$y_7 + y_8 + y_9 + y_{10} + 2y_{11} + 2y_{12} + 3y_{13} + y_{18} + y_{19} + 2y_{20} + y_{23} \leq 5$
$x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + 2x_{19} + 2x_{20} \leq 8$
$y_{14} + y_{15} + y_{16} + y_{17} + y_{18} + y_{19} + y_{20} + 2y_{21} + 2y_{22} + 2y_{23} + 3y_{24} \leq 8$
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15}$
$+ x_{16} + x_{17} + x_{18} + x_{19} + x_{20} = 3$
$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11} + y_{12} + y_{13} + y_{14} + y_{15}$
$+ y_{16} + y_{17} + y_{18} + y_{19} + y_{20} + y_{21} + y_{22} + y_{23} + y_{24} = 3$
$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, y_1, y_2,$
$y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}, y_{12}, y_{13}, y_{14}, y_{15}, y_{16}, y_{17}, y_{18}, y_{19}, y_{20}, y_{21}, y_{22}, y_{23},$
$y_{24} \geq 0$

Based on the optimal solution obtained, the first stage used the second cutting pattern and the 10th cutting pattern. The second stage can use the 23rd cutting pattern and the 24th cutting pattern with a 3-inch long plate cut four times to the size of 2 inches. Two 3-inch long plates cut nine times by 2 inches.

5. Conclusion

Based on the results, the *N*-Sheet model that formulated from PG algorithm can be seen in Model (6). The constraints assured that the item from the first stage can be reused in the second stage. All of the constraints can fulfill the demand. The optimal solution from *N*-Sheet Model by using Branch and Cut produced the minimum trim loss amount 6 inches from all the stocks.

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